

Newton's Law of Universal Gravitation

The greatest moments in science are when two phenomena that were considered completely separate suddenly are seen as just two different versions of the same thing. Newton's explanation that the motion of the moon around the Earth (and the planets around the sun) and of an apple falling to the ground are two expressions of the same phenomenon, gravity, represents a wonderful instance of this sort of synthesis.

Newton conjectured that objects near the surface of the Earth fall towards the center of the Earth as a result of a universal attraction of all matter towards all other matter. He then considered that the circular motion of the moon around the Earth could then be explained if that same force of gravity were supplying the force towards the center of the moon's circular orbit, the Earth. That force of gravity would provide the unbalanced force necessary for circular motion.

Universal Gravity

Newton postulated that all matter in the universe is attracted to all other matter. He named this force of attraction "gravity" and he formulated the following expression to describe how the force of gravity depends on the mass of each object and the distance between their centers.

$$F_G = \frac{Gm_1m_2}{r^2}$$

In this equation, "G" is a constant that would need to be discovered by experiment, "m₁" and "m₂" are the masses of the two objects and "r" is the distance between their centers. It doesn't matter which mass you call "m₁" and which one you call "m₂": That's a result of Newton's third law which states that the force on one object will be equal in size to the force on the other.

Newton had to invent calculus in order to show that the distance between two spherical objects, the "r" in his equation, should be measured between the centers of the objects. He was able to show that spherical objects act, from the perspective of gravitational force, as if all of their mass were located at a point at their center. As a result, the distance from a spherical mass, such as the Earth or the sun, must always be measured from its center.

It took more than a hundred years before Henry Cavendish was able to directly measure the value of "G". The modern accepted value of "G" is:

$$G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$$

Example 1

What is the force between a 5.0kg spherical object and a 10 kg spherical object whose centers are located 2.5m apart?

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$F_G = (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.0\text{kg})(10\text{kg})/(2.5\text{m})^2$$

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$$F_G = 5.3 \times 10^{-10} \text{ N}$$

This is a very small force...which is why the force of gravity between even large objects in our everyday life is very small. However it can be large if very large masses are involved. That's also why scientific notation is so important in solving problems in this chapter.

Example 2

What is the average gravitational force between the Earth and the sun? The mass of the Earth is 6.0×10^{24} kg, the mass of the sun is 2.0×10^{30} kg and the average distance between their centers is 1.5×10^{11} m.

$$F_G = Gm_1m_2/r^2$$

$$F_G = (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(6.0 \times 10^{24} \text{ kg})(2.0 \times 10^{30} \text{ kg})/(1.5 \times 10^{11} \text{ m})^2$$

$$F_G = 36 \times 10^{21} \text{ N}$$

$$F_G = 3.6 \times 10^{22} \text{ N towards each other's center}$$

Example 3

What is the distance between two objects whose masses are 28,000 kg and 50,000 kg if the gravitational force between them is 50N?

$$F_G = Gm_1m_2/r^2$$

To solve for r, first multiply both side by r^2

$$(F_G)(r^2) = Gm_1m_2$$

Then divide both sides by F_G

$$r^2 = Gm_1m_2 / F_G$$

And take the square root of both sides

$$r = (Gm_1m_2 / F_G)^{1/2}$$

Now substitute in the given values and solve

$$r = ((6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(2.8 \times 10^4 \text{ kg})(5.0 \times 10^4 \text{ kg})/(50 \text{ N}))^{1/2}$$

$$r = (1.86 \times 10^{-3})$$

$$r = 18.6 \times 10^{-4}$$

$$r = 4.3 \times 10^{-2} \text{ m}$$

$$r = 4.3 \text{ cm}$$

Example 4

The force between two objects of equal mass whose centers are separated by 15cm is 2×10^{-4} N. What is the mass of the objects?

$$F_G = Gm_1m_2/r^2$$

First let $m = m_1 = m_2$

$$F_G = Gmm/r^2$$

$$F_G = Gm^2/r^2$$

Then multiply both sides by r^2 and divide both sides by G

$$m^2 = (F_G)(r^2)/G$$

Then take the square root of both sides

$$m = ((F_G)(r^2)/G)^{1/2}$$

$$m = (r)(F_G/G)^{1/2}$$

Substituting the given values

$$m = (0.15 \text{ m})((2 \times 10^{-4} \text{ N})/(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2))^{1/2}$$

$$m = (0.15 \text{ m})((0.3 \times 10^7 \text{ kg}^2/\text{m}^2))^{1/2}$$

$$m = (0.15 \text{ m})((3 \times 10^6 \text{ kg}^2/\text{m}^2))^{1/2}$$

$$m = (0.15 \text{ m})(1.73 \times 10^3 \text{ kg/m})$$

$$m = 0.26 \times 10^3 \text{ kg}$$

$$m_1 = m_2 = 260 \text{ kg}$$

Falling apples: The surface gravity of planets

Since Newton was able to show that the gravitational force of a spherical object can be thought of as emanating from a point at its center, it can be seen that an apple sitting near the surface of the Earth is actually a great distance from the gravitational center of the Earth. In fact, any object near the surface of the Earth is always a distance from the Earth's center given by the radius of the Earth, $R_E = 6.4 \times 10^6 \text{ m}$. From this it's possible to determine the acceleration of all objects near the surface of the Earth.

$$\Sigma \mathbf{F} = m\mathbf{a}$$

For an unsupported object of mass "m" near the surface of the Earth, there will be an unbalanced gravitational force given by $F_G = Gm_1m_2/r^2$

$$GM_E m / R_E^2 = ma$$

In this equation, M_E is the mass of the Earth and R_E is its radius, the distance from the center to the surface of the Earth. Canceling the mass of the unsupported object (which shows that our result will be independent of mass)

$$GM_E / R_E^2 = a$$

but this just represents "g", the acceleration of unsupported objects near the surface of the Earth. Since we'll be calculating g for other circumstances, let's call "g" near the surface of the Earth "g_E"

$$g_E = GM_E / R_E^2$$

plugging in the currently accepted values for "G" and the mass and radius of the Earth yields

$$g_E = (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(6.0 \times 10^{24} \text{ kg}) / (6.4 \times 10^6 \text{ m})^2$$
$$g_E = 9.8 \text{ m/s}^2$$

This is the result that had been measured for some time and shows that Newton's more general Universal Theory of Gravity is consistent with prior observation. However, there is nothing in that analysis that is unique to the Earth, except for the figures that we used for the Earth's mass and radius. The same basic expression will yield the acceleration of unsupported objects near the surface of any planet.

The acceleration due to gravity of unsupported objects near a planet whose mass is "M" and whose radius "R" is given by

$$\mathbf{g} = \mathbf{GM}/\mathbf{R}^2$$

Example 5

Calculate g for Saturn given that the mass of Saturn is $5.7 \times 10^{26} \text{ kg}$ and its radius is $6.0 \times 10^7 \text{ m}$.

$$g = GM/R^2$$

$$g = (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.7 \times 10^{26} \text{ kg}) / (6.0 \times 10^7 \text{ m})^2$$

$$g = 1.05 \times 10^1 \text{ m/s}^2$$

$$g_{\text{Saturn}} = 10.5 \text{ m/s}^2$$

So while Saturn is much more massive than the Earth, its gravity is only a little larger because of its larger radius.

Gravitational Field

Especially when thinking about large massive objects like planets or stars, it's convenient to think of the force on one object due to the gravitational attraction of another in a slightly different way. We can think of the first object as creating a "gravitational field" and the other object reacting to that field. This

gravitational field is the “g” that we are so familiar with. For instance, we can equally well think of the force on an object of mass m near the surface of the Earth as being given by

$$F_G = GM_E m / R_E^2$$

$$F_G = m (GM_E / R_E^2)$$

or by

$$F_G = mg, \text{ where } g = GM_E / R_E^2$$

These describe exactly the same situation, but once you compute g near the Earth as being 9.8 m/s^2 , it's a lot easier to calculate the force of gravity on all objects near the surface of the Earth, their weight, by just using $F_G = mg$, rather than going back to the equation $F_G = GM_E m / R_E^2$, each time.

Looking at it this way, we can think of each object as creating a gravitational field which extends throughout all locations in space.

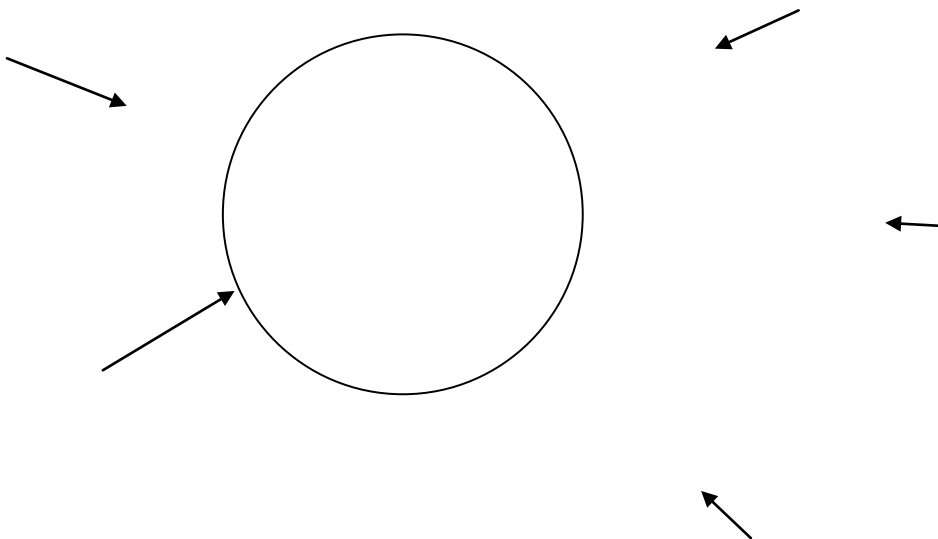
Gravitational Field

When a spherical object creates a gravitational field, the size of the field at a particular location in space is given by

$$g = GM / r^2$$

where M is the mass of the object creating the field and r is the distance from that object's center. The units of gravitational field are acceleration, m/s^2 .

The direction of the gravitational field is the direction that a small test mass would move if placed in that location, towards the center of the spherical object creating the field. The illustration shown below describes the gravitational field of a spherical object, for instance a planet. The direction of the arrows shows the direction of the gravitational field, while their length shows the relative size of the field. Notice that the field gets weaker with increasing distance and that it always points towards the center of the planet.



In the case of the gravitational field of the Earth, its direction will always point towards the center of the Earth. At locations that are on the surface of the Earth, the magnitude of the field is 9.8 m/s^2 . However, since the field will fall off as $1/r^2$, it will diminish at distances above the surface of the Earth.

Example 6

What is the gravitational field of the Earth at a distance of $8.0 \times 10^6 \text{ m}$ above its surface?

The first step in any problem like this is to figure out how far the location under study is from the center of the object that's creating the gravitational field. That's the "r" in the equation. A typical mistake is to use the distance to the surface of the object instead of the distance to its center. For instance, in this example we were given the distance from the surface of the Earth, let's call that the height of the object. We have to add the radius of the Earth to that in order to obtain the distance of that location from the center of the Earth, $r = h + R_E$.

The radius the Earth is $6.4 \times 10^6 \text{ m}$, so the surface of the Earth is just that far from the center of the Earth, and the particular location under study is a height of $8.0 \times 10^6 \text{ m}$ above the surface. So this location is $14.4 \times 10^6 \text{ m}$ from the center of the Earth, $r = 14.4 \times 10^6 \text{ m}$. With this information, there are two ways to calculate "g" at that location.

The first way uses the mass of the Earth and the distance directly:

$$g = GM/r^2$$

$$g = (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(6.0 \times 10^{24} \text{ kg})/(14.4 \times 10^6 \text{ m})^2$$

$$g = 1.9 \text{ m/s}^2 \text{ towards the center of the Earth}$$

The second way takes advantage of the fact that we know g for the surface of the Earth. So we just have to compute how much further away this location is from the center of the Earth than is the surface of the Earth and substitute that into the equation. In this case, $r = 14.4 \times 10^6 \text{ m}$ and $R_E = 6.4 \times 10^6 \text{ m}$, so $r = (14.4/6.4)R_E = 2.25 R_E$. We can then substitute that into the equation and get:

$$g = GM/r^2$$

Substituting in our expression for r

$$g = GM_E/(2.25 R_E)^2$$

Squaring that expression

$$g = GM_E/(5.06 R_E^2)$$

Regrouping to obtain (GM_E/R_E^2)

$$g = (GM_E/R_E^2) (1/5.06)$$

Substituting: $g_E = GM_E/R_E^2 = 9.8 \text{ m/s}^2$

$$g = (9.8 \text{ m/s}^2) / 5.06$$

$$g = 1.9 \text{ m/s}^2 \text{ towards the center of the Earth}$$

Example 7

What is the gravitational field of the Earth at a height that is 5 times the radius of the Earth above its surface?

In this case, the location under study is $5R_E$ above the Earth's surface. We have to add an additional distance of R_E , in order to reach the Earth's center, so

$$r = 6 R_E, \text{ or algebraically:}$$

$$r = h + R_E$$

$$r = 5R_E + R_E$$

$$r = 6R_E$$

Having found “r”, the second method that we used above will prove simplest.

$$g = GM/r^2$$

Substituting $r = 6 R_E$

$$g = GM_E/(6 R_E)^2$$

Then squaring everything in the denominator

$$g = GM_E/(36 R_E^2)$$

Regrouping to get the term “ (GM_E/R_E^2) ” alone

$$g = (GM_E/R_E^2) (1/36)$$

Then, using $g_E = 9.8 \text{ m/s}^2 = GM_E/R_E^2$

$$g = (9.8 \text{ m/s}^2) / 36$$

$$g = 0.27 \text{ m/s}^2 \text{ towards the center of the Earth}$$

Orbital Motion

Newton based much of his theory on the work of Johannes Kepler, which was based on the astronomical observations of Tycho Brahe. Newton concluded that in order for the planets to proceed in their orbits around the sun there must be a force attracting them towards the sun. Kepler writings could be summarized into three laws:

- **Kepler’s first law** states that the orbits of the planets are elliptical, with the sun at one focus.
- **Kepler’s second law** states that if you extend an imaginary line between the sun and a planet, that that line would sweep out equal areas in equal times. In other words, the closer a planet is to the sun, the faster it moves.
- **Kepler’s third law** states that there is a specific relationship between the orbital period and orbital radius for each planet in the solar system. Specifically, for all planets orbiting the sun the ratio of their orbital period squared divided by their orbital radius cubed, T^2/r^3 , yields the same number.

Newton showed that these laws could only hold if there was a force attracting the planets towards the sun. That force would have to be proportional to the mass of the orbiting planet and would have to diminish as $1/r^2$. By applying his third law, he also reasoned that if the force were proportional to the mass of the planet, then it must also be proportional to the mass of the sun. The result was consistent with his Law of Universal Gravitation:

$$F_G = GMm/r^2$$

This equation can be shown to be consistent with Kepler’s third law. We’ll use the approximation that the orbits of the planets are nearly circular so that we can use our earlier analysis of circular motion: It turns out that the same result is obtained for elliptical orbits.

We know from our earlier analysis that an object can only move in circular motion if there is a net force directed towards the center of that circle. If no such force were present, it would move in a straight line, not in a circle. We also know that the acceleration needed to keep an object moving in a circle is v^2/r . If we apply this to the planets orbiting the sun, we know that there must be a net force directed towards the sun and that the size of that force must be equal to ma , or in the case of circular motion, $m(v^2/r)$. If the force were supplied by gravity, then it must be given by $F = GMm/r^2$.

$$\Sigma F = ma$$

Assuming gravitational force is responsible and that the motion is circular yields

$$F_G = m(v^2/r)$$

Substituting in Newton's formula for gravitational force

$$GMm/r^2 = mv^2/r$$

Canceling the mass of the orbiting planet (showing that that mass of the orbiting object doesn't matter) and one of the r's

$$GM/r = v^2$$

Now substituting $v = 2\pi r/T$

$$GM/r = (2\pi r/T)^2$$

Squaring everything on the right side of the equation

$$GM/r = 4\pi^2 r^2/T^2$$

Now solve for T^2/r^3

$$T^2/r^3 = 4\pi^2/GM$$

Everything on the right hand side of this equation, $4\pi^2/GM$, is a constant. Also, at no point in this analysis did we indicate which planet we were discussing...so this analysis is true for any planet orbiting the sun. This is consistent with Kepler's third law which states that the ratio of T^2/r^3 is the same for the orbits of all the planets.

In fact, this analysis will hold for any orbital system, for all objects orbiting the Earth, all objects orbiting Saturn, etc. All that changes is that the mass of the object being orbited becomes "M" in the above equation. The mass of the orbiting object doesn't matter.

All orbiting objects obey the relation that

$$T^2/r^3 = 4\pi^2/GM$$

Where T is the period of the orbit, r is the radius of the orbit, G is the Universal Gravitation Constant and M is the mass of the object being orbited.

The fact that T^2/r^3 is constant for all the planets was known before Newton developed his law of universal gravitation. It's what led him to the conclusion that the gravitational force must decrease as $1/r^2$; otherwise the above analysis would predict the wrong result. It would take many years before the constant "G" was measured by Henry Cavendish...but evidence for this law of Universal Gravitation came even before that. It came from the parallel development of a law that would predict the falling of objects near the surface of a planet.

Example 8

Now let's determine the acceleration of the moon towards the Earth and see if it's consistent with Newton's Theory of Universal Gravitation. The key is that the radius of the moon's orbit is 60 times the radius of the Earth...so $r = 60R_E$. Given this, we can calculate the gravitational field at that distance from the Earth. That will represent the acceleration of any object at that location, including the moon, towards the Earth. We can then compare that to the centripetal acceleration of the moon based on the period and radius of its orbit.

First, let's calculate the gravitational field due to the Earth at a distance equal to the moon's distance from the Earth, the radius of the moon's orbit.

$$g = GM/r^2$$

Substituting $r = 60 R_E$

$$g = GM_E/(60 R_E)^2$$

Then squaring everything in the denominator

$$g = GM_E/(3600 R_E^2)$$

Regrouping to get the term " (GM_E/R_E^2) " alone

$$g = (GM_E/R_E^2) (1/3600)$$

$$\text{Then, using } g_E = 9.8 \text{ m/s}^2 = GM_E/R_E^2$$

$$g = (9.8 \text{ m/s}^2) / 3600$$

$$g = 0.0027 \text{ m/s}^2$$

$$\mathbf{g = 2.7 \times 10^{-3} \text{ m/s}^2 \text{ towards the center of the Earth}}$$

Now, let's calculate the acceleration of the moon towards the Earth using its period, 27.3 days, and the radius of its orbit, 60 R_E .

$$a = v^2/r$$

$$\text{Substituting } v = 2\pi r/T$$

$$a = (2\pi r/T)^2/r$$

$$a = 4\pi^2 r/T^2$$

$$T = (27.3 \text{ days})(24 \text{ hr/day})(3600 \text{ s/hr}) = 2.36 \times 10^6 \text{ s}$$

$$r = 60 R_E = 60(6.4 \times 10^6 \text{ m}) = 3.8 \times 10^8 \text{ m}$$

$$a = 4\pi^2(3.8 \times 10^8 \text{ m})/(2.36 \times 10^6 \text{ s})^2$$

$$\mathbf{a = 2.7 \times 10^{-3} \text{ m/s}^2 \text{ towards the center of the Earth}}$$

This is a striking confirmation of Newton's Theory of Universal Gravitation. It shows clearly that the force holding the moon in orbit is exactly the same as the force that makes an apple fall to the ground : the gravitational field of the Earth.

Example 9: Geosynchronous Satellites

It's often important to have a satellite maintain a constant position above the equator of the Earth. Otherwise, satellite antennas on the ground, for instance for satellite television, could not always point to the same location in space. For a satellite to remain above the same location on the Earth it must complete one orbit of the Earth in 24 hours, so that it orbits at the same rate as the Earth rotates. What height above the ground must geosynchronous satellites orbit?

The simplest way to solve this problem is to compare the orbit of a geosynchronous satellite to something else orbiting the Earth, the moon. We know the period and the orbital radius of the moon and by Kepler's Third Law we know that T^2/r^3 will be the same for all objects orbiting the Earth.

$$T_m^2/r_m^3 = T_{gs}^2/r_{gs}^3$$

Where the subscript "m" is for the moon and "gs" is for a geosynchronous satellite. Now we solve for r_{gs} . First, multiply both sides by r_{gs}^3 .

$$r_{gs}^3 (T_m^2/r_m^3) = T_{gs}^2$$

Then multiply both sides by (r_m^3/T_m^2)

$$r_{gs}^3 = (r_m^3/T_m^2) (T_{gs}^2)$$

Substituting $r_m = 60R_E$, $T_m = 27.3 \text{ days}$ and $T_{gs} = 1 \text{ day}$

$$r_{gs}^3 = ((60R_E)^3/(27.3 \text{ days})^2)(1 \text{ day})^2$$

$$r_{gs}^3 = 216000R_E^3/745$$

$$r_{gs}^3 = 290R_E^3$$

$$r_{gs} = 6.6R_E$$

This gives us the distance that the satellite must be from the center of the Earth. However, to calculate its distance above the surface of the Earth we need to subtract the radius of the Earth, R_E .

$$h = r - R_E$$

$$h = 6.6R_E - R_E$$

$$h = 5.6 R_E$$

To get an answer in meter, we just substitute the value of R_E

$h = 5.6 (6.4 \times 10^6 \text{ m})$
 $h = 36 \times 10^6 \text{ m}$
 $h = 36,000 \text{ km}$ above the surface of the Earth

Example 10: Weighing the Earth

In 1798, the value of “G” was measured for the first time. This was accomplished by Henry Cavendish, more than a century after Newton’s theories were published. This was a difficult feat because the value of “G” is so small. But he was able to use a delicate instrument to determine the gravitational force between two accurately measured spherical masses. His measurement is often referred to as “weighing the Earth” as it allowed for the mass of celestial objects, including the Earth, to be accurately determined. Cavendish determined that:

$$G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$$

With that information let’s determine the mass of the Earth.

$$G = GM_e/R_e^2$$

Solving for M_e

$$M_e = gR_e^2/G$$

Now let’s put in actual numbers

$$\begin{aligned}
 M_e &= (9.8 \text{ m/s}^2) (6.4 \times 10^6 \text{ m})^2 / (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2) \\
 M_e &= (9.8 \text{ m/s}^2) (41 \times 10^{12} \text{ m}^2) / (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2) \\
 M_e &= (401 \times 10^{12} \text{ m}^2) / (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2) \\
 M_e &= 6.0 \times 10^{24} \text{ kg}
 \end{aligned}$$

Launching a Satellite and Weightlessness

There are two steps required to launch a satellite. First, you must get it to its specified height, and then you must give it the correct velocity; both with respect to its size as well as its direction. The direction of the velocity must be tangent to its orbit, and perpendicular to the line between the satellite and the center of the sphere being orbited. The magnitude depends on the gravitational field at that height, g , in that g at that height must be equal to the centripetal acceleration of the satellite.

$$g = a_c$$

Substituting expression for g and a_c

$$GM/r^2 = v^2/r$$

Canceling one r and taking the square root of both sides

$$v = (GM/r)^{1/2}$$

Thus the velocity that a satellite requires in order to maintain its orbit is a function only of the mass of the object being orbited and the distance from its center.

The velocity of an object in circular orbit is given by:

$$\mathbf{v = (GM/r)^{1/2} \text{ tangent to its orbit}}$$

where M is the mass of the object being orbited and r is the distance from its center.

It’s important to recognize that the direction of the velocity is critical. If the satellite is launched to its orbital height by being shot vertically upwards, then the velocity that it must be given to go into orbit will be perpendicular to its launch velocity. So, at the moment it reaches its highest point, and momentarily stops, its rockets must fire in a direction perpendicular to the path that it was traveling to reach that point. That gives the satellite the velocity in the correct direction in order to maintain its orbit.

If the satellite is given too much or too little velocity for its given height, based on the above formula, then the orbit will be elliptical rather than circular. If its velocity is too large at the location where the rockets are fired then the orbit will be at its lowest altitude at that location, the perigee of the satellites orbit. If too little, that will be the highest point of its orbit, its apogee.

We can also see from this analysis why object appear weightless in space. It's not that gravity is not present; we know from our earlier discussion that the gravitational field of an object extends through all of space, falling off as $1/r^2$, but never becoming zero. However, all of the objects in a spacecraft have the same velocity and are at the same distance from the center of the Earth. As a result, they are all in orbit together. If you took away the spacecraft, the occupants and all the objects in the spacecraft would continue to orbit the Earth in exactly the same way. So if you tried to stand on a scale, the scale, the "floor" of the spacecraft and you would all just float together in orbit...you'd exert no "weight" on the scale...hence you appear "weightless". The actual force of gravity acting on you is not much different than when you are standing on the surface of the earth.

Example 11

Determine the force of gravity acting on a 100 kg person standing on the surface of the Earth and in orbit aboard the International Space Station. Calculate the percentage difference.



The International Space Station orbiting the Earth at a height of 218 miles

On the surface of the Earth

$$F_G = mg$$

$$\text{On the surface of the Earth, } g = 9.8 \text{ m/s}^2$$

$$F_G = (100\text{kg})(9.8 \text{ m/s}^2)$$

$$F_G = 980\text{N}$$

In the orbiting International Space Station

First, we must calculate its distance from the center of the Earth. Its orbital height is 218 miles, or $0.35 \times 10^6 \text{ m}$, above the surface of the earth. To this, must be added the distance from the surface to the center of the earth, R_E , which is $6.4 \times 10^6 \text{ m}$. So its distance from the center of the Earth is given by:

$$r = h + R_E$$

$$r = 0.35 \times 10^6 \text{ m} + 6.4 \times 10^6 \text{ m}$$

$$r = 6.75 \times 10^6 \text{ m}$$

which is not very different than R_E , $6.4 \times 10^6 \text{ m}$.

Now we can calculate g at that location.

$$g = GM/r^2$$

$$g = (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(6.0 \times 10^{24} \text{ kg})/(6.75 \times 10^6 \text{ m})^2$$

$$g = 8.8 \text{ m/s}^2$$

Now we can use that value of g to calculate F_G

$$F_G = (100\text{kg})(8.8 \text{ m/s}^2)$$

$$F_G = 880\text{N}$$

So a 100 kg astronaut would feel a gravitational force of 980N standing on the Earth and a force of 880N in orbit on the International Space Station. That difference of 100N represents a percentage decrease of $100\text{N}/980\text{N} = 10\%$. The fact that he or she would feel a reduction of 10% in gravitational force would hardly explain why the astronaut would float in the cabin, as shown below. That is explained by the fact that both the space station and the astronaut are falling around the planet in orbit together.

