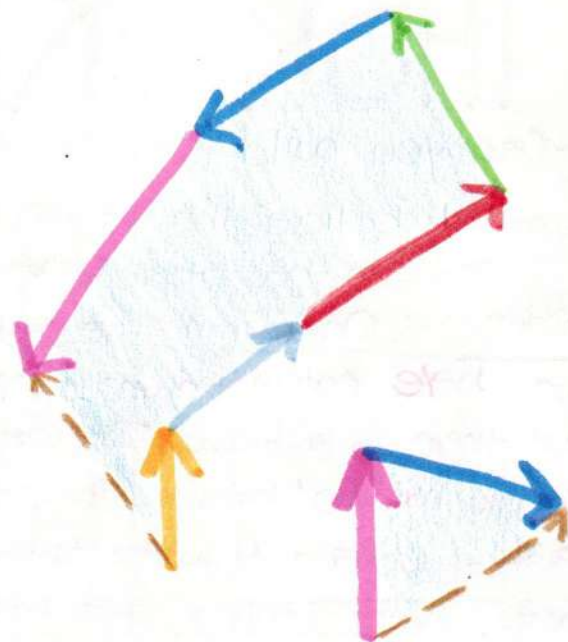
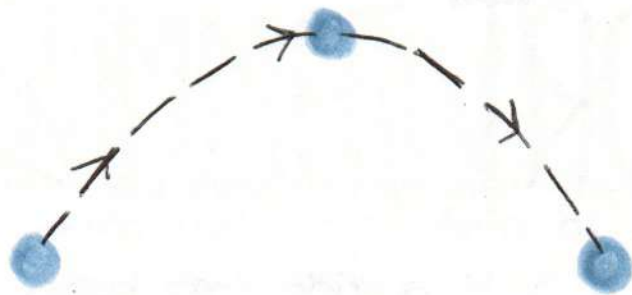




UNIT-3

COMPLEX MOTION

TASK INTRODUCTION

You have to determine the flight path of a ride in which people are shot out of a cannon.

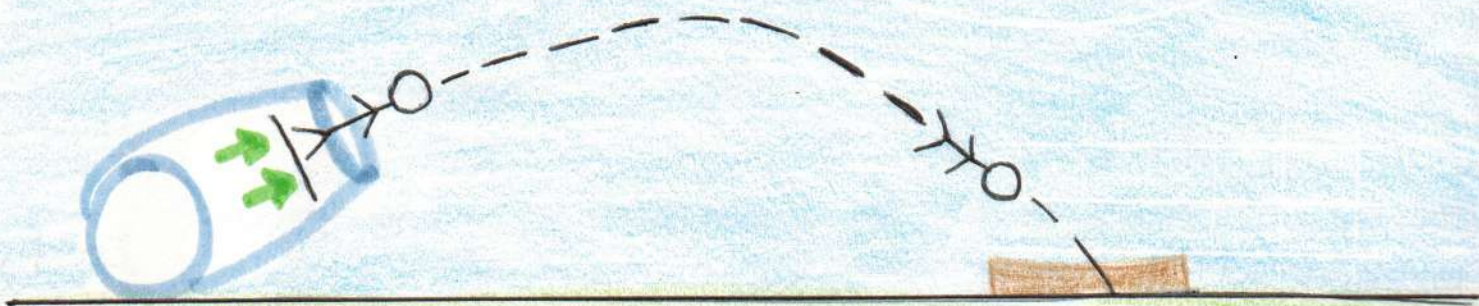


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ESSENTIAL

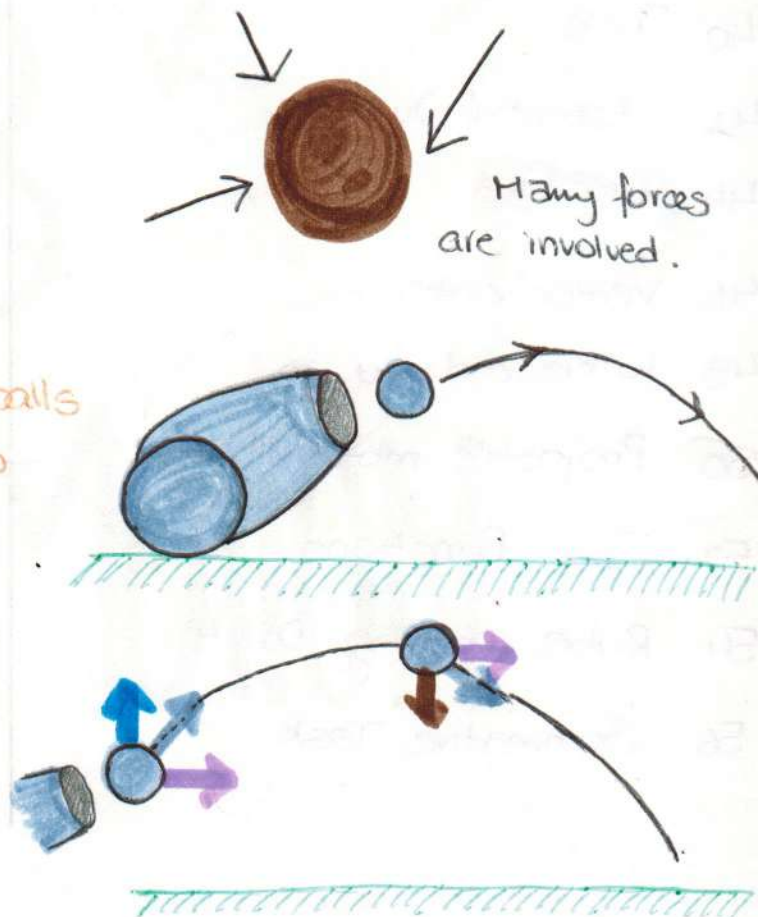
FIRST THOUGHTS

I think that complex motion is different from the motion we learned in Unit-2 in terms of number of forces acting on a single object at a given time.

Examples Projectile motion-

Say we are launching cannon balls from a cannon at an angle to the ground.

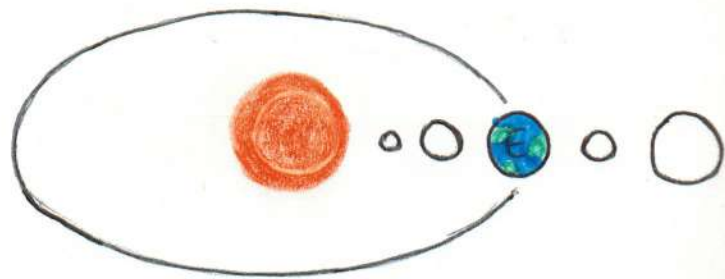
There is a force that pushes the ball to the right & another force pushes it up. So it goes in between these vectors. Gravity also acts on the ball.



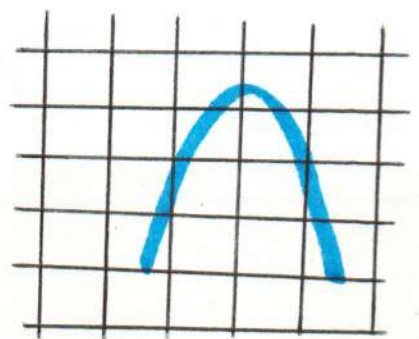
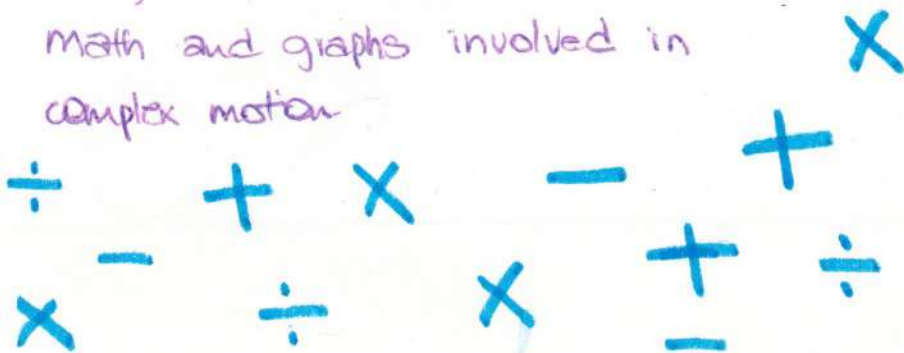
Many forces are involved.

Earth's revolution around sun-

There are a lot of forces involved, one of these is the gravity of the sun. Gravities of other planets play a role as well.



Also, I think there would be more math and graphs involved in complex motion

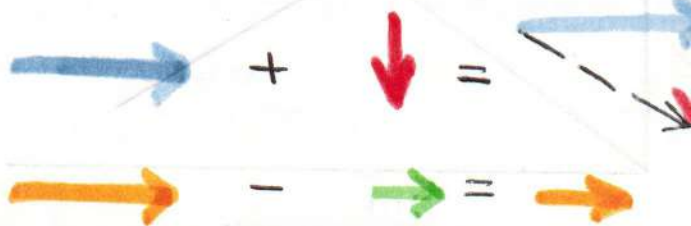


QUESTION

Q How is complex motion different?

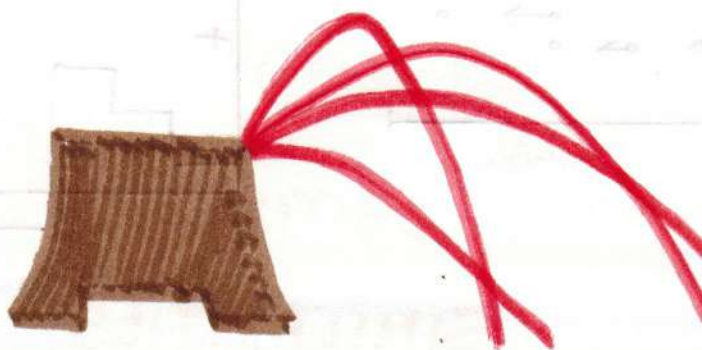
LAST THOUGHTS

Complex motion is different from motion we learned in Unit-2 because we started to add and subtract vectors.

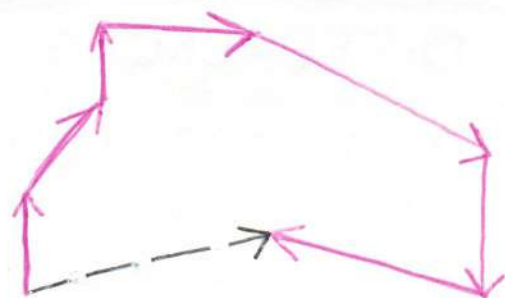


We also started to talk about projectile motion.

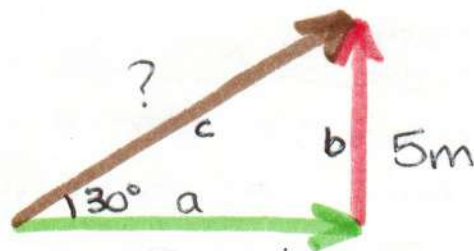
We launched marbles from marble launcher and predicted its landing spot.



Vectors can be added to precisely find the ending location and then find the resultant vector



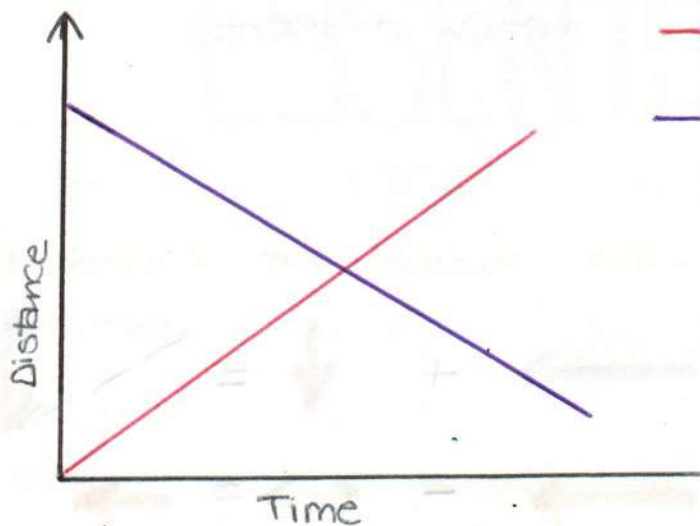
In complex motion, trigonometric functions can be used to calculate the resultant vector if a right angled triangle is formed by the vectors.



$$\sin 30^\circ = \frac{b}{c}$$
$$c = \frac{5}{\sin 30^\circ} = \frac{5}{\frac{1}{2}}$$

$$c = 10 \text{ m}$$

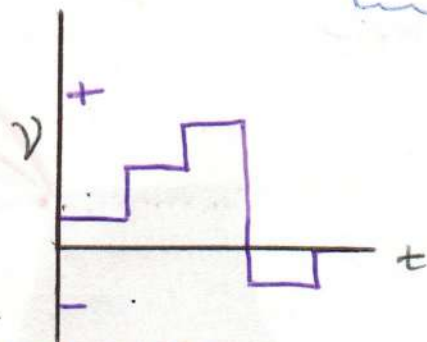
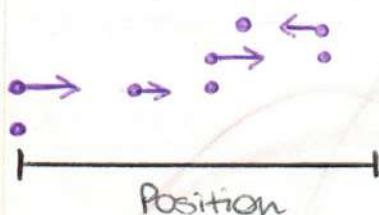
MOTION



- Car travels to the right at 1m/s
- Car travels to the left at 1m/s

Both direction and magnitude are involved in this example

Involve d, t, v, a



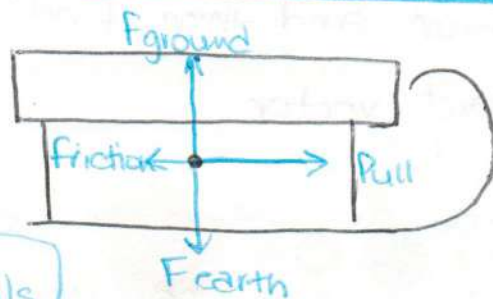
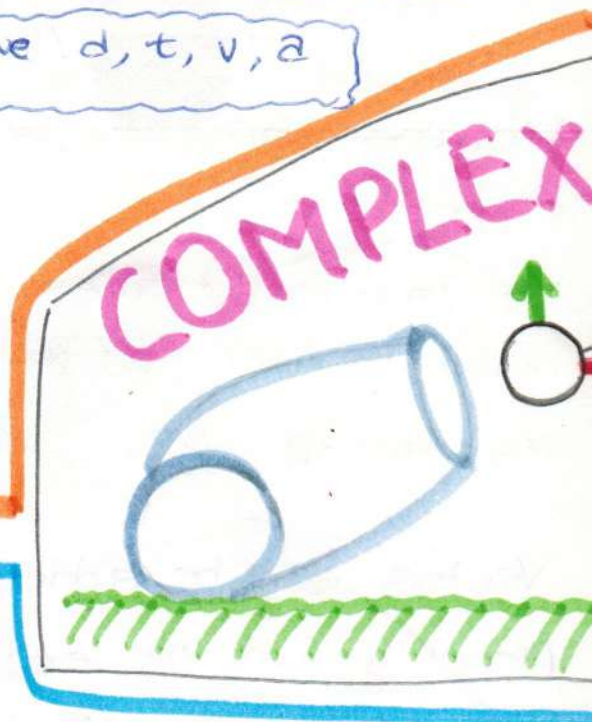
SIMILARITIES

DIFFERENCES

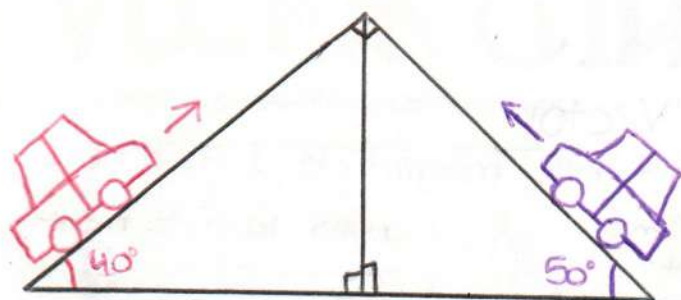
Free body Diagrams used to add vectors. These diagrams analyze for affecting motion of 1 object.

Arrows are placed with all tails starting at center of the object

Trigonometric Functions were not involved in this unit about motion.



COMPLEX MOTION



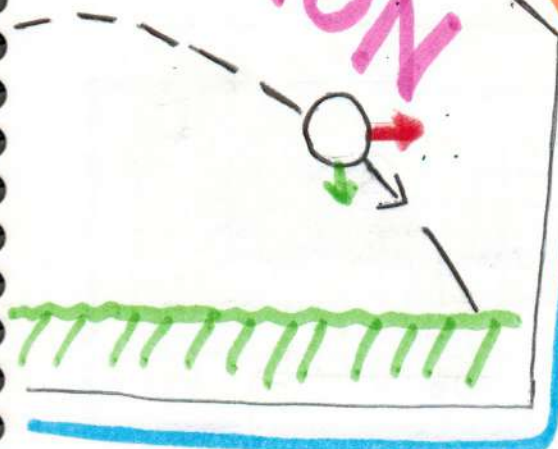
- Car travels uphill at 40° angle to the ground at 2m/s

- Car travels uphill at 50° angle to the ground at 2m/s

Both direction & magnitude are involved

Involve d, t, v, a

MOTION



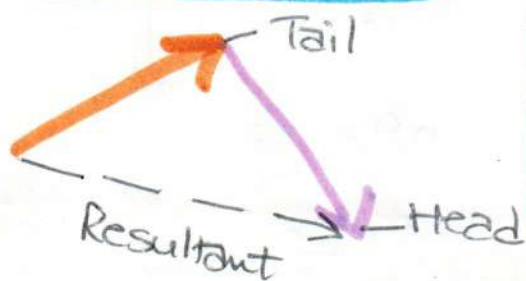
$> =$ Direction

Magnitude

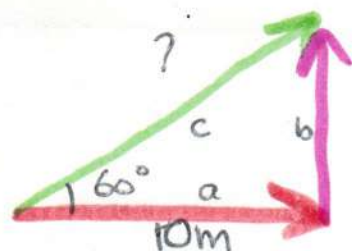
SIMILARITIES

DIFFERENCES

- Arrows can be used to add vectors and find the resultant vector. Arrows are placed with one arrow next to the head of the arrow that precedes it. Direction of each arrow is never changed.



Trigonometric functions can be used to find the resultant vector from 2 given vectors, when a right angled triangle is formed



$$\cos 60^\circ = \frac{10}{c}$$

$$c = \frac{10}{\cos 60^\circ} = \frac{c}{20m}$$

VECTOR NOTES

Scalar

← Both are measurements →

Vector

Has magnitude (size)



100mL



30°C

moves 5m



moves 5m to right



5m

runs 10m to right



10m

Forces are drawn as vectors using arrows.

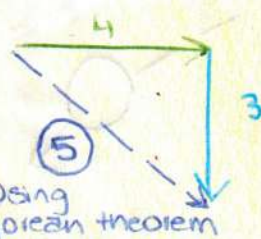
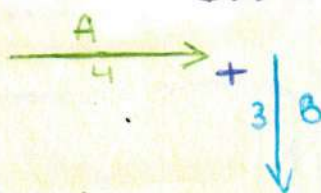
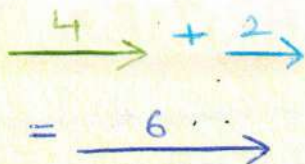


Where arrow points is the direction.



The length of the arrows is the magnitude.

Vector Addition



Using Pythagorean theorem

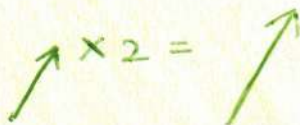
Vector Subtraction



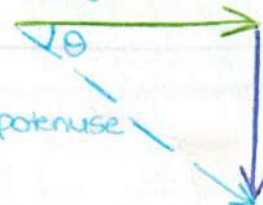
$A - B =$



Vector multiplication



adjacent

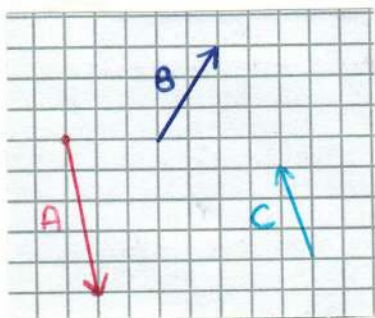


opposite

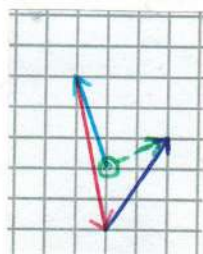
$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

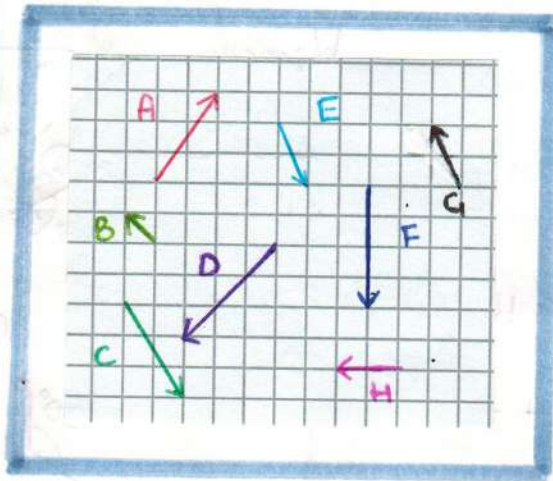


$A + B + C =$



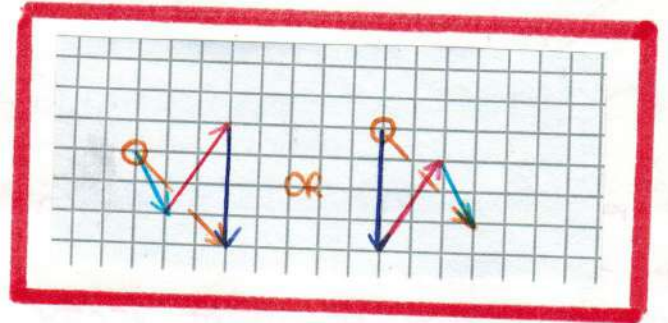
Resultant vector

VECTOR ADDITION WS

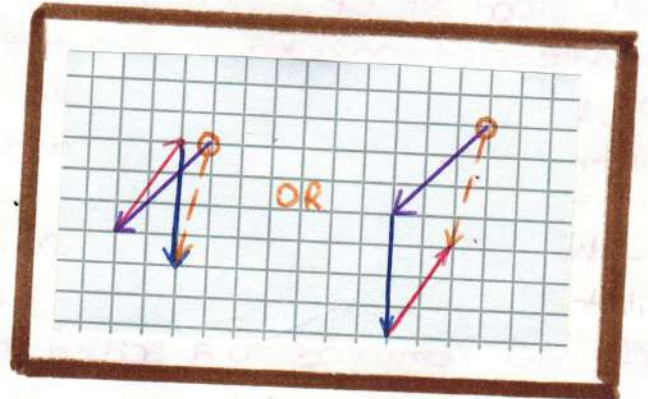


GIVEN VECTORS

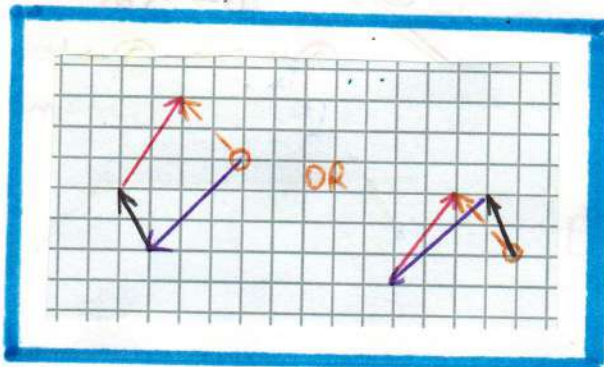
$$A + E + F$$



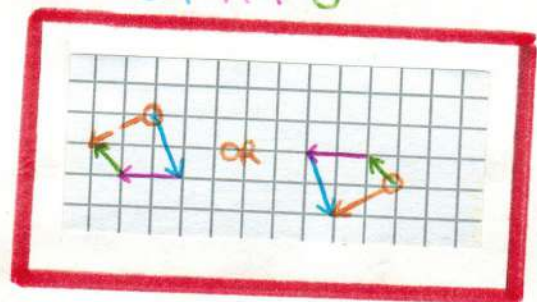
$$D + A + F$$



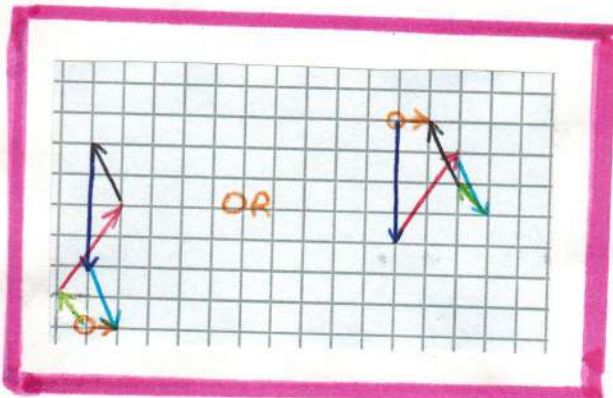
$$D + G + A$$



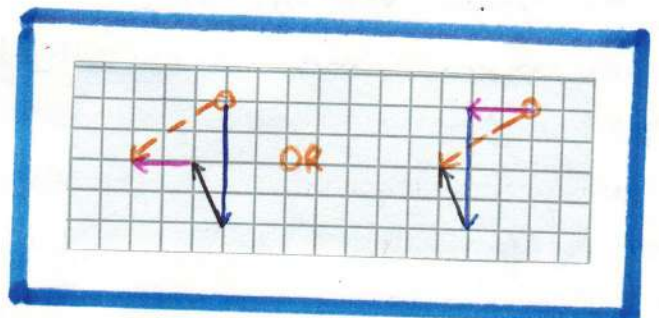
$$E + H + B$$



$$F + E + G + A + B$$



$$F + G + H$$



WHERE DID YOU GO?

Question - Can you precisely calculate where you will end up using given directions?

Evidence - We marked starting point, and then we marked north towards any direction.

- Then we simply followed instruction on lab page.

- We used string to find direction and chalk & ruler to mark it.

- For finding the resultant vector, we put a marker in standing position at SP & one person stood at EP. Then we put a marker in middle and adjusted it so that end point (EP), (SP), & middle marker are in one line.

- We then traced this line & measured its magnitude & direction.

Claim - We calculated the end point & connected it to starting point pretty precisely as compared to a scaled drawing.

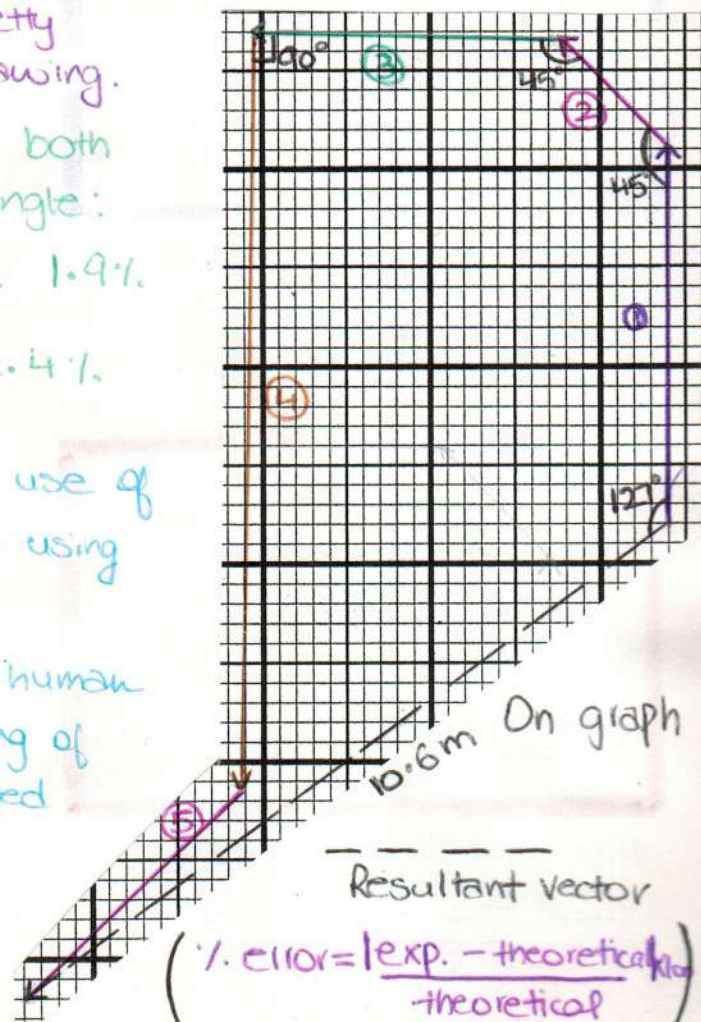
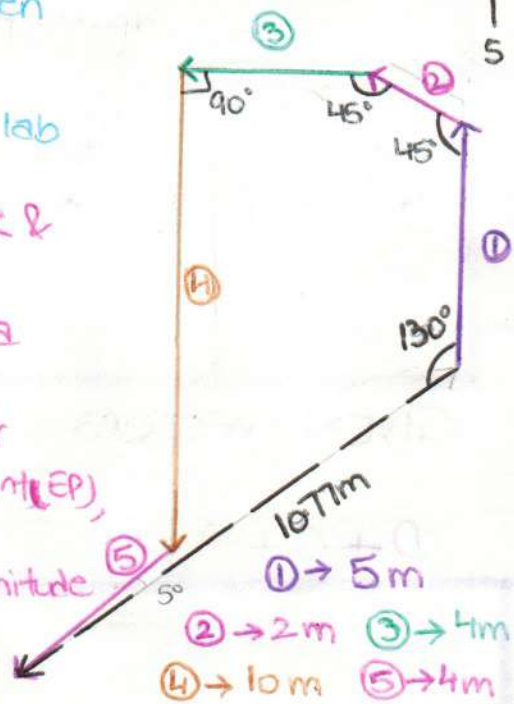
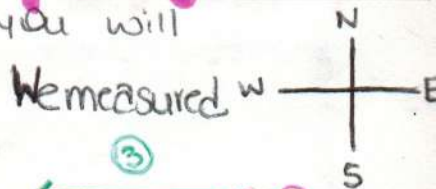
Reasoning - We got precise values for both length of resultant vector & the angle:

$$\text{Length \% error} = \left| \frac{10.8\text{m} - 10.6\text{m}}{10.6\text{m}} \right| \times 100 = 1.9\%$$

$$\text{Angle \% error} = \left| \frac{130 - 127}{127} \right| \times 100 = 2.4\%$$

- Accuracy was possible due to the use of string & protractor to mark angles & using ruler to draw vector lines.

- Error in the experiment is due to human error or due to differences in scaling of ruler used for graph & meter stick used in the experiment.



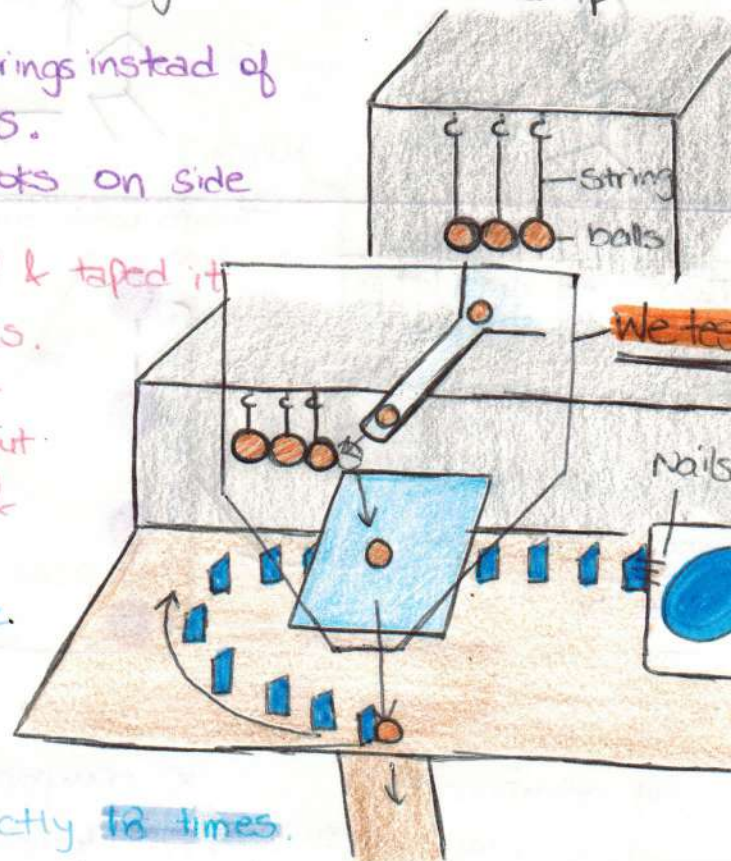
$$\% \text{ error} = \left| \frac{\text{exp.} - \text{theoretical}}{\text{theoretical}} \right| \times 100$$

RUBE GOLDBERG DAY-3

Problem— Make the newton's cradle work in a better way and make ping pong ball travel straight down the ramp

Evidence— • This time we used strings instead of tape to suspend the balls.

- Also, we suspended them from 3 hooks on side counter.
- We had to make a new funnel & taped it to the counter, above the balls.
- We adjusted it to an angle at which a ping pong ball coming out of funnel hits Newton's cradle & makes it work.
- We let the ball roll through the funnel 20 times.
- It worked for Newton's Cradle 17 times (making cradle work)
- Ball rolled down the ramp correctly 18 times.



Claim— Newton's Cradle worked & ball rolled down ramp the way we want.

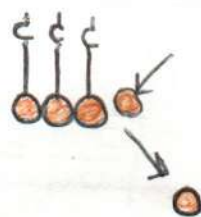
Reasoning— • Newton's Cradle works because the 3 ping pong balls are in contact with each other.

• The funnel is adjusted such that the ping pong ball changes its path by about 100° to continue rolling down the ramp.

• We will make a new funnel out of cardboard as it will be easier to adjust.

• Also, we might increase height of the ramp to make sure the ping pong ball acquires enough energy to topple the dominoes.

• The highlighted part of our set up works because it did what we wanted it to do 17-18 times out of 20 trials.

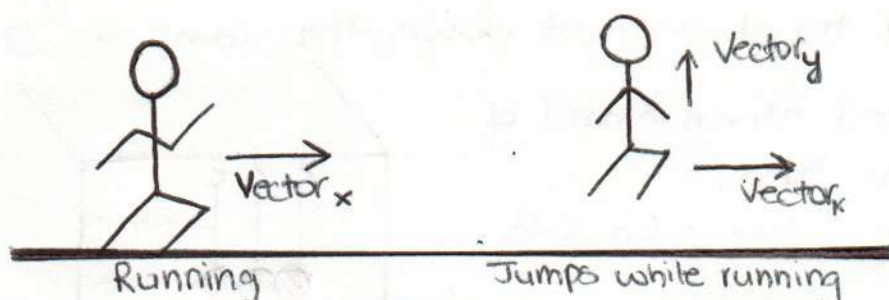


Problem for next time— Ping-pong ball must have enough energy to topple dominoes and the last domino with nails must pop the ball.

PROJECTILE MOTION

Vectors are broken into components to avoid vector multiplication.

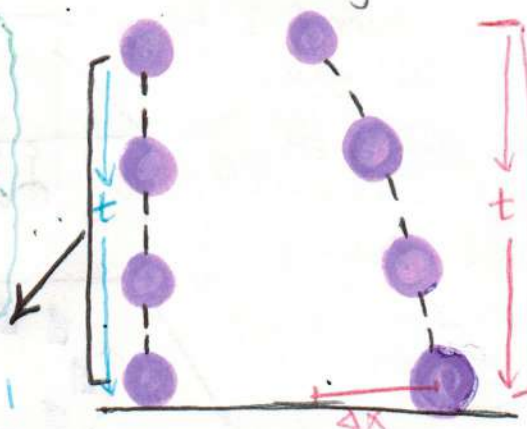
PM Follows parabolic path



Two balls are let go to fall at same time.
① has no horizontal velocity, while ② has it.
Both touch ground at same time.

$$V_i = 0$$

This is motion in 1 dimension.



Motion in 2-D

Hits ground at same time.
But also moves horizontally.

$$\Delta x = v \cdot t$$

y component

$$a_y = 9.81 \text{ m/s}^2$$

$$d = 0.70 \text{ m}$$

$$v_{yi} = 0 \text{ m/s}$$

$$d = \frac{1}{2} a_y \cdot t^2$$

$$t = \sqrt{\frac{2d}{a_y}} = 0.38 \text{ s}$$

x component

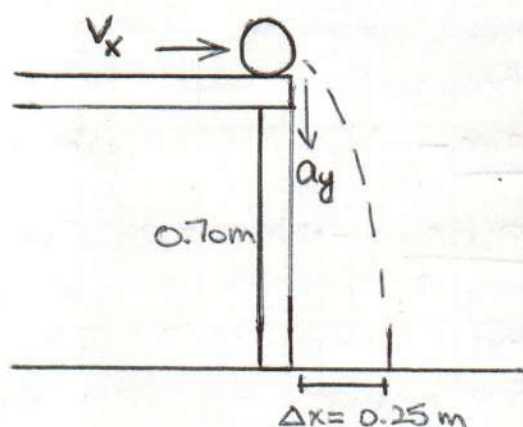
Ball travels 0.25m

in 0.38 s.

$$\Delta x = v_x \cdot t$$

$$v_x = \frac{\Delta x}{t}$$

$$v_x = 0.66 \text{ m/s}$$



y component

$$a_y = 9.81 \text{ m/s}^2$$

$$d = 5.4 \text{ m}$$

$$v_{yi} = 0$$

$$d = \frac{1}{2} a_y \cdot t^2$$

$$t = \sqrt{\frac{2d}{a_y}} = 1.2 \text{ s}$$

x component

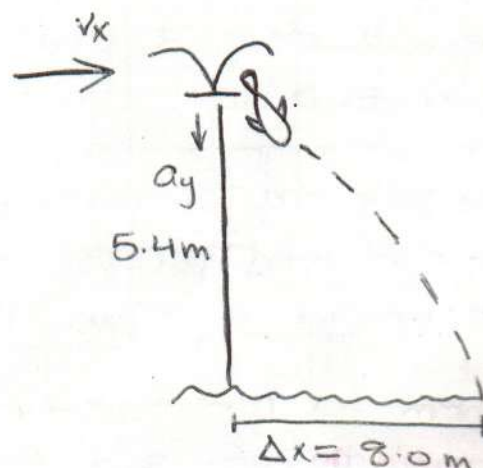
Fish travels 8.0m

in 1.2 s

$$\Delta x = v_x \cdot t$$

$$v_x = \frac{\Delta x}{t}$$

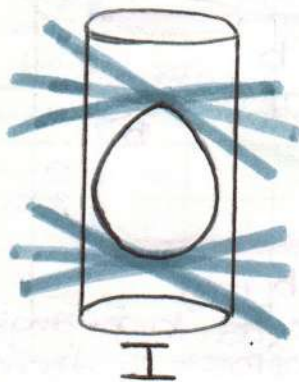
$$v_x = 6.7 \text{ m/s}$$



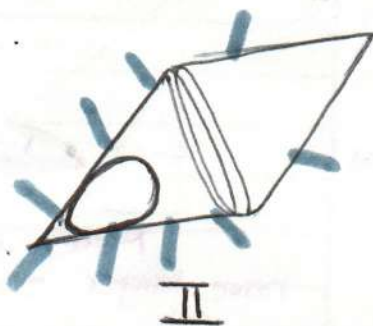
EGG PROJECT DAY-2

Q - Can you create a structure that houses an egg and when thrown into a wall prevents it from breaking? Use 4 sheets of paper and 30 cm of tape.

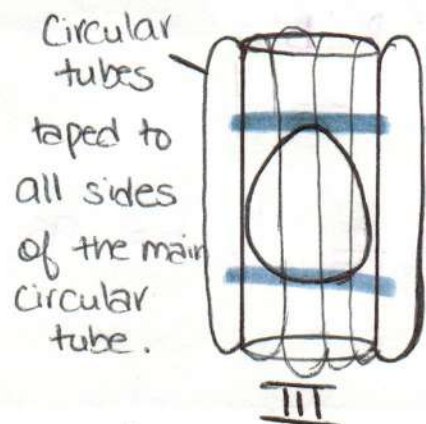
Evidence - We designed 3 structures, that look this.



I



II



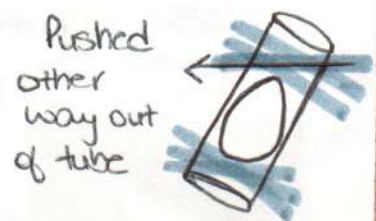
Circular tubes taped to all sides of the main circular tube.

III

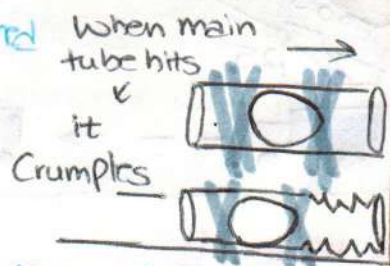
— Represents straw like structure made out of paper.

- Built structure II & put in a golf ball in it. Threw it at wall, the same structure did not bounce back for first 2 throws.
- Built structure I, put an egg in it. Threw same design at wall, the egg broke 0 times.

Reasoning - • For structure I, when it is thrown at wall, sometimes when a single straw hits the wall, it gets pushed out of the main tube or main tube is torn a little where that straw was inserted.



- But when the tube hits the wall (this happened once) it crumpled the way we want. Also there are straws on both sides (top & bottom) of egg to protect it.



- We need a better way to tape holes where the straws have been inserted.

Q for next time — Does the IIIrd structure work?

TRIG FUNCTIONS



$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{a}{c} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{b}{c}$$

$$\tan \theta = \frac{\sin}{\cos} = \frac{\text{opp}}{\text{adj}} = \frac{a}{b}$$

Find 'a' & 'b'

$$\sin 35 = \frac{a}{95}$$

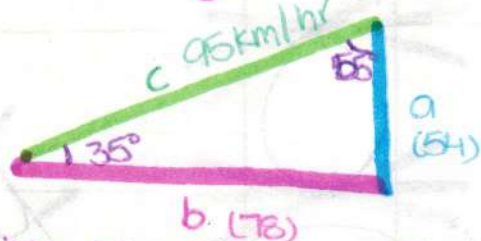
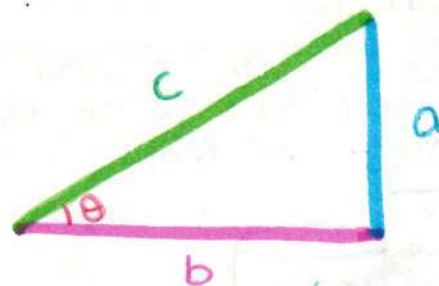
$$a = 95 \sin 35$$

$$a = 54$$

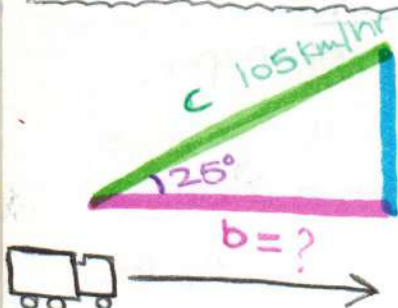
$$\cos 35 = \frac{b}{95}$$

$$b = 95 \cos 35$$

$$b = 78$$



Makes sense as larger angles have larger sides opposite to them.

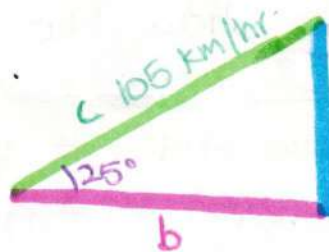


$$\cos 25 = \frac{b}{105}$$

$$b = 105 \cdot \cos 25$$

$$b = 95$$

The truck must travel at 95 km/hr to stay under the air plane

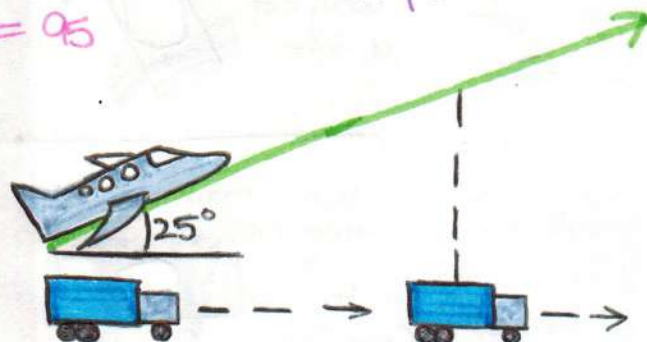


$$\sin 25 = \frac{a}{105}$$

$$a = 105 \cdot \sin 25$$

$$a = 44$$

The vertical component of the air-plane is 44.



There is no horizontal component

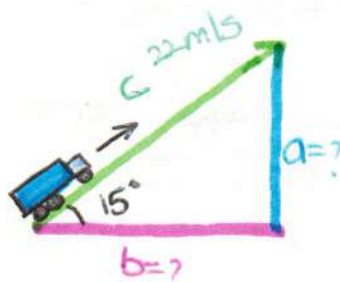
Vertical component is 5.



$$\cos 15 = \frac{b}{22}$$

$$b = 22 \cdot \cos 15$$

$$b = 21$$



$$\sin 15 = \frac{a}{22}$$

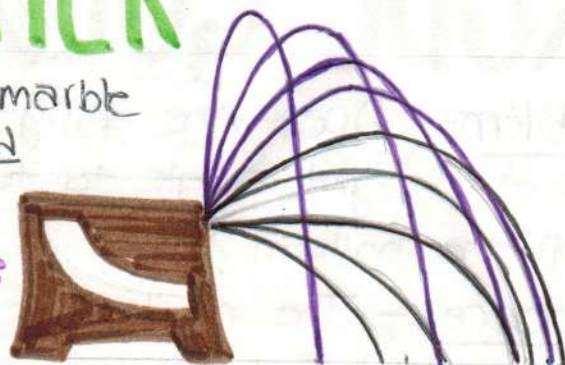
$$a = 22 \cdot \sin 15$$

$$a = 5.7$$

MARBLE LAUNCHER

Question- Can you predict where a marble will end up when launched

Evidence - Using marble launcher we launch marble at every 10° from 0° to 80° and use 1st & 2nd power for each angle.



Angle	1st Power		2nd Power	
	Distance	Averaged	Distance	Average
0°	0.94, 0.92, 0.92	0.93 m	1.20, 1.18, 1.18	1.19 m
10°	1.21, 1.29, 1.27	1.26 m	1.61, 1.60, 1.61	1.61 m
20°	1.64, 1.62, 1.66	1.64 m	2.05, 2.0, 2.04	2.03 m
30°	1.70, 1.72, 1.68	1.70 m	2.42, 2.41, 2.36	2.40 m
40°	1.88, 1.89, 1.91	1.89 m	2.74, 2.8, 2.75	2.76 m
50°	1.78, 1.76, 1.76	1.76 m	2.85, 2.85, 2.85	2.85 m
60°	1.69, 1.58, 1.64	1.64 m	2.45, 2.4, 2.41	2.42 m
70°	1.26, 1.26, 1.24	1.25 m	1.8, 1.81, 1.79	1.80 m
80°	0.64, 0.65, 0.61	0.63 m	0.90, 0.88, 0.88	0.89 m

- Then we placed cup at averaged for 70° and 1st power and launched marble. We were not able to catch the marble, except for once out of 5 tries
- We tried 60° and were not able to land the

marble in the cup even once out of 5 tries.

Claim - We could not predict where the marble would end up after being launched at a set angle and power.

Reasoning - • When we launched a marble thrice at same angle and power setting it did not hit the same point over and over.

- The launcher is not very precise.
- Also, we did not make sure that the launcher wasn't moving from its position.
- Even a little change in launcher's position made a huge difference in landing spot of marble.



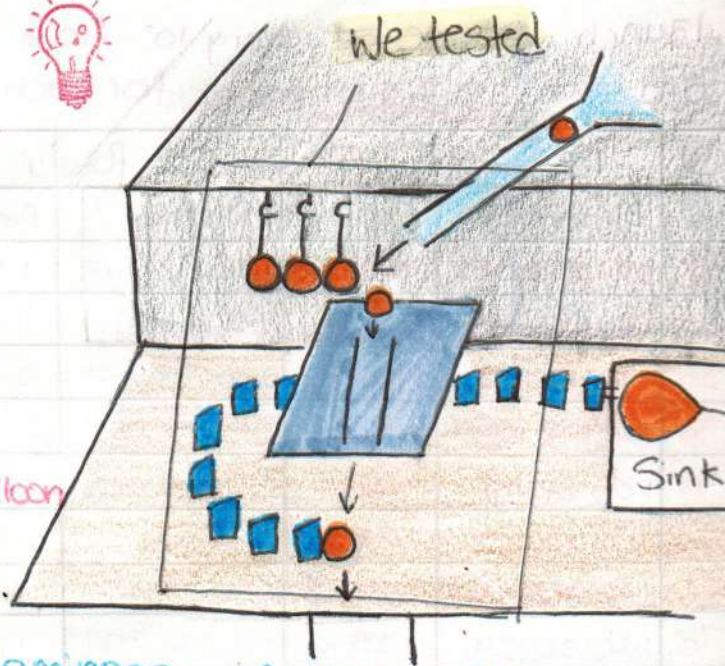
RUBE GOLDBERG DAY-4

Question— Does the ping pong ball hit the first domino at the right angle to topple it and does the last domino pop the balloon?

Evidence — The newton's cradle worked 19 out of 20 times it was tested.

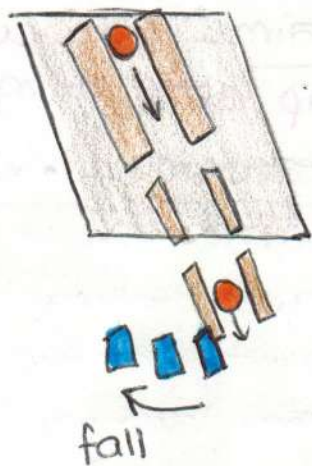


- The ball travelled the correct path and toppled the first domino 9 out of 10 times it was tested.
- We couldn't test popping balloon with nails on the last domino.



Claim— Ping pong ball topples dominoes in the right way, but we couldn't test popping the balloon.

Reasoning— After the ball from funnel hit Newton's Cradle, it sometimes bounced off of the board. We created a wall using ice cream sticks to make sure it travels straight down the board.



The position of first domino was adjusted such that ping pong ball hits it from side and topples it.

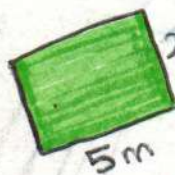
Because the highlighted portion worked at least 90% of the times, we can count it a success.

Problem for next time— Make sure that nails attached to the last domino pops the balloon

TEST REVIEW

VECTORS & SCALARS

Scalar - has magnitude, but no direction.



Vector - has both magnitude & direction.



5m



20m

MOTION IN 1 DIMENSION

Distance - change in position

Velocity - change in distance over time.

Has both direction and magnitude.



in 1 sec.

Acceleration - change in velocity over given amount of time.
Positive a means increase in v,
negative a means decrease in v

PROJECTILE MOTION

vectors are broken into x & y components to avoid vector multiplication.



y component

$$a_y = 9.81 \text{ m/s}^2 \quad d = 0.70 \text{ m}$$

$$v_{yi} = 0 \text{ m/s}$$

$$d = \frac{1}{2} a_y t^2$$

$$t = \sqrt{\frac{2d}{a_y}} = 0.38 \text{ s}$$

x component

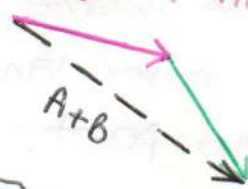
$$\Delta x = 0.25 \text{ m}$$

$$t = 0.38 \text{ s}$$

$$v_x = \frac{\Delta x}{t} = 0.66 \text{ m/s}$$

VECTOR ADDITION (A+B)

Place vectors together head-tail.
Keep angle & magnitude same.



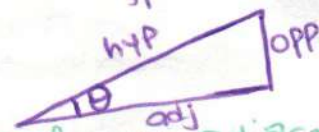
Can use trig-functions to find resultant vector

TRIGONOMETRIC FUNCTIONS

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

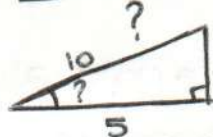
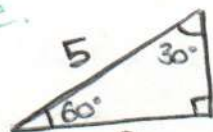


If there is a side & the adjacent angle given then the trig function including that side can be used to find every other side.

Can use Sin or Cos

Here find $\sin \theta = \frac{5}{10}$

Then plug $\frac{5}{10}$ into $\sin^{-1}()$ to find angle



SUMMATIVE TASK

Question - Can you predict where a marble lands after being launched at a specific angle and power setting, by landing it in a cup?

- We started off with 65° angle and power setting of 2nd power.
- We launched the marble about 10 times but it never landed close to the previous point.

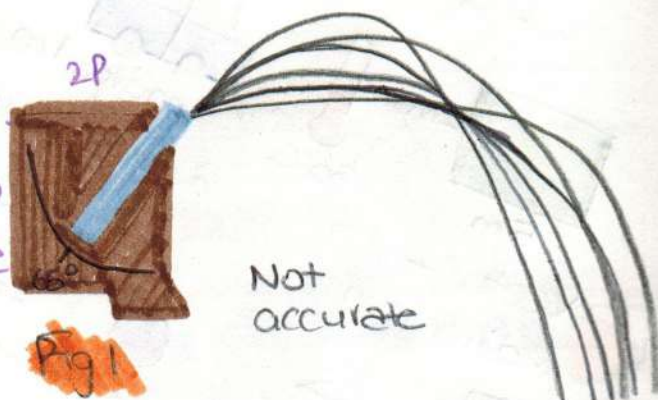


Fig 2 Paper-tube

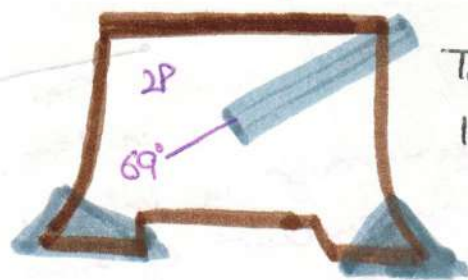
We also changed the angle to 69° .

- We taped the launcher, now the marble started to land closer to the previous landing points. But marble got in only once out of 10 times.

Marble always goes in

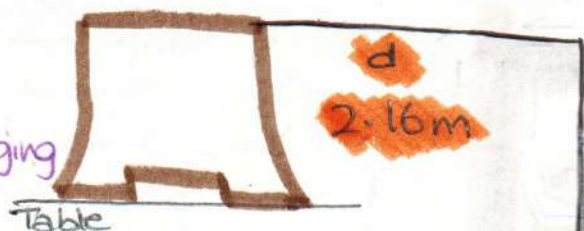


- Next up, we tried using a tube made out of paper to try to extend the length of launched and improve its accuracy. This didn't help.



- We now added a funnel made out of paper on the cup. Accuracy improved. Marble went into the cup 8 out of 10 times.

- We used a string to find the distance between the launcher & the cup. A weight was attached at end of string with weight hanging directly above the cup.



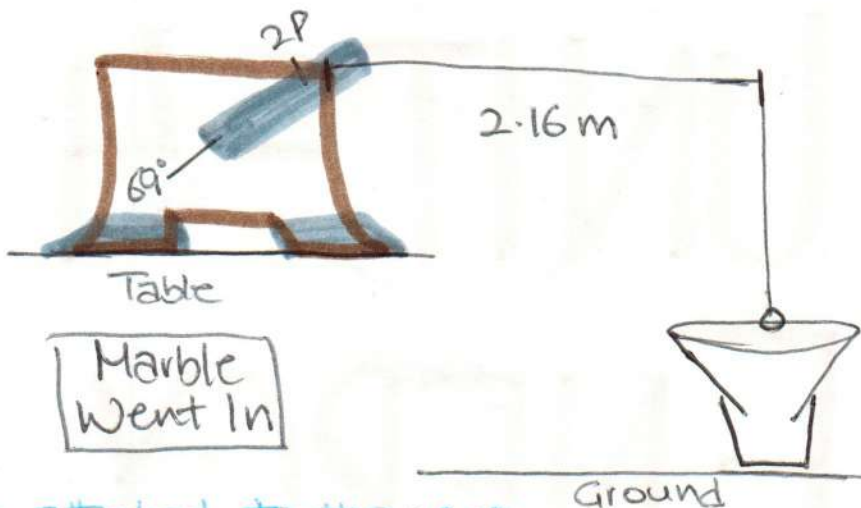
TASK REPORTST



Question - Can you land a marble in a cup after it is launched at a specific angle and power setting?

Evidence - We adjusted the launcher to 69° angle and power setting to 2nd P.

The launcher was taped to the table and cup at distance 'd' (as from fig. 5) from table, on the ground

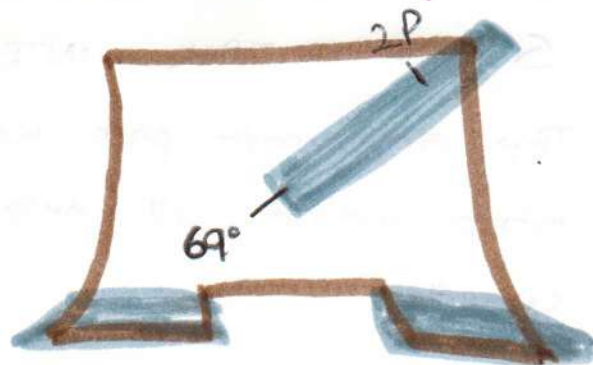


Funnel (as in fig. 4) was attached to the cup.

When launched, ~~the marble went into the cup~~

Claim - Yes, a marble can be launched into a cup from a marble launcher.

Reasoning - Angle was adjusted to 69° as we found the result using angle close to 70° consistent. (As from Pg. 53)



- Second power level was also more consistent than other power levels
- By adding a funnel to the cup, the area where marble could land increased and so did our success rate.
- Taping launcher to table provided it stability. And taping cup did the same thing.
- Using string, we could measure distance between launcher and the cup.