Chapter 1

Which of the following functions has a graph that is symmetric about the origin?

(A) $f(x) = x \sin x$ **(B)** $f(x) = x^5 + 3x + 1$

(C) $f(x) = x^3 - 4x$ **(D)** $f(x) = \sec x$

The domain of the function $f(x) = \sqrt{4 - x^2}$ is

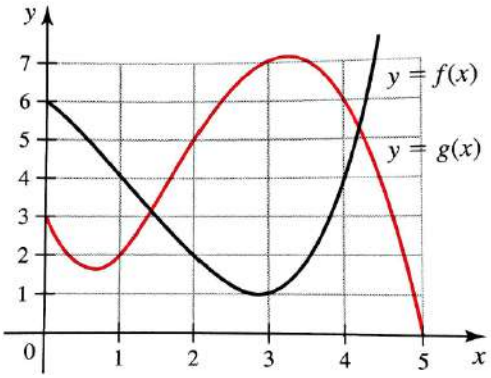
(A) $|x| \geq 2$.

(B) $-2 < x < 2$.

(C) $-2 < x \leq 2$.

(D) $-2 \leq x \leq 2$.

Consider the graphs of the functions f and g shown below.



Which of the following statements is true?

(A) $f(g(1)) = 2$ and $g(f(1)) = 6$.

(B) $f(g(0)) = 1$ and $g(f(0)) = 6$.

(C) $f(g(2)) = 7$ and $g(f(2)) = 5$.

(D) $f(g(3)) = 1$ and $g(f(3)) = 2$.

4

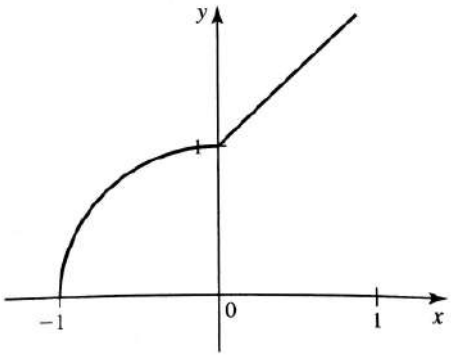
8. The graph of f in the figure consists of a quarter circle of radius 1 and a line segment with slope 1. Which of the following functions is a correct definition of f on the interval $[-1, 1]$?

(A) $f(x) = \begin{cases} 1 + x & \text{for } -1 \le x \le 0 \\ \sqrt{1 - x^2} & \text{for } 0 < x \le 1 \end{cases}$

(B) $f(x) = \begin{cases} \sqrt{1 - x^2} & \text{for } -1 \le x \le 0 \\ 1 - x & \text{for } 0 < x \le 1 \end{cases}$

(C) $f(x) = \begin{cases} \sqrt{1 - x^2} & \text{for } -1 \le x \le 0 \\ 1 + x & \text{for } 0 < x \le 1 \end{cases}$

(D) $f(x) = \begin{cases} \sqrt{x^2 - 1} & \text{for } -1 \le x \le 0 \\ 1 + x & \text{for } 0 < x \le 1 \end{cases}$



5

The functions f and g have values shown in the table. For which values of x does $f(g(x)) = g(f(x))$?

x	1	2	3	4	5
$f(x)$	3	4	5	1	2
$g(x)$	2	1	4	5	3

(A) 1, 2, 3, 4, and 5

(B) 1, 3, and 4 only

(C) 1, 4, and 5 only

(D) 1 and 4 only

Simplify the expression $6e^{2\ln 4+3}$.

(A) $96e^3$ **(B)** $24e^2$ **(C)** $48e^2$ **(D)** $48e^3$

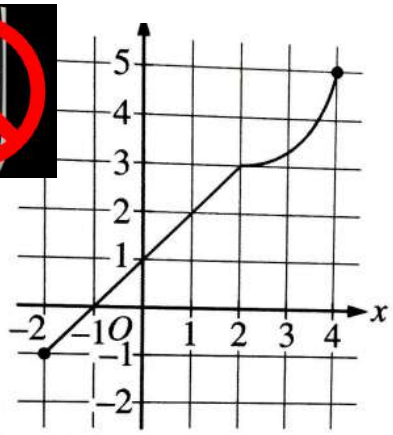
Where does the curve $y = e^{-x} - 2$ intersect the x- and y-axes?

(A) $x = \ln\frac{1}{2}, y = -2$ **(B)** $x = \ln 2, y = -1$

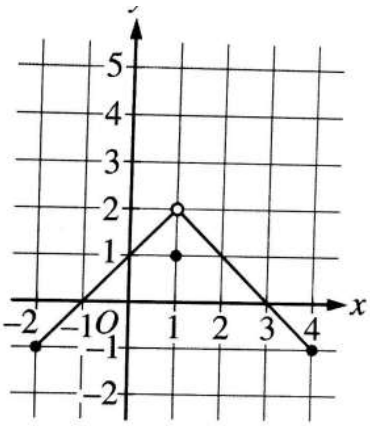
(C) $x = \ln\frac{1}{2}, y = -1$ **(D)** $x = \ln 2, y = -2$

Estimate the smallest positive root of $f(x) = x^2e^{-x} - \frac{1}{5}$.

(A) -0.371 **(B)** 0.605 **(C)** 4.708 **(D)** 3.816



Graph of f



Graph of g

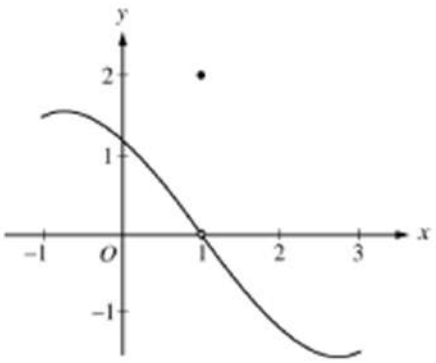
he graphs of the functions f and g are shown above. The value of $\lim_{x \to 1} f(g(x))$ is

A) 1

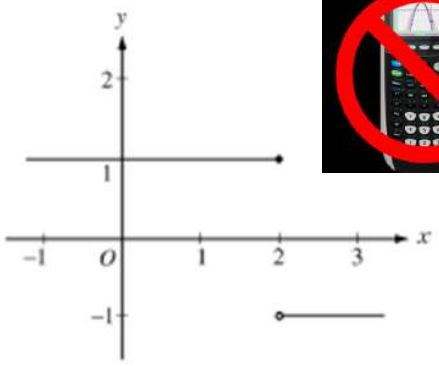
B) 2

C) 3

D) nonexistent



Graph of f



Graph of g

15. The graphs of the functions f and g are shown in the figures above. Which of the following statements is false?

(A) $\lim_{x\to 1} f(x) = 0$

(B) $\lim_{x\to 2} g(x)$ does not exist.

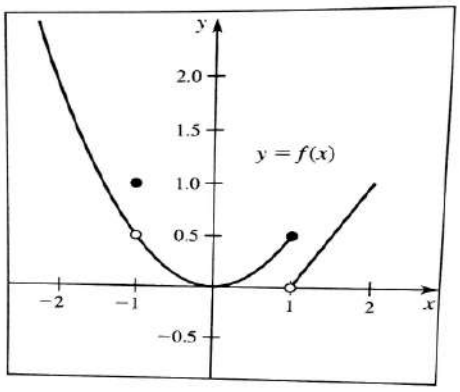
(C) $\lim_{x\to 1} (f(x)g(x+1))$ does not exist.

(D) $\lim_{x\to 1} (f(x+1)g(x))$ exists.

$\lim_{x\to -5} f(x) = 4$	$\lim_{x\to 5} f(x) = 2$	$\lim_{x\to 5} g(x) = 5$

86. The table above gives selected limits of the functions f and g. What is $\lim_{x\to 5} (f(-x) + 3g(x))$?

(A) 19 (B) 17 (C) 13 (D) 9



1. Which of the following statements is true?

(A) $f(-1)$ does not exist.

(B) $\lim_{x \to -1} f(x) = \frac{1}{2}$.

(C) $\lim_{x \to -1} f(x) = f(-1)$.

(D) f is continuous at $x = -1$.

2. Evaluate $\lim_{x \to 1} f(x)$.

(A) $\frac{1}{2}$ **(B)** 1 **(C)** 0 **(D)** The limit does not exist.

3. The average rate of change of the function $f(x) = x^2 - 3x$ on the interval $[2, x]$ is 3. Find x.

(A) 6 **(B)** 5 **(C)** 4 **(D)** 1

4. Given that $\lim_{x \to a} f(x) = A$, which of the following statements must be true?

(A) $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$

(B) $f(a) = A$

(C) f is continuous at a.

(D) $\lim_{x \to a^+} f(x)$ is not necessarily equal to A.

Evaluate $\lim_{x \to \infty} \dfrac{4x^4 - 3x + 2}{3x^4 + x^2 - 1}$.

(A) $\dfrac{3}{4}$ **(B)** 0 **(C)** $\dfrac{4}{3}$ **(D)** The limit does not exist.

5. Evaluate $\lim_{x \to 2} \frac{\sqrt{3x^2 - x + 6}}{\sin \pi x + \cos 2\pi x}$.

(A) 2 **(B)** −4 **(C)** 4 **(D)** The limit does not exist.

What are the vertical asymptotes of $f(x) = \frac{2(x^2 - 1)}{x^2 + x - 2}$?

(A) $x = 1$ and $x = -2$ **(B)** $x = -2$
(C) $x = 1$ **(D)** $x = -1$

Evaluate $\lim_{x \to 0} \frac{\sin^2 x}{1 - \cos x}$.

(A) -2 **(B)** 0 **(C)** 2 **(D)** The limit does not exist.

Evaluate $\lim_{x \to -\infty} \frac{2e^{2x} - e^{-x}}{3e^{-x} + 4e^{2x}}$.

(A) $-\frac{1}{3}$ **(B)** $\frac{1}{2}$ **(C)** 0 **(D)** $\frac{1}{3}$

11. For what constant k is the function

$$f(x) = \begin{cases} \dfrac{x^3 - 6x^2 + 8x}{x - 2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$$

continuous at $x = 2$?

(A) 5 **(B)** -4 **(C)** 4 **(D)** -2

14. Suppose f and g are continuous functions for all real numbers with values given in the table. Let $h(x) = f(g(x)) + 4$.

x	$f(x)$	$g(x)$
1	8	4
2	12	2
3	15	3
4	20	1

Which of the following statements is true?

(A) $h(1) = 24$.
(B) A number c exists with $3 < c < 4$ such that $h(c) = 13$.
(C) $h(2) = 28$.
(D) **A** and **B** are true.

12. Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2+8x}-x\right)$

(A) 3 **(B)** 0 **(C)** 4
(D) The limit does not exist.

13. Evaluate $\lim_{x\to-\infty}\left(\frac{x}{\sqrt{x^2+1}}\right)$

(A) -1 **(B)** 0 **(C)** $\frac{1}{2}$ **(D)** 1

Describe the behavior of $f(x) = \dfrac{\sqrt{9x^6 - 1}}{2x^3 + 1}$ as x becomes large and positive.

(A) The function values become large and positive.
(B) The function values approach 0.
(C) The function values approach 3/2.
(D) The function values approach 9/2.

Describe the behavior of $f(x) = e^{-1/x}$ as x approaches 0.

(A) $\lim_{x \to 0} f(x) = 1$

(B) $\lim_{x \to 0^+} f(x) = 1$ and $\lim_{x \to 0^-} f(x) = 0$

(C) $\lim_{x \to 0^+} f(x) = \infty$ and $\lim_{x \to 0^-} f(x) = 0$

(D) $\lim_{x \to 0^+} f(x) = 0$ and $\lim_{x \to 0^-} f(x) = \infty$

Chapter 3

Questions 1 and 2 relate to the function f shown in the figure.

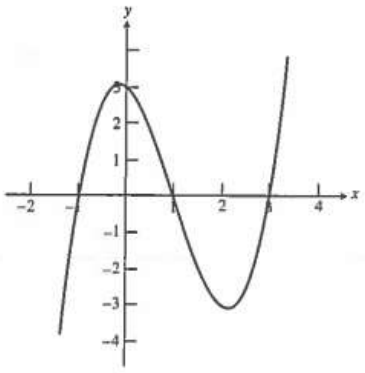
1. Give the approximate value(s) of the x-intercepts of f'.

   (A) $\{3\}$

   (B) $\{-1, 1, 3\}$

   (C) $\{0, 2\}$

   (D) $\{-3, 3\}$



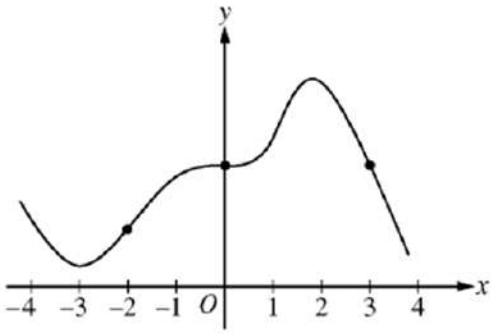
2. Over what interval(s) will the graph of f' have only negative values?

   (A) $(-\infty, \infty)$

   (B) $(-\infty, 0) \cup (2, \infty)$

   (C) $(0, 2)$

   (D) $(-\infty, -1) \cup (1, 3)$



Graph of f

76. The graph of a differentiable function f is shown in the figure above. Which of the following is true?

(A) $f'(-2) < f'(0) < f'(3)$

(B) $f'(-2) < f'(3) < f'(0)$

(C) $f'(3) < f'(-2) < f'(0)$

(D) $f'(3) < f'(0) < f'(-2)$

90. For any function f, which of the following statements must be true?

I. If f is defined at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

II. If f is continuous at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

III. If f is differentiable at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

(A) III only

(B) I and II only

(C) II and III only

(D) I, II, and III

7. A normal line to the graph of a function f at point $(x, f(x))$ is the line perpendicular to the tangent line at the same point. An equation of the normal line to the curve $y = 2x^3 + x$ at the point where $x = 2$ is

$(A)\ y = 25x + \frac{2}{25}$

$(B)\ y = \frac{1}{25}x + \frac{452}{25}$

$(C)\ y = -\frac{1}{25}x + \frac{452}{25}$

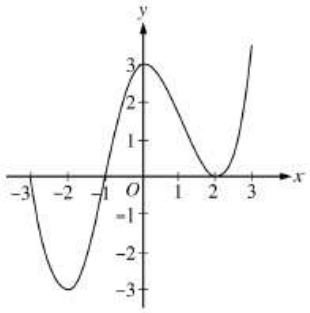
$(D)\ y = -\frac{1}{25}x + 18$

2018 #1 10/23/18

1. If f is the function given by $f(x) = \frac{4}{x} + 5x - 1$, then $f'(2) =$

(A) 4 (B) 6 (C) 7 (D) 11

2018 #76 10/23/18



Graph of f'

76. The graph of f', the derivative of the function f, is shown above for $-3 \le x \le 3$. On what intervals is f increasing?

(A) $[-3, -1]$ only (B) $[-1, 3]$ (C) $[-2, 0]$ and $[2, 3]$ (D) $[-3, -1]$ and $[1, 3]$

2018 #6 10/24/18

6. If $y = x^2\left(e^x - 1\right)$, then $\dfrac{dy}{dx} =$

(A) $2xe^x$

(B) $2xe^x - 2x$

(C) $x^2e^x + 2xe^x - 2x$

(D) $x^2e^x + 2xe^x - x^2 - 2x$

2018 #10

10/24/18

10. If $\frac{dy}{dx} = x^4 - 2x^3 + 3x - 1$, then $\frac{d^3y}{dx^3}$ evaluated at $x = 2$ is

(A) 11 (B) 24 (C) 26 (D) 125

2017 #3

10/25/18

3. If $f(x) = \frac{5 - x}{x^3 + 2}$, then $f'(x) =$

(A) $\frac{-4x^3 + 15x^2 - 2}{\left(x^3 + 2\right)^2}$

(B) $\frac{-2x^3 + 15x^2 + 2}{\left(x^3 + 2\right)^2}$

(C) $\frac{2x^3 - 15x^2 - 2}{\left(x^3 + 2\right)^2}$

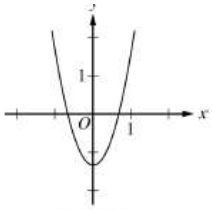
(D) $\frac{4x^3 - 15x^2 + 2}{\left(x^3 + 2\right)^2}$

2017 # 12 10/25/18

12. The number of gallons of water in a storage tank at time t, in minutes, is modeled by $w(t) = 25 - t^2$ for $0 \le t \le 5$. At what rate, in gallons per minute, is the amount of water in the tank changing at time $t = 3$ minutes?

(A) 66 (B) 16 (C) -3 (D) -6

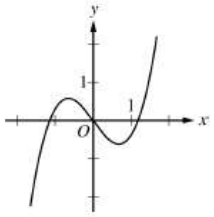
2018 #8
10/26/18



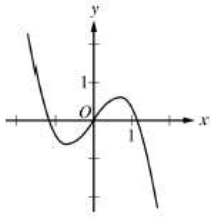
Graph of f''

8. The graph of f'', the second derivative of the function f, is shown above. Which of the following could be the graph of f?

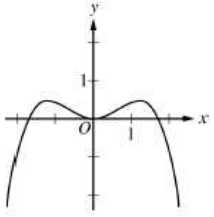
(A)



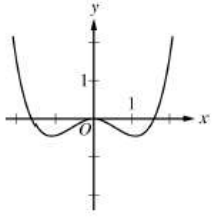
(B)



(C)



(D)



2018 #77 10/26/18

77. The rate at which water leaks from a tank, in gallons per hour, is modeled by R, a differentiable function of the number of hours after the leak is discovered. Which of the following is the best interpretation of $R'(3)$?

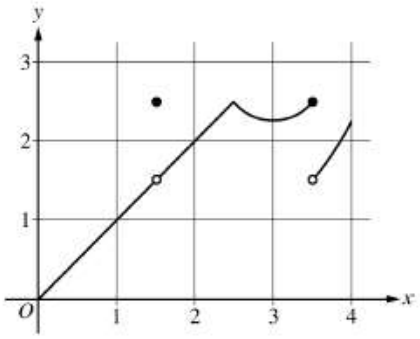
(A) The amount of water, in gallons, that has leaked out of the tank during the first three hours after the leak is discovered

(B) The amount of change, in gallons per hour, in the rate at which water is leaking during the three hours after the leak is discovered

(C) The rate at which water leaks from the tank, in gallons per hour, three hours after the leak is discovered

(D) The rate of change of the rate at which water leaks from the tank, in gallons per hour per hour, three hours after the leak is discovered

2018 #79 10/31/2018



Graph of f

79. The graph of the function f is shown above. Of the following intervals, on which is f continuous but not differentiable?

(A) $(0, 1)$ (B) $(1, 2)$ (C) $(2, 3)$ (D) $(3, 4)$

2017 #1 10/31/2018

1. If $f(x) = \left(2x^2 + 5\right)^7$, then $f'(x) =$

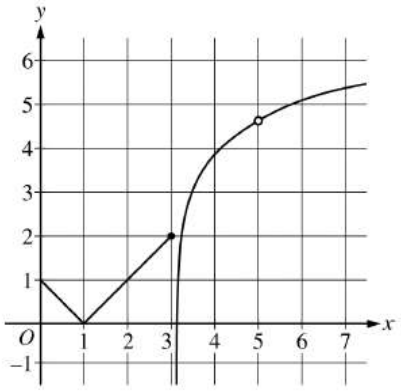
(A) $7(4x)^6$

(B) $7\left(2x^2 + 5\right)^6$

(C) $14x^2\left(2x^2 + 5\right)^6$

(D) $28x\left(2x^2 + 5\right)^6$

2016 #76 11/1/2018

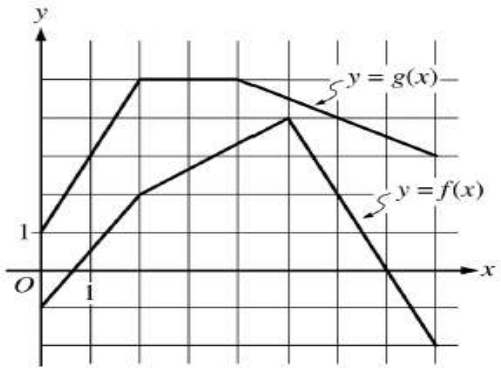


Graph of f

76. The graph of a function f is shown above. Which of the following limits does not exist?

(A) $\lim_{x \to 1^-} f(x)$ (B) $\lim_{x \to 1} f(x)$ (C) $\lim_{x \to 3^-} f(x)$ (D) $\lim_{x \to 3} f(x)$ (E) $\lim_{x \to 5} f(x)$

2016 #91 11/1/2018



91. The graphs of f and g are shown above. If $h(x) = f(x)g(x)$, then $h'(6) =$

(A) -9 (B) -7 (C) 1 (D) 7 (E) 9

2016 #4 11/6/2018

4. If $y = \left(\frac{x}{x+1}\right)^5$, then $\frac{dy}{dx} =$

(A) $5(1+x)^4$ (B) $\frac{x^4}{(x+1)^4}$ (C) $\frac{5x^4}{(x+1)^4}$ (D) $\frac{5x^4}{(x+1)^6}$ (E) $\frac{5x^4(2x+1)}{(x+1)^6}$

2016 #1 11/6/2018

1. If $y = \cos 2x$, then $\frac{dy}{dx} =$

(A) $-2\sin 2x$ (B) $-\sin 2x$ (C) $\sin 2x$ (D) $2\sin 2x$ (E) $2\sin x$

2015 11/7/2018

2. If $f(x) = \sqrt{x} + \frac{3}{\sqrt{x}}$, then $f'(4) =$

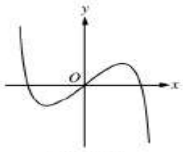
(A) $\frac{1}{16}$ (B) $\frac{5}{16}$ (C) 1 (D) $\frac{7}{2}$ (E) $\frac{49}{4}$

2015 11/7/18

7. If $f(x) = x^2 - 4$ and g is a differentiable function of x, what is the derivative of $f(g(x))$?

(A) $2g(x)$ (B) $2g'(x)$ (C) $2xg'(x)$ (D) $2g(x)g'(x)$ (E) $2g(x) - 4$

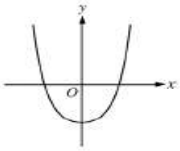
2016 11/8/2018



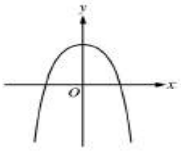
Graph of f

11. The graph of the function f is shown in the figure above. Which of the following could be the graph of f', the derivative of f?

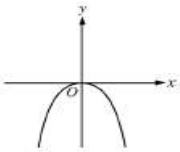
(A)



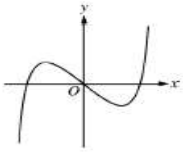
(B)



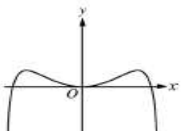
(C)



(D)



(E)



2015 11/8/2018

$$f(x) = \begin{cases} x^2 \sin(\pi x) & \text{for } x < 2 \\ x^2 + cx - 18 & \text{for } x \geq 2 \end{cases}$$

5. Let f be the function defined above, where c is a constant. For what value of c, if any, is f continuous at $x = 2$?

(A) 2 (B) 7 (C) 9 (D) $4\pi - 4$ (E) There is no such value of c.

2017 #8 11/13/18

8. If f is the function given by $f(x) = e^{x/3}$, which of the following is an equation of the line tangent to the graph of f at the point $(3 \ln 4, 4)$?

(A) $y - 4 = \frac{4}{3}(x - 3 \ln 4)$

(B) $y - 4 = 4(x - 3 \ln 4)$

(C) $y - 4 = 12(x - 3 \ln 4)$

(D) $y - 3 \ln 4 = 4(x - 4)$

2016 11/13/18

82. If f is a continuous function such that $f(2) = 6$, which of the following statements must be true?

(A) $\lim_{x \to 1} f(2x) = 3$

(B) $\lim_{x \to 2} f(2x) = 12$

(C) $\lim_{x \to 2} \dfrac{f(x) - f(2)}{x - 2} = 6$

(D) $\lim_{x \to 2} f\left(x^2\right) = 36$

(E) $\lim_{x \to 2} (f(x))^2 = 36$

2016 11/27/18

6. The slope of the line tangent to the graph of $y = \ln(1 - x)$ at $x = -1$ is

(A) -1 (B) $-\dfrac{1}{2}$ (C) $\dfrac{1}{2}$ (D) $\ln 2$ (E) 1

11/28/18

If y = arctan (cos x), then $\frac{dy}{dx}$ =

a. $\frac{-\sin x}{1+\cos^2 x}$ b. $-(\operatorname{arc\,sec}(\cos x))^2 \sin x$ c. $(\operatorname{arc\,sec}(\cos x))^2$

d. $\frac{1}{(\arccos x)^2+1}$ e. $\frac{1}{1+\cos^2 x}$

11/28/18

The slope of the line tangent to the curve $y^2 + (xy + 1)^3 = 0$ at (2, -1) is

a. $-\frac{3}{2}$ b. $-\frac{3}{4}$ c. 0 d. $\frac{3}{4}$ e. $\frac{3}{2}$

$2yy' + 3(xy+1)^2 \cdot (xy' + y) = 0$

$2(-1)y' + 3(2\cdot -1+1)^2 \cdot (2y' - 1) = 0$

$-2y' + 3(1) \cdot (2y' - 1) = 0$

$-2y' + 6y' - 3 = 0$

$4y' = 3$

$y' = 3/4$

11/29/18

x	f(x)	f ´(x)	g(x)	g ´(x)
-1	6	5	3	-2
1	3	-3	-1	2
3	1	-2	2	3

The table above gives values for f, f ´, g, and g ´ at selected values of x. If h(x) = f(g(x)), then h ´(1) =

a. 5 b. 6 c. 9 d. 10 e. 12

$h'(1) = f'(g(1)) \cdot g'(1)$

$= f'(-1) \cdot 2$

$= 5 \cdot 2$

11/29/18

Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of g ´(2)?

a. $\frac{1}{13}$ b. $\frac{1}{4}$ c. $\frac{7}{4}$ d. 4 e. 13

$g'(x) = \frac{1}{f'(g(x))}$

$g'(2) = \frac{1}{f'(g(2))} = \frac{1}{f'(1)} = \frac{1}{4}$

2016 #7

7. For which of the following pairs of functions f and g is $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ infinite?

(A) $f(x) = x^2 + 2x$ and $g(x) = x^2 + \ln x$

(B) $f(x) = 3x^3$ and $g(x) = x^4$

(C) $f(x) = 3^x$ and $g(x) = x^3$

(D) $f(x) = 3e^x + x^3$ and $g(x) = 2e^x + x^2$

(E) $f(x) = \ln(3x)$ and $g(x) = \ln(2x)$

2017 # 20

20. Let f be the function given by $f(x) = \frac{x-2}{2|x-2|}$. Which of the following is true?

(A) $\lim_{x\to 2} f(x) = \frac{1}{2}$

(B) f has a removable discontinuity at $x = 2$.

(C) f has a jump discontinuity at $x = 2$.

(D) f has a discontinuity due to a vertical asymptote at $x = 2$.

2016 #22

22. A particle moves along the x-axis so that at time $t \geq 0$, the acceleration of the particle is $a(t) = 15\sqrt{t}$. The position of the particle is 10 when $t = 0$, and the position of the particle is 20 when $t = 1$. What is the velocity of the particle at time $t = 0$?

(A) -14 (B) 0 (C) 5 (D) 6 (E) 10

2016

26. Let f be the function given by $f(x) = x^3 + 5x$. For what value of x in the closed interval $[1,3]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) $\sqrt{\frac{7}{3}}$ (B) $\sqrt{\frac{13}{3}}$ (C) $\sqrt{5}$ (D) $\sqrt{6}$ (E) $\sqrt{\frac{19}{3}}$

2016

77. Let f be a function that is continuous on the closed interval $[1, 3]$ with $f(1) = 10$ and $f(3) = 18$. Which of the following statements must be true?

(A) $10 \le f(2) \le 18$

(B) f is increasing on the interval $[1, 3]$.

(C) $f(x) = 17$ has at least one solution in the interval $[1, 3]$.

(D) $f'(x) = 8$ has at least one solution in the interval $(1, 3)$.

(E) $\int_1^3 f(x)dx > 20$

2016

88. The height above the ground of a passenger on a Ferris wheel t minutes after the ride begins is modeled by the differentiable function H, where $H(t)$ is measured in meters. Which of the following is an interpretation of the statement $H'(7.5) = 15.708$?

(A) The Ferris wheel is turning at a rate of 15.708 meters per minute when the passenger is 7.5 meters above the ground.

(B) The Ferris wheel is turning at a rate of 15.708 meters per minute 7.5 minutes after the ride begins.

(C) The passenger's height above the ground is increasing by 15.708 meters per minute when the passenger is 7.5 meters above the ground.

(D) The passenger's height above the ground is increasing by 15.708 meters per minute 7.5 minutes after the ride begins.

(E) The figure shows a graph in a coordinate plane. The horizontal axis is labeled t, and the vertical axis is labeled v of t. There are evenly spaced tick marks labeled 1 to 6 along the horizontal axis. There are evenly spaced tick marks

und 7.5 minutes after the ride begins.

2018 12/6/18

14. If f is a differentiable function and $y = \sin\left(f\left(x^2\right)\right)$, what is $\frac{dy}{dx}$ when $x = 3$?

(A) $\cos(f'(9))$

(B) $6\cos(f(9))$

(C) $f'(9)\cos(f(9))$

(D) $6f'(9)\cos(f(9))$

$\frac{dy}{dx} = \cos(f(x^2)) \cdot f'(x^2) \cdot 2x$

$\left.\frac{dy}{dx}\right|_{x=3} = \cos(f(9)) \cdot f'(9) \cdot 6$

Chapter 4

2018 12/6/18

16. Let f be the function given by $f(x) = x^3 - 6x^2 - 15x$. What is the maximum value of f on the interval $[0, 6]$?

(A) 0 (B) 5 (C) 6 (D) 8

2018 12/12/18

x	10	11	12	13	14
$f(x)$	5	2	3	6	5

26. The table above gives values of the continuous function f at selected values of x. If f has exactly two critical points on the open interval $(10, 14)$, which of the following must be true?

(A) $f(x) > 0$ for all x in the open interval $(10, 14)$.

(B) $f'(x)$ exists for all x in the open interval $(10, 14)$.

(C) $f'(x) < 0$ for all x in the open interval $(10, 11)$.

(D) $f'(12) \neq 0$

2018 12/12/18

x	3	7
$h(x)$	7	22
$h'(x)$	5	10

90. Selected values of the increasing function h and its derivative h' are shown in the table above. If g is a differentiable function such that $h(g(x)) = x$ for all x, what is the value of $g'(7)$?

(A) $-\frac{1}{10}$ (B) $\frac{1}{10}$ (C) $\frac{1}{5}$ (D) $\frac{7}{5}$

2018 12/13/18

80. The first derivative of the function f is defined by $f'(x) = \left(x^2 + 1\right)\sin(3x - 1)$ for $-1.5 < x < 1.5$. On which of the following intervals is the graph of f concave up?

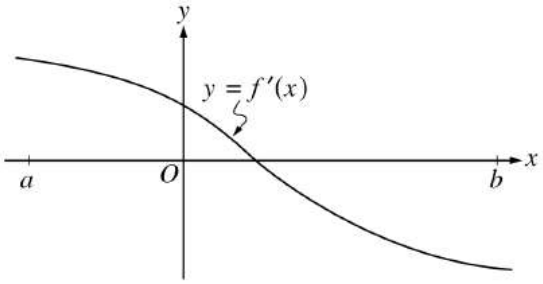
(A) $(-1.5, -1.341)$ and $(-0.240, 0.964)$

(B) $(-1.341, -0.240)$ and $(0.964, 1.5)$

(C) $(-0.714, 0.333)$ and $(1.381, 1.5)$

(D) $(-1.5, -0.714)$ and $(0.333, 1.381)$

2017 12/13/18



27. The graph of f', the derivative of the function f, is shown in the figure above. Which of the following statements must be true?

I. f is continuous on the open interval (a, b).

II. f is decreasing on the open interval (a, b).

III. The graph of f is concave down on the open interval (a, b).

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

2018 12/17/18

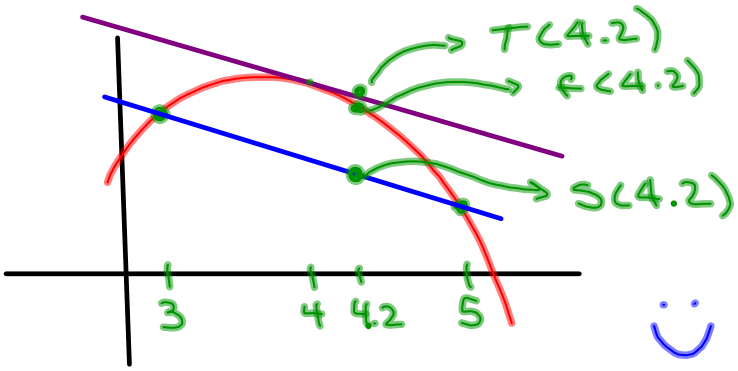
88. Let f be a twice-differentiable function such that $f''(x) < 0$ for all x. The graph of $y = S(x)$ is the secant line passing through the points $(3, f(3))$ and $(5, f(5))$. The graph of $y = T(x)$ is the line tangent to the graph of f at $x = 4$. Which of the following is true?

(A) $f(4.2) < S(4.2) < T(4.2)$

(B) $f(4.2) < T(4.2) < S(4.2)$

(C) $S(4.2) < f(4.2) < T(4.2)$

(D) $T(4.2) < f(4.2) < S(4.2)$



2018 12/17/18

t (minutes)	0	1	5	6	8
$g(t)$ (cubic feet per minute)	12.8	15.1	20.5	18.3	22.7

1. Grain is being added to a silo. At time $t = 0$, the silo is empty. The rate at which grain is being added is modeled by the differentiable function g, where $g(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Selected values of $g(t)$ are given in the table above.

   (a) Using the data in the table, approximate $g'(3)$. Using correct units, interpret the meaning of $g'(3)$ in the context of the problem.

2017 12/19/18

$$f''(x) = x(x-1)^2(x+2)^3$$
$$g''(x) = x(x-1)^2(x+2)^3 + 1$$
$$h''(x) = x(x-1)^2(x+2)^3 - 1$$

86. The twice-differentiable functions f, g, and h have second derivatives given above. Which of the functions f, g, and h have a graph with exactly two points of inflection?

   (A) g only

   (B) h only

   (C) f and g only

   (D) f, g, and h

2017 12/19/18

x	1	2	3	4	5
$f(x)$	9	4	0	−3	−5

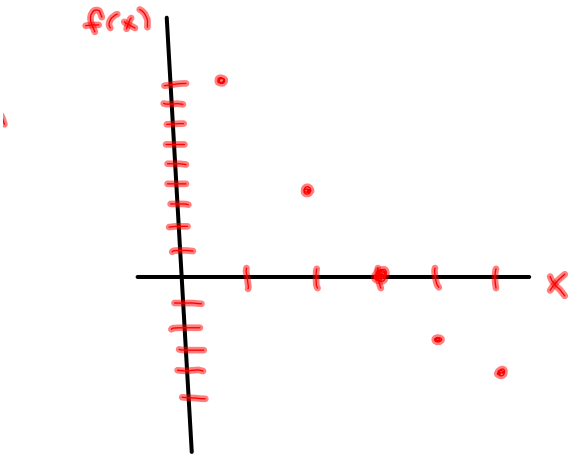
87. The table above gives values of a function f at selected values of x. If f is twice-differentiable on the interval $1 \le x \le 5$, which of the following statements could be true?

(A) f' is negative and decreasing for $1 \le x \le 5$.

(B) f' is negative and increasing for $1 \le x \le 5$.

(C) f' is positive and decreasing for $1 \le x \le 5$.

(D) f' is positive and increasing for $1 \le x \le 5$.



2017 12/20/18

16. Let f be the function defined by $f(x) = -3 + 6x^2 - 2x^3$. What is the largest open interval on which the graph of f is both concave up and increasing?

(A) $(0, 1)$ (B) $(1, 2)$ (C) $(0, 2)$ (D) $(2, \infty)$

$f'(x) = 12x - 6x^2$

$= 6x(2 - x)$

$f''(x) = 12 - 12x$

$= 12(1 - x)$

2018 12/20/18

18. Let f be the function defined by $f(x) = \sqrt[3]{x}$. What is the approximation for $f(10)$ found by using the line tangent to the graph of f at the point $(8, 2)$?

(A) $\frac{11}{6}$ (B) $\frac{25}{12}$ (C) $\frac{13}{6}$ (D) $\frac{7}{3}$

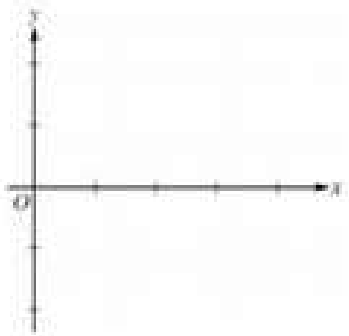
2005 FR #4 1/3/19

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.



3. $\lim_{x\to 0} \frac{\sin x \cos x}{x}$ is

(A) -1 (B) 0 (C) 1 (D) $\frac{\pi}{4}$ (E) nonexistent

L'H : $\lim_{x\to 0} \frac{-\sin^2 x + \cos^2 x}{1}$

$= \frac{(-\sin 0)^2 + (\cos 0)^2}{1} = 1$

26. $\lim_{x\to\infty} \frac{\ln\left(e^{3x}+x\right)}{x} =$

(A) 0 (B) 1 (C) 3 (D) ∞

L'H : $\lim_{x\to\infty} \frac{\frac{3e^{3x}+1}{e^{3x}+x}}{1}$

$= \lim_{x\to\infty} \frac{3e^{3x}+1}{e^{3x}+1}$

L'H : $\lim_{x\to\infty} \frac{9e^{3x}}{3e^{3x}} = \frac{9}{3} = 3$

12. Given that $3x - \tan y = 4$, what is $\frac{dy}{dx}$ in terms of y ?

(A) $\frac{dy}{dx} = 3\sin^2 y$

(B) $\frac{dy}{dx} = 3\cos^2 y$

(C) $\frac{dy}{dx} = 3\cos y \cot y$

(D) $\frac{dy}{dx} = \frac{3}{1+9y^2}$

18. Let f be the function defined by $f(x) = \sqrt[3]{x}$. What is the approximation for $f(10)$ found by using the line tangent to the graph of f at the point $(8, 2)$?

(A) $\frac{11}{6}$ (B) $\frac{25}{12}$ (C) $\frac{13}{6}$ (D) $\frac{7}{3}$

19. Let f be the function given by $f(x) = 2\cos x + 1$. What is the approximation for $f(1.5)$ found by using the line tangent to the graph of f at $x = \frac{\pi}{2}$?

(A) -2 (B) 1 (C) $\pi - 2$ (D) $4 - \pi$

21. If $f(x) = \ln x$, then $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$ is

(A) $\frac{1}{3}$ (B) e^3 (C) $\ln 3$ (D) nonexistent

79. A file is downloaded to a computer at a rate modeled by the differentiable function $f(t)$, where t is the time in seconds since the start of the download and $f(t)$ is measured in megabits per second. Which of the following is the best interpretation of $f'(5) = 2.8$?

(A) At time $t = 5$ seconds, the rate at which the file is downloaded to the computer is 2.8 megabits per second.

(B) At time $t = 5$ seconds, the rate at which the file is downloaded to the computer is increasing at a rate of 2.8 megabits per second per second.

(C) Over the time interval $0 \le t \le 5$ seconds, 2.8 megabits of the file are downloaded to the computer.

(D) Over the time interval $0 \le t \le 5$ seconds, the average rate at which the file is downloaded to the computer is 2.8 megabits per second.

80. The function f has first derivative given by $f'(x) = x^4 - 6x^2 - 8x - 3$. On what intervals is the graph of f concave up?

(A) $(2, \infty)$ only

(B) $(0, \infty)$

(C) $(-1, 2)$

(D) $(-\infty, -1)$ and $(3, \infty)$

83. Let f be a function with derivative given by $f'(x) = \dfrac{x^3 - 8x^2 + 3}{\sqrt{x^3 + 1}}$ for $-1 < x < 9$. At what value of x does f attain a relative maximum?

(A) -0.591 (B) 0 (C) 0.638 (D) 7.953

88. Let f be the function defined by $f(x) = \ln(x^2 + 1)$, and let g be the function defined by $g(x) = x^5 + x^3$. The line tangent to the graph of f at $x = 2$ is parallel to the line tangent to the graph of g at $x = a$, where a is a positive constant. What is the value of a ?

(A) 0.246 (B) 0.430 (C) 0.447 (D) 0.790

1. A particle moves along the x-axis so that its velocity at time t is given by $v(t) = \dfrac{t^6 - 13t^4 + 12}{10t^3 + 3}$. At time $t = 0$, the initial position of the particle is $x = 7$.

   (a) Find the acceleration of the particle at time $t = 5.1$.

2016

9. Let f be the function with derivative given by $f'(x) = \dfrac{-2x}{\left(1 + x^2\right)^2}$. On what interval is f decreasing?

   (A) $[0, \infty)$ only

   (B) $(-\infty, 0]$ only

   (C) $\left[-\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right]$ only

   (D) $(-\infty, \infty)$

   (E) There is no such interval.

13. $\frac{d}{dx}\left(\tan^{-1}x + 2\sqrt{x}\right) =$

(A) $-\frac{1}{\sin^2 x} + \frac{1}{\sqrt{x}}$

(B) $\frac{1}{\sqrt{1-x^2}} - 4\sqrt[3]{x}$

(C) $\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x}}$

(D) $\frac{1}{1+x^2} - 4\sqrt[3]{x}$

(E) $\frac{1}{1+x^2} + \frac{1}{\sqrt{x}}$

16. The first derivative of the function f is given by $f'(x) = 3x^4 - 12x^3$. What are the x-coordinates of the points of inflection of the graph of f ?

(A) $x = 3$ only

(B) $x = 4$ only

(C) $x = 0$ and $x = 2$

(D) $x = 0$ and $x = 3$

(E) $x = 0$ and $x = 4$

x	$f(x)$
-1	-30
0	-2
3	10
5	18

21. The table above gives selected values for a twice-differentiable function f. Which of the following must be true?

(A) f has no critical points in the interval $-1 < x < 5$.

(B) $f'(x) = 8$ for some value of x in the interval $-1 < x < 5$.

(C) $f'(x) > 0$ for all values of x in the interval $-1 < x < 5$.

(D) $f''(x) < 0$ for all values of x in the interval $-1 < x < 5$.

(E) The graph of f has no points of inflection in the interval $-1 < x < 5$.

27. If $e^{xy} - y^2 = e - 4$, then at $x = \frac{1}{2}$ and $y = 2$, $\frac{dy}{dx} =$

(A) $\frac{e}{4}$ (B) $\frac{e}{2}$ (C) $\frac{4e}{8-e}$ (D) $\frac{4e}{4-e}$ (E) $\frac{8-4e}{e}$

28. Let f be the function defined by $f(x) = x^3 + x^2 + x$. Let $g(x) = f^{-1}(x)$, where $g(3) = 1$. What is the value of $g'(3)$?

(A) $\frac{1}{39}$ (B) $\frac{1}{34}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) 39

85. The function f is defined on the open interval $0.4 < x < 2.4$ and has first derivative f' given by $f'(x) = \sin\left(x^2\right)$. Which of the following statements are true?

I. f has a relative maximum on the interval $0.4 < x < 2.4$.

II. f has a relative minimum on the interval $0.4 < x < 2.4$.

III. The graph of f has two points of inflection on the interval $0.4 < x < 2.4$.

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

87. The first derivative of the function g is given by $g'(x) = \cos\left(\pi x^2\right)$ for $-0.5 < x < 1.5$. On which of the following intervals is g decreasing?

(A) $-0.5 < x < 0$

(B) $0 < x < 1$

(C) $0.707 < x < 1.225$

(D) $1.225 < x < 1.414$

(E) $1.414 < x < 1.5$

88. The height above the ground of a passenger on a Ferris wheel t minutes after the ride begins is modeled by the differentiable function H, where $H(t)$ is measured in meters. Which of the following is an interpretation of the statement $H'(7.5) = 15.708$?

(A) The Ferris wheel is turning at a rate of 15.708 meters per minute when the passenger is 7.5 meters above the ground.

(B) The Ferris wheel is turning at a rate of 15.708 meters per minute 7.5 minutes after the ride begins.

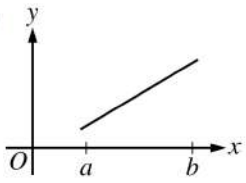
(C) The passenger's height above the ground is increasing by 15.708 meters per minute when the passenger is 7.5 meters above the ground.

(D) The passenger's height above the ground is increasing by 15.708 meters per minute 7.5 minutes after the ride begins.

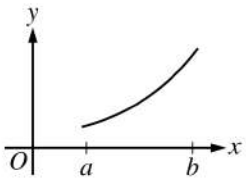
(E) The passenger is 15.708 meters above the ground 7.5 minutes after the ride begins.

90. Let f be a twice-differentiable function on the open interval (a, b). If $f'(x) > 0$ on (a, b) and $f''(x) < 0$ on (a, b), which of the following could be the graph of f?

(A)



(B)



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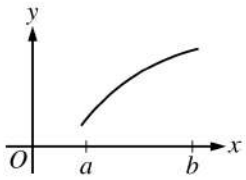

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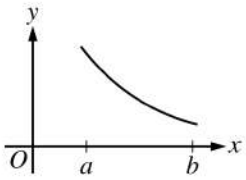

(E)