Worksheet 5. Applying the Fundamental Theorem of Calculus: Exercises

Work Problems 1–3 by both methods.

1. $y' = 2 + \frac{1}{x^2}$ and $y(1) = 6$. Find $y(3)$.

2. $f'(x) = \cos(2x)$ and $f(0) = 3$. Find $f\left(\frac{\pi}{4}\right)$.

3. Water flows into a tank at a rate of $\frac{dW}{dt} = \frac{1}{75}(600 + 20t - t^2)$ where $\frac{dW}{dt}$ is measured in gallons per hour and t is measured in hours. If there are 150 gallons of water in the tank at time $t = 0$, how many gallons of water are in the tank when $t = 24$?

Work Problems 4–10 using the Fundamental Theorem of Calculus and your calculator.

4. $f'(x) = \cos(x^3)$ and $f(0) = 2.3$ Find $f(1)$.

5. $f'(x) = e^{-x^2}$ and $f(5) = 1$. Find $f(2)$.

6. A particle moving along the x-axis has position $x(t)$ at time t with the velocity of the particle given by $v(t) = 5\sin(t^2)$ At time $t = 6$, the particle's position is (4, 0). Find the position of the particle when $t = 7$.

7. Let $F(t)$ represent a bacteria population which is 4 million at time $t = 0$. After t hours, the population is growing at an instantaneous rate of 2^t million bacteria per hour. Find the total increase in the bacteria population during the first three hours, and find the population at $t = 3$ hours.

8. A particle moves along a line so that at any time $t \geq 0$ its velocity is given by $v(t) = \frac{t}{1 + t^2}$. At time $t = 0$, the position of the particle is $s(0) = 5$. Determine the position of the particle at $t = 3$.

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9. Let f be the function whose graph passes through the point (3, 6) and whose derivative is given by:

$$f'(x)=\frac{1+e^x}{x^2}.$$ Find $f(3.1)$.

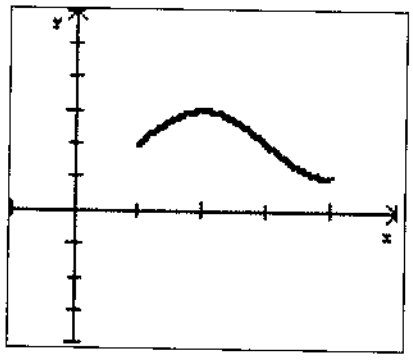
10. (Multiple Choice) If f is the antiderivative of $\frac{x^2}{1+x^5}$ such that $f(1)=5$, then $f(4)=$

(a) 4.988 (b) 5 (c) 5.016 (d) 5.376 (e) 5.629

In Problems 11–13, use the Fundamental Theorem of Calculus and the given graph. Each tick mark on the axes below represents one unit.

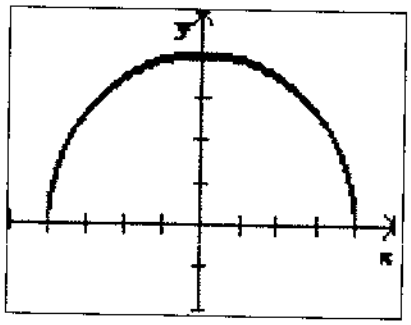
11. The graph of f' is shown at right.

$\int_1^4 f'(x)dx = 6.2$ and $f(1)=3$. Find $f(4)$.



12. The graph of f' is the semicircle shown at right.

Find $f(-4)$ given that $f(4)=7$.



13. The graph of f', consisting of two line segments and a semicircle, is shown at right. Given that $f(-2)=5$, find:

(a) $f(1)$

(b) $f(4)$

(c) $f(8)$