Work and Energy

7.1 Work and energy

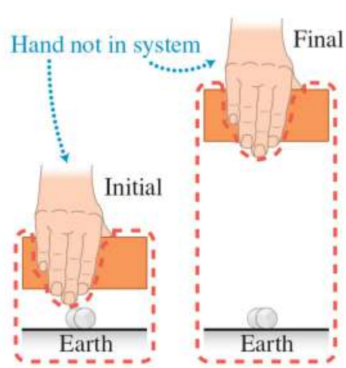
7.1.1 Observe and find a pattern

Lab or class: *Equipment:* Brick, chalk, slingshot, cart. For experiment b. tape the chalk to a wall so that the cart hits it when it collides with the wall.

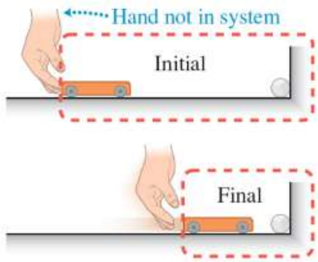
Below you see the description of several experiments. You can either repeat the experiments with your group members or read their descriptions here.

All three experiments involve a well-defined system and a process in which the system changes from an initial state to a final state. At the end of this process, we find that the system has the potential to do something it couldn't do before—to crush a piece of chalk into many pieces. In each chase, the chalk-crushing ability (CCA) increases due to the intervention of an external agent (you). For each situation, work with the members of your group to draw arrows indicating the direction of the external force that you (outside the system) exert on a system object and the displacement of the object while you exert the force in order to increase the CCA of the system. Fill in the table that follows to help you complete this activity.

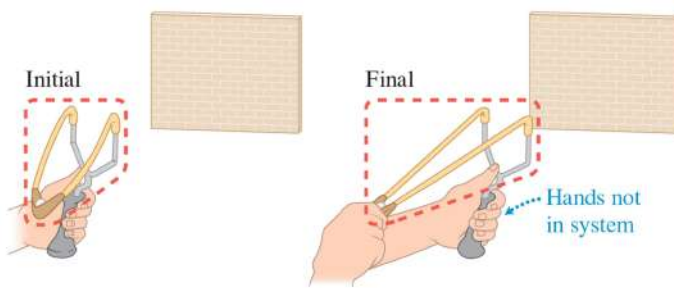
a. The system includes a brick with a flat bottom, Earth, and a piece of chalk. You (outside the system) pull up on brick so that slowly rises 0.5 m above the piece of chalk. After this lifting process, you release the brick. It falls and breaks the chalk.



b. The system includes a low friction cart that can roll on a floor and a piece of chalk that is placed by the vertical wall. You (outside the system) continuously push the cart over a set distance so that it rolls faster and faster toward the chalk on the wall and breaks the chalk when it hits it.



c. The system includes a slingshot that holds a piece of chalk. Your instructor (outside the system) slowly pulls back on the sling. When she/he releases the sling, the chalk shoots out at high speed and hits the wall, causing the chalk to break.



Experiment	**a.**	**b.**	**c.**
Draw arrows indicating the direction of the force you exerted on the system $\vec{F}_{\text{Y on S}}$ and the displacement $\vec{d}$ of the object in the system while you were exerting the force.			

d. Look for a pattern in what was done to the system to increase its chalk-crushing ability (CCA). Then, devise a new physical quantity to describe this pattern. Be explicit.

7.1.2 Observe and find a pattern

In Activity 7.1.1, you found that the external force you exerted on an object in a system *increased* the ability of the system to smash a piece of chalk. The force you exerted on the object in the system was always in the direction of the displacement of that object. Suppose that a friend outside the system decides to save the chalk in the first two experiments by exerting with her hand an opposing force on the brick or on the cart after they are released. In each case your friend pushes on the moving object opposite to the direction of its velocity. Describe for parts **a.** and **b.** of Activity 7.1.1 the direction of the force your friend exerts on the moving object relative to the displacement of the object as she stops it—in other words, what does she do to *reduce* the chalk-crushing ability of the system?

a. The system includes a 1-kg brick with a flat bottom and a string attached to the top, Earth, and a piece of chalk. You (outside the system) pull up on the string so that the 1-kg brick slowly rises 0.5 m above the piece of chalk. After this lifting process, you release the block, and it starts falling. Your friend then starts pushing upward on the falling brick, slows it down, and the brick does not break the chalk.

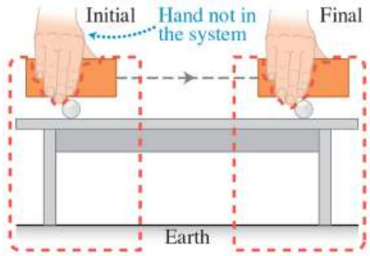
b. The system includes a 1-kg dynamics cart that can roll on a low-friction horizontal dynamics track and a piece of chalk that is taped to the fixed vertical end of the track. You (outside the system) push the cart so that it rolls faster and faster. Before the cart reaches the chalk, your friend pushes on it opposite its displacement. This causes the cart to slow down and stop so that it does not break the chalk.

c. Discuss with your group: how could you modify the definition of the quantity you devised in Activity 7.1.1 to account for the system's loss of chalk-crushing ability thanks to your friend's intervention? Put your group's ideas down on a whiteboard.

7.1.3 Observe and find a pattern

Consider a system that includes Earth and a 1-kg brick.

a. You (outside the system) hold the brick so that it stays about 2 cm above a table. A piece of chalk is placed on the table under the brick. If you release the brick and it falls on the chalk, the chalk does not break (it's too close to the chalk).



b. Next you slowly walk about 1 m beside the table, continually keeping the brick 2 cm above the surface. After you have walked the 1 m, the brick hangs over a second identical piece of chalk. Draw the force exerted by you on the brick and the displacement of the brick as you walked the 1 m.

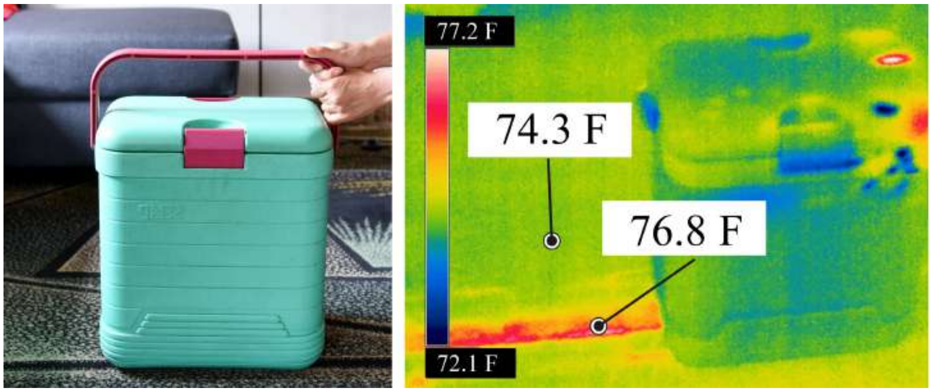
c. Discuss with your group whether the *vertical* force that you exerted on the brick while moving it *horizontally* above the tabletop caused the system to have a better chance of crushing the second piece of chalk than the first piece. Work with your group members to revise the quantity you devised in the last two activities to account for this result. Your revision will involve the angle between the external force exerted on the object in the system and the object's displacement. We call this physical quantity *work.*

7.1.4 Construct a mathematical model

Work together with your group to construct a mathematical equation that relates changes in chalk-crushing ability, ΔCCA, to the new physical quantity you devised in Activities 7.1.1–7.1.3. Compare your equation with another group's equation and discuss and resolve any differences or inconsistencies.

7.1.5 Observe and find a pattern

A system consists of a heavy cooler and a rough horizontal surface on which it sits. You (outside the system) pull on the handle of a cooler so that it moves slowly along the surface at constant velocity. You do positive work by pulling the cooler for about 5 m. Your friend takes a photo of the cooler and the surface using a thermal camera (see photo at right below). Describe how the system (cooler and surface) is different after you do the work than before the cooler started moving. Notice that the chalk-crushing ability of the cooler did not change (ΔCCA = 0), and yet you did work on it. Does this fit with the equation you just invented in Activity 7.1.4? Discuss with your group how you could modify the equation you came up with in Activity 7.1.4 to account for this anomalous result. *Hint*: You may need to invent a new physical quantity. Put your group's revised equation on a whiteboard and compare your ideas with those from another group.



7.1.6 Describe

You do work on a system to change its ability to do something—for example, to crush chalk or to make the touching surfaces of objects in a system warm. In Activities 7.1.1 through 7.1.5, the work done on the system by the external force caused different types of changes in the system. For each situation below, discuss with your group members how you can describe each type of change in the system as a result of the work done on it, and come up with a name for it.

a. The external force caused the block to move higher above Earth's surface.

b. The external force caused the cart to move faster and faster.

c. The external force caused the slingshot to stretch.

d. The external force caused the surfaces of the touching objects to warm.

7.1.7 Pose a problem

Describe with your group members a real-life situation in which an external force does the following:

a. positive work on a system;

b. positive work on a system, but with a value that is less than in part a.;

c. negative work on a system;

d. zero work even though an object in the system moves.

Note that your situations should be different from the scenarios we've already encountered in Activities 7.1.1–7.1.5

7.1.8 Apply

Lab: *Equipment:* children toys that allow for the processes described below.

For each item below, describe one real-life experiment that is consistent with the work–energy process.

a. Positive work causes an increase in the gravitational potential energy of the system.

b. Positive work causes an increase in the kinetic energy of the system.

c. Positive work causes an increase in the elastic potential energy of the system.

d. Kinetic energy in the system is converted to gravitational potential energy.

e. Kinetic energy in the system is converted to elastic potential energy.

f. Gravitational potential energy in the system is converted to internal energy.

g. Gravitational potential energy in the system is converted to elastic potential energy.

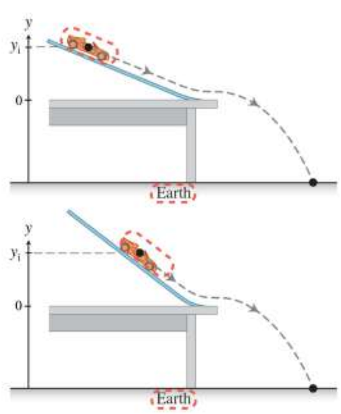
7.2 Energy is a conserved quantity

7.2.1 Observe and explain

Lab: *Equipment per group:* A track that that looks like the one in Testing Experiment Table 7.3, Hot Wheels car, meter stick, lab stand.

Work with your group members to put the experiment together, decide what data to record and how to record them.

a. Place the car on the track as shown in the figure below and note the height of the car above the table. Let the car go and record the location where it lands on the floor. Repeat the experiment to make sure the car lands reliably (same location) on the floor.



b. Change the steepness of the track but release the car from the same height above the table and record the location where it lands on the floor. Repeat the experiment to make sure the car lands reliably (same location) on the floor.

c. Repeat the experiment by changing the steepness again.

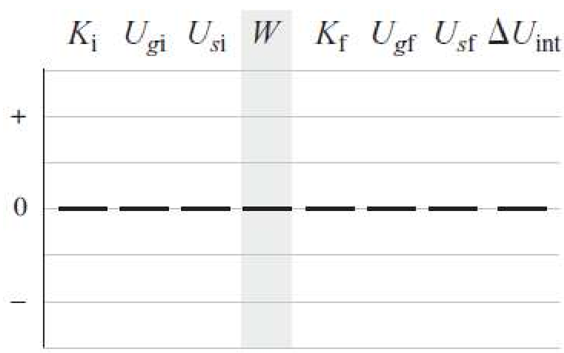
d. Present the results in a table. What pattern do you find? Discuss with your group members how you can explain this pattern using your knowledge of gravitational and kinetic energy. Provide your group's explanation.

7.2.2 Represent and reason

Work with your group member to analyze the following experiment: A rope pulls a skier, initially at rest, up a hill. *Initial state:* A skier is at rest at the bottom of the hill. *Final state:* The skier is moving at moderate speed at the top of the hill. *System:* Includes the skier, rope, and Earth but excludes the motor that pulls the rope up the hill. Ignore friction.

a. Draw a sketch showing initial and final states.

b. Construct a qualitative work–energy bar chart using the grid below, or on your whiteboard.



7.2.3 Represent and reason

Below you read the description of one experiment.

a. Draw a sketch showing initial and final states.

b. Construct a qualitative work-energy bar chart for each of the systems listed below.

Experiment: You are lifting a heavy suitcase.

Initial state: The suitcase at rest *right* above the ground.

Final state: The suitcase is moving at constant speed *v,* at a distance *y* from the ground.

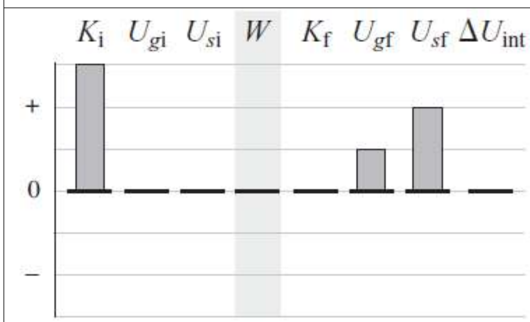
System 1: The suitcase (Earth does work on the ball here).

System 2: The suitcase and Earth.

System 3: The suitcase, Earth and you.

7.2.4 Bar-chart Jeopardy

Work with your group members to describe in words a process (the system, its initial and final states, and any work done on the system) that is consistent with the qualitative work–energy bar chart shown below. There are many possible choices.

Bar chart for a process	**Describe in words one possible consistent process.**
	

7.3 Quantifying gravitational potential and kinetic energies

7.3.1 Find the relationships (show but not do)

To develop mathematical expressions for gravitational potential energy and kinetic energy, we analyze the following situation: a cable lifts a block from vertical position y_i to vertical position y_f. When at position y_i, the block is moving up at speed v_{yi}, and when at position y_f it is moving up at greater speed v_{yf}.

a. Sketch the situation. Include a labeled vertical axis.

b. Write an expression for the work the cable does on the block during its displacement $y_f - y_i$.

c. Draw a force diagram for the block. Use it to find an expression for the force that the cable exerts on the block in terms of its mass *m*, acceleration *a*, and the gravitational constant *g*. Substitute this expression into the expression in part **b.**

d. Use a kinematics equation to convert the acceleration a in the equation from part **b.** into an expression involving the block's speeds v_i and v_f and its displacement $y_f - y_i$.

e. Substitute the expression from part **d.** into the expression for the force that the cable exerts on the block found in part **c**.

f. Substitute the new expression in part **e.** for the force that the cable exerts on the block into the expression for work in part **b**.

g. Examine the expressions that you derived in parts **e.** and **f**. Do you see that the work that the cable did on the block equals the sum of the changes of two quantities: $mgy_f - mgy_i$ and $\frac{1}{2}mv_{yf}^2 - \frac{1}{2}mv_{yi}^2$? Discuss how these expressions can be used to write an expression for the gravitational potential energy of the block–Earth system and an expression for the kinetic energy of the block.

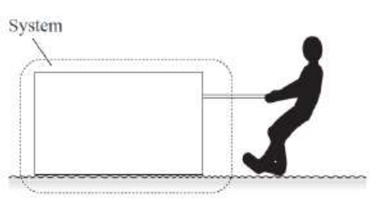
7.5 Friction and energy conversion

7.5.1 Reading exercise

Read Section 7.5 in the textbook and answer Review Question 7.5.

7.5.2 Represent and reason

Work with your group, using a whiteboard, to determine an expression for the change in internal energy due to friction in a system that consists of a crate and a rough horizontal surface on which it slides. You, outside the system, pull on a rope attached to the crate so that it moves slowly at constant velocity (see figure below). At the end of the process, the bottom of the crate and the surface on which it was moving have become warmer.

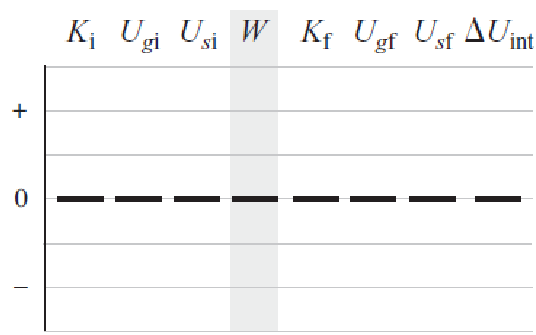


a. Write an expression for the work done on the system by the external force of the rope on the crate as the rope pulls the crate a distance s across the surface exerting a force $\vec{F}_{\text{R on C}}$.

b. Choose the crate alone as an object of interest and draw a force diagram for the crate. Apply Newton's second law for the horizontal *x*-axis. How are $\vec{F}_{\text{R on C}}$ (the force the rope exerts on the crate) and $\vec{f}_{\text{k S on C}}$ (the kinetic friction force that the surface exerts on the crate) related?

c. Now, combine parts **a.** and **b.** to write an expression for the work done by the force $\vec{F}_{\text{R on C}}$ on the system that consists of a crate and a rough horizontal surface on which it slides. Is it positive or negative?

d. Represent the process that you analyzed in part **c.** with a bar chart.



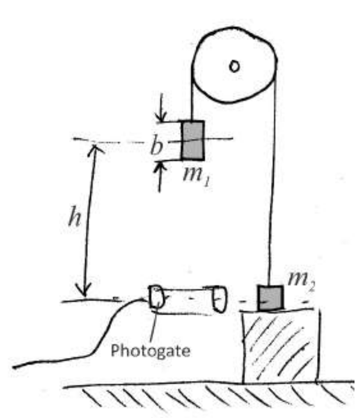
e. Examine the bar chart. Write an expression for the change in internal energy of the system and decide whether it increases or decreases.

7.6 Skills for analyzing processes using the work-energy principle

7.6.1 Reason

Summarize the results of the activities in Sections 7.3-7.5 to construct a generalized work–energy relationship—a relationship between the initial energy of a system, the external work done on the system, and the final energy of the system. Put your ideas on a whiteboard and compare with another group.

Atwood machine problem



Atwood machine consists of two objects that are connected by a light string, which passes over the light pulley. In our case $m_1 > m_2$. The picture on the right shows the initial state. The teacher will release the object 2. Determine the velocity of object 1 as it passes through the photogate (final state).

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a. Represent the process by drawing a work-energy bar chart. Choose both objects and Earth as the system.

b. Based on the bar chart that you drew, write an equation describing the generalized work-energy principle.

c. Using the equation that you wrote in the previous step derive the expression for the velocity of the object 1 as it passes through the photogate. Indicate any assumptions that you made. Evaluate the equation.

d. Determine the speed of the object 1 as it passes through the photogate in our experiment knowing that m1 = …., m2= …., h=….. and b= …..

Workshop leader will perform the experiment. The data acquisition system will measure the time interval Δt during which the photogate was blocked (i.e. the time needed for the object 1 to pass through the photogate).

e. Is the speed that you calculated in step d. consistent with the measured data? Explain.

Back to the need to know

How to draw work-energy bar-charts representing going up and downstairs? What to choose as a system?

Initial state – bottom of the step, final state top of the step.

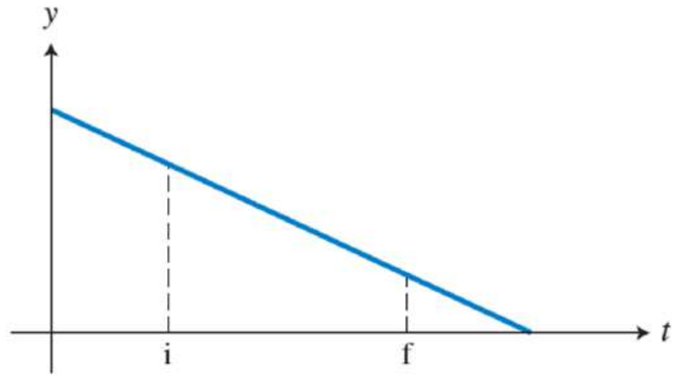
Initial state – top of the step – final state bottom of the step

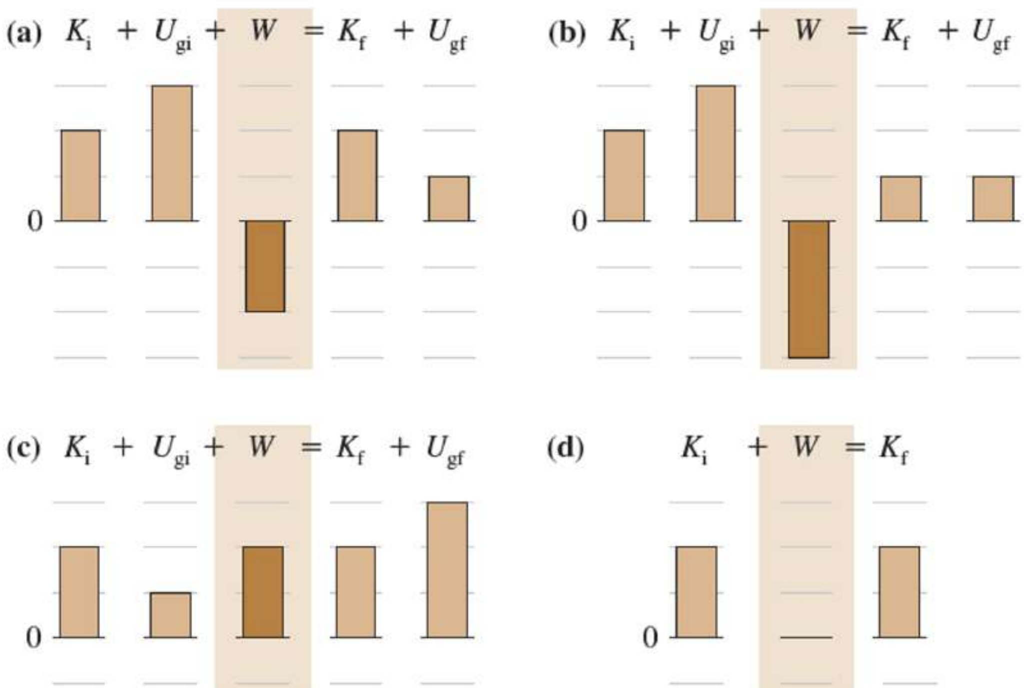
New type of problems

<u>Switching between different representations</u>

11. The graph in **Figure Q7.11** shows the time dependence of the vertical displacement of a lead ball with marked initial and final states. Choose all the work-energy bar charts (a) to (d) that can represent this process (multiple answers may be correct). Note that the y-axis can point either up or down.

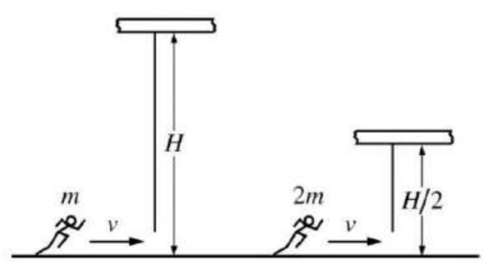
FIGURE Q7.11





<u>Chose the right answer with the best explanation</u>

An athlete with mass m running at speed v grabs a light rope that hangs from a ceiling of height H and swings to a maximum height of h_1. In another room



with a lower ceiling of height *H/2*, a second athlete with mass *2m* running at the same speed v grabs a light rope hanging from the ceiling and swings to a maximum height h_2. How does the maximum height reached by the two athletes compare, and why?

A. The first athlete reaches a greater height, because this athlete swings on a longer rope.
B. The second athlete reaches a greater height, because this athlete has a greater mass.
C. The two athletes reach the same height, because the effect of the rope length offsets the effect of the athletes' masses.
D. The two athletes reach the same height, because the athletes run with the same speed.

On-line games based on ISLE approach

Go to https://universeandmore.com/energy and play as many games as time permits.