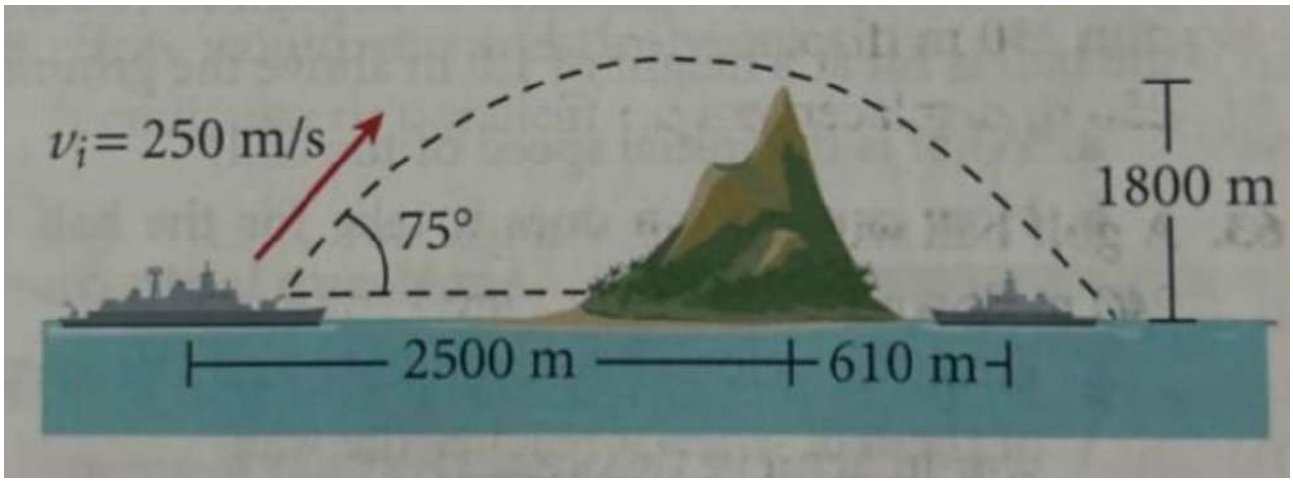
Unit 3 Ch 1 Projectile Motion

An Army Observation Post (OP) is located at the top of an island's 1800 m high mountain peak. An OP is a camouflaged location setup for watching the movement of enemy forces or to radio in effective artillery fire. They spot an enemy ship 610 m away from their OP. They radio to a friendly Naval ship that is 2500 m away on the other side of the island, hidden from the view of the enemy. The OP calls in fire support and the friendly naval ship fires a projectile at the enemy ship on the other side of the island as shown in the diagram below.

If the ship shoots the projectile with an initial velocity of 250 ms^{-1} at an angle of 75°, how close to the enemy ship does the projectile land? How close vertically does the projectile come to the OP on the peak of the mountain?



Bonus Question

If sounds can travel in air at 343 ms^{-1}, how long after the friendly ship fires will it take the OP at the top of the mountain to hear the launch, and how long after the impact will they hear the explosion?

SELF ASSESSMENT RUBRIC

Syllabus Criteria	Approaching Grade Standard	At Grade Standard	Above Grade Standard
Apply vector analysis to determine horizontal and vertical components of projectile motion	I needed to look at the hints to get me started OR I discussed this question with another person to help get to a solution	I can select, apply and manipulate appropriate formulae to solve numerical problems involving several steps	I can select, apply and manipulate appropriate formulae to solve complex numerical problems, and analyse the validity of the solution
Solve problems involving projectile motion	I needed to look at the hints to get me started OR I discussed this question with another person to help get to a solution	I can apply principles to interpret problems, and make plausible predictions in unfamiliar, including real-world, contexts	I can apply principles to interpret complex problems, and make reasoned, plausible predictions in unfamiliar, including real-world, contexts

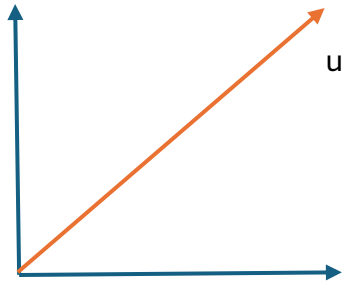
To improve my work, I need to:

Unit 3 Ch 1 Projectile Motion

A good summary of projectile motion can be found here:

https://www.youtube.com/watch?v=cILWOL9ucIA

Using the formulae in the data book for our course, break the initial motion in to x and y componenets



u = 250 ms^{-1}

u_y = 250 x Sin75°

= 241.5 ms^{-1}

u_x = 250 x Cos75°

= 64.7 ms^{-1}

Question 1: How far from the enemy ship does the projectile land?

❶ *Find the time in the air*

$$v_y = u_y + gt$$
$$0 = 241.5 = -9.8 \times t$$
$$t = \frac{-241.5}{-9.8}$$
$$t = 24.6\ s$$

This is the one way journey up, so double to get the total time in the air:

$$24.6x2 = 49.3\ s$$

❷ *Find the horozontal range*

$$s_x = u_x t$$
$$s_x = 64.7 \times 49.3$$
$$s_x = 3189.7\ m$$

❸ *Find the distance from the enemy ship*

The friendly ship is 2500 m + 610 m from the enemy ship, or 3110 m away.

The difference in these two numbers is how far the projectile lands from the enemy ship.

$3181.7\ m\ - 3110\ m = 79.7\ m$ beyond the enemy ship

Question 2: How far above the mountain (the OP) will the projectile pass?

❶ *Find the time for the projectile to reach a horozontal range of 2500 m*

$$s_x = u_x t$$
$$2500 = 64.7 \times t$$
$$t = \frac{2500}{64.7}$$
$$t = 38.6\ s$$

❷ *Find the vertical range (s_y) traveled at this time*

$$s_y = u_y t + \frac{1}{2} g t^2$$
$$s_y = 241.5 \times 38.6 + \frac{1}{2} \times -9.8 \times 38.6^2$$
$$s_y = 9321.9 - 7300.8$$
$$s_y = 2015.1\ m$$

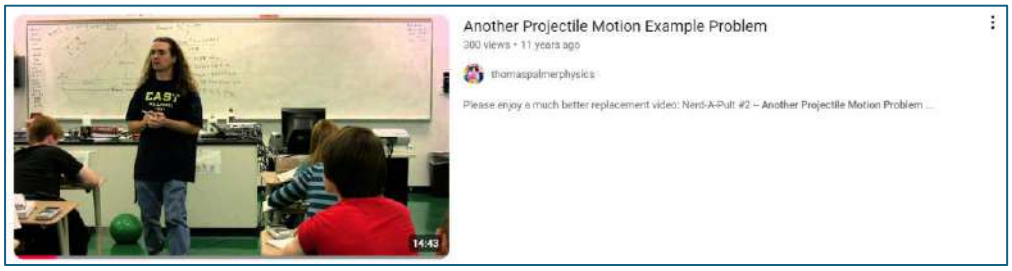
❸ *The differnece between the height of the mountain and the projectile height is how close vertically the projectile will come to the peak (the OP).*

$$2015.1\ m - 1800\ m = 215.1\ m$$

That's quite close if you are in the OP …

An alternative answer, using different equations can be found here:

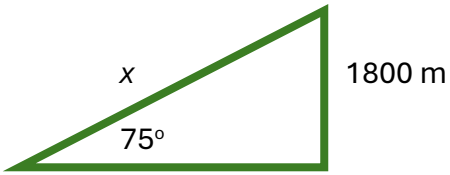
https://www.youtube.com/watch?v=354zLDfcKis&list=PLPN8ldQPmg2cVuoY2Vh8jIdDPqLQVkkbL&index=10



Bonus Question 2: How long after the friendly ship fires will it take the OP at the top of the mountain to hear the launch?

This question was asked to give some real world context to the question. The launch will happen, the OP could possible see the launch, and it will take more time for the sound to reach them than the light. When the OP sees the projectile land approximately 80 m beyond the enemy ship, they are likely to radio the friendly ship to adjust fire (drop) by 80 m. They would then wait for the projectile to be loaded and then wait another 49.3 seconds to see if the fire was accurate. To find the time is takes for the sound to reach the OP on the top of the mountian, first we must find out how far away they are from the friendly ship.

❶ *Find the distance from the friendly ship to the OP at the top of the mountain.*



Use Pythagoras to find x, the distance from the ship to the OP.

$$x^2 = 2500^2 + 1800^2$$
$$x = \sqrt{9490000}$$
$$x = 3080.6\ m$$

❷ *Calculate the time for the sound to travel this distance*

$$t = \frac{distance}{speed}$$
$$t = \frac{3080.6\ m}{343\ ms^{-1}}$$
$$t = 8.98\ s$$

They could hear the launch almost 9 seconds after they see it happen.

Bonus Question 2: How long after the impact will they hear the explosion?

This question is done the same way.

$$x^2 = (610 + 79.7)^2 + 1800^2$$
$$x = \sqrt{375686}$$
$$x = 1927.6\ m\ from\ the\ impact\ site$$

$$t = \frac{distance}{speed}$$
$$t = \frac{1927.6\ m}{343\ ms^{-1}}$$
$$t = 5.62\ s$$

They hear the impact/explosion more than 5 seconds after it occurs.