Reading - The Centripetal Force Requirement

To understand the importance of a centripetal force, it is important to have a sturdy understanding of the Newton's first law of motion - **the law of inertia**. That is:

... objects in motion tend to stay in motion with the same speed and the same direction unless acted upon by an unbalanced force.

According to Newton's first law of motion, it is the natural tendency of all moving objects to continue in motion in the same direction that they are moving ... unless some form of unbalanced force acts upon the object to deviate its motion from its straight-line path. Moving objects will tend to naturally travel in straight lines; an unbalanced force is only required to cause it to turn. Thus, the presence of an unbalanced force is required for objects to move in circles.
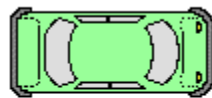
Inertia, Force and Acceleration for an Automobile Passenger

Imagine that you are a passenger in a car at a traffic light. The light turns green and the driver accelerates from rest. The car begins to accelerate forward and your seat begins to push you forward, yet your body begins to lean backwards. According to Newton's first law (the law of inertia), your body being at rest, tends to stay at rest until a net force pushes you. It certainly might seem to you as though your body were experiencing a backwards force causing it to accelerate backwards, yet you would have a difficult time identifying any object pushing your body backwards. Indeed there isn't one. The feeling of being thrown backwards is merely the reaction that your body feels of the seat pushing on you. The car is accelerating and pushes you forward leaving you with the false feeling of being pushed backwards.

Now imagine that you are in the same car moving along at a constant speed approaching a stoplight. The driver applies the brakes, the wheels of the car lock, and the car begins to skid to a stop. Your body, being in motion, tends to continue in motion while the car is skidding to a stop, however, your seatbelt then pushes you backwards and subsequently slows you down with the car. It certainly might seem to you as though your body were experiencing a forwards force causing it to accelerate forwards. Yet you would once more have a difficult time identifying any object pushing you forward. Indeed there is no physical object pushing you forwards. The feeling of being thrown forwards is merely the unbalanced force acting on you by the seatbelt. You once again have the false feeling of being pushed in a direction opposite to your acceleration.
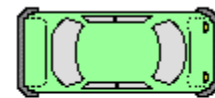
These two driving scenarios are summarized by the following graphic. In each case (the car starting from rest and the moving car braking to a stop) the direction passengers lean is opposite the direction of the acceleration. This is merely the result of the passenger's inertia - the tendency to resist a motion. The passenger's lean is not an acceleration in itself but rather the tendency to maintain (the car seat or the seat belt) push you. The tendency of a passenger's body to maintain its state of rest or motion while the surroundings (the car) accelerate is often misconstrued as an acceleration and becomes particularly problematic when we consider the third possible inertia experience of a passenger in a moving automobile - the left hand turn.

Car Starts from Rest — A passenger at rest would remain at rest, thus causing the "sensation of a backwards acceleration."

Moving Car Brakes to Stop — A passenger in motion would remain in motion, thus causing the "sensation of a forwards acceleration."

Suppose that on the next part of your travels the driver of the car makes a sharp turn to the left at a constant speed and that during the turn, the car travels in a circular-type path. If you are sitting on the passenger side of the car (without a seatbelt), then as the car turns you will slide across your seat until the passenger-side door of the car hits you. This phenomenon might cause you to think that you are being accelerated outwards away from the center of the circle. In reality, this situation is similar to the previous two because the only object that keeps you in the car is the force provided by the car door on you directed towards the center of the circular path. The sensation of an outward force and an outward acceleration is a false sensation. There is no physical object capable of pushing you outwards. You are once more left with the false feeling of being pushed in a direction that is opposite your acceleration. The unbalanced force and the acceleration are both directed towards the center of the circle about which the car is turning and is provided, in this

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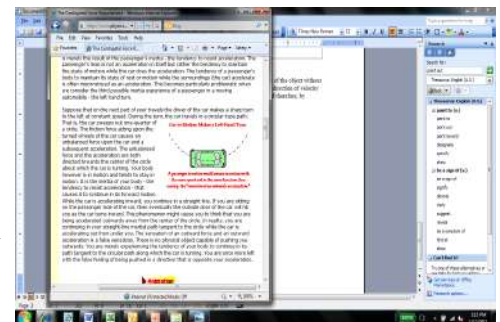
case, by the passenger side door. Your body, however, is in motion and tends to stay in motion; if the car door was not there and you weren't wearing your seatbelt then you would have continued in a straight-line path tangent to the circle while the car is accelerating out from under you.
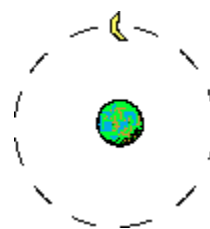
The Centripetal Force and Direction Change

Any object moving in a circle (or along a circular path) experiences a **centripetal force**. That is, there is some physical force pushing or pulling the object towards the center of the circle. This is the centripetal force requirement. The word *centripetal* is merely an adjective used to describe the direction of the force. "Centri-" means center and "-petal" means towards. **We are not introducing a new *type* of force but rather describing the direction of the net force acting upon the object that moves in the circle.** Whatever the object, if it moves in a circle, there is some force acting upon it to cause it to deviate from its straight-line path, accelerate inwards and move along a circular path. Three such examples of centripetal force are shown below.

As a car makes a turn, the force of friction acting upon the turned wheels of the car provides centripetal force required for circular motion.
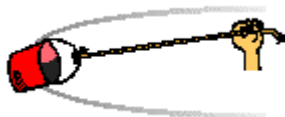
As a bucket of water is tied to a string and spun in a circle, the tension force acting upon the bucket provides the centripetal force required for circular motion.

As the moon orbits the Earth, the force of gravity acting upon the moon provides the centripetal force required for circular motion.

The centripetal force for uniform circular motion alters the direction of the object without altering its speed. The idea that an unbalanced force can change the direction of velocity but not its magnitude still indicates that the velocity has changed and therefore, by definition, the object has accelerated.

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Mathematical Representation

Honors: Geometric Proof for $F_{net-c} = mv^2/r$

Consider the circle of radius **r** below. For a small θ, the arc length **s** $\approx$ the chord Δ**x**.
The radii form the sides of an isosceles triangle with base Δx equal to the product of velocity and time. $\Delta x = \vec{v}t$
Now consider the circle below in which the velocity vectors for two positions are shown. Assume that the central angle θ is the same in both circles.
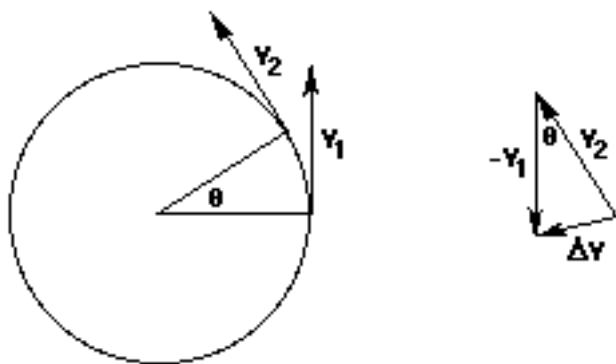
To the right of the circle is a diagram showing the difference of the two vectors as the sum of v_2 and $-v_1$. Because the stopper was traveling at constant speed, the magnitudes of the two velocity vectors are the same, making this second triangle also isosceles. The vertex angle θ is the same as the angle formed by the two radii in the previous diagram.   The Δx & Δv values are determined using the same Δt, therefore the two triangles are similar with a vertex of Θ.

Because the triangles are similar, the bases are proportional to the sides. Thus, substitute vΔt for Δx rearrange terms and multiply both sides by v to get

$$\frac{\Delta \vec{v}}{\Delta x} = \frac{\vec{v}}{r} \qquad \frac{\Delta \vec{v}}{\vec{v} \Delta t} = \frac{\vec{v}}{r} \qquad \frac{\Delta \vec{v}}{\Delta t} = \frac{v^2}{r} \qquad \vec{a}_c = \frac{v^2}{r}$$

The vector sum Δv, divided by the Δt, is the acceleration. Taken at the mid-time of the interval, it is directed toward the center of the circle.

then $\quad F_{net-c} = m\dfrac{v^2}{r} \qquad$ from Newton's 2nd law (F_{net} = ma).

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