KEY

9-A *Introduction to Sequences through exploration of Patterns*

#1-3: Find the next 4 terms of the sequence and describe the pattern.

1. 2,4,8,16,32,…

   64, 128, 256, 512

   doubles (times 2)

2. 66,50,34,18,…

   2, -14, -30, -46

   subtract 16

3. 1, 1, 2, 3, 5, 8, 13, …

   21, 34, 55, 89

   Add previous two terms

#4-5: Complete the table for the sequence. Then write a rule for the n^th^ term.

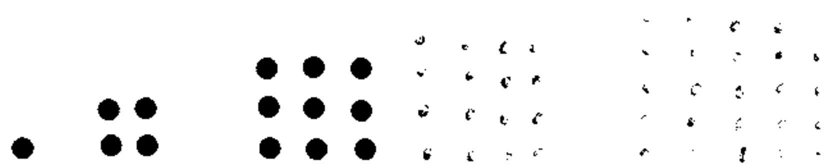
Term	1	2	3	4	5	6	7
Value	1	3	5	7	9	11	13

Rule for n^th^ term: $a_n = 2n - 1$

Term	1	2	3	4	5	6	7
Value	160	80	40	20	10	5	2.5

Rule for the n^th^ term: $a_n = 160(\frac{1}{2})^{n-1}$

#6 - #7: Draw the next 2 figures and describe the pattern in words.

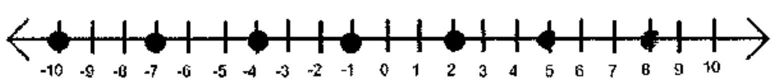
6. 

The number of dots in each row & column increases by 1

7. 

Add one triangle, flipping its orientation from the previous triangle

8. Plot the next 2 terms and describe the pattern.



Add 3

#9-12

#9 - 14: Use the description to write or draw the next 5 terms.

9. Start with 6, add 4 each time.

10, 14, 18, 22, 26

10. Start with –2, multiply by –3 each time.

6, -18, 54, -162, 486

11. Start with 8, divide by 4 each time.

2, 1/2, 1/8, 1/32, 1/128

12. The first term is 2. The second term is 3. The next term is the product of the previous 2 terms.

6, 18, 108, 1944, 209952

13. *n* is the number of the term. Description: 3*n*+2 — Direction should state where n should start at. Let n start at 1.

5, 8, 11, 14, 17

14. A term in a sequence is 7. If the terms go up by 5 write the next 3 terms and the 3 previous terms.

-8, -3, 2, 7, 12, 17, 22

#15 - #16: Fill in the table based on the description and answer the question presented.

15. You borrow \$500 from a relative, and you agree to pay back the debt at a rate of \$15 per month. Fill in the table to indicate the amount still owed at the beginning of each month.

# of months	1	2	3	4	5	6	7
Amount still owed	\$500	485	470	455	440	425	410

How long will it take you to pay back the \$500 completely?

$0 = -15n + 515$

$-515 = -15n$

$n = 34.33$ months

→ $\boxed{\text{35 months}}$

16. You are saving for a bike that costs \$300. You can save \$10 per week. You have started with \$25 the beginning of the 1st week.

# of weeks	1	2	3	4	5	6	7
Amount of money saved	\$25	35	45	55	65	75	85

How long will it take you to have enough saved to buy the bike?

$300 = 10n + 15$

$285 = 10n$

28.5 weeks $= n$

→ $\boxed{\text{29 weeks}}$

KEY

9-B	*Recursive and Explicit (Closed) Rules*

1. Describe in words what the Recursive rule means.

   Using previous term(s) to find the next term

2. Describe in words what the Explicit rule means.

   Using the term number to find the value of that term

#3-5: Write a Recursive and Explicit rule for each sequence:

3. Sequence: $3, -1, -5, -9 \ldots$

Recursive: $a_1 = 3$
$a_n = a_{n-1} - 4$

Explicit:
$a_n = -4n + 7$

4. Sequence: $5, 10, 15, 20 \ldots$

Recursive: $a_1 = 5$
$a_n = a_{n-1} + 5$

Explicit:
$a_n = 5n$

5. Sequence: $1, 4, 9, 16 \ldots$

Recursive: $a_1 = 1$
$a_n = 2n - 1 + a_{n-1}$

Explicit:
$a_n = n^2$

#6-8: Write the first five terms in each Recursive sequence. (The first term is given for you, so include that term and four more.)

6. $a_1 = 3, a_n = 2a_{n-1}$

   3, 6, 12, 24, 48

7. $a_1 = 6, a_n = \frac{1}{2}(a_{n-1}) + 4$

   6, 7, 7.5, 7.75, 7.875

8. $a_0 = -2, a_n = (a_{n-1})^2 + 3$

   -2, 7, 52, 2707, 7327852

#9-11: Write the first five terms in each Explicit sequence. ~~(The first term is given for you, so include that term and four more.)~~ Use $n=1$ for the first term

9. $a_n = 1 - n$

0, -1, -2, -3, -4

10. $a_n = 13.5 + \frac{1}{2}n$

14, 14.5, 15, 15.5, 16

11. $a_n = 2(10)^{n-1}$

2, 20, 200, 2000, 20000

#12-17: Match the pattern with its recursive form (in the table on the right). Think about the *now / previous* and *now / next* that the recursive rule is representing.

12.

x	0	1	2	3
y	5	8	11	14

C

13. 5, 10, 20, 40, …

B

14. John had 3 sons, each of sons his sons had 3 sons, their sons had 3 son. ~~If this pattern continues, how many sons will be in the 6th generation?~~
their sons each had 3 sons.

A

15.

x	1	2	3	4	5
y	10	2	$\frac{2}{5}$	$\frac{2}{25}$	$\frac{2}{125}$

H

16. Add the previous two terms.

D

17. …, –1, 0.2, –0.04, 0.008, –0.0016, 0.00032

F

Choose from the options below:
A. $a_1 = 1,\ a_{n+1} = a_n \cdot 3$
B. $a_0 = 5,\ a_n = 2a_{n-1}$
C. $a_0 = 5,\ a_{n+1} = a_n + 3$
D. $a_0 = 2,\ a_1 = 7,\ a_n = a_{n-1} + a_{n-2}$
E. $a_1 = 10,\ a_n = \frac{2}{5}a_{n-1}$
F. $a_1 = ?,\ a_n = -0.2a_{n-1}$
G. $a_1 = 1,\ a_{n+1} = a_n \cdot 6$
H. $a_1 = 10,\ a_n = \frac{a_{n-1}}{5}$

18. The owner of Spirit Cell has hundreds of outdated cell phones to sell. Each phone will begin selling at $200 ~~each~~ the first week. The phones will be discounted by 25% each week until they are sold.

a) Complete the table for the price per phone for the first 5 weeks.

Week	0	1	2	3	4	5
Cost per phone	200	150	112.50	84.38	63.28	47.46

b) Write a recursive rule to model the situation.

$a_0 = 200$

$a_n = 0.75 \cdot a_{n-1}$

c) When is the first time that the phone will cost less than $10?

12 weeks

d) Will the phone ever be free? Explain.

no; the value will get closer and closer to 0, but never reaches → asymptote at y=0

19. Malmburg's Nursery owns 7000 white pine trees. Each year, they plan to sell 12% of the trees.

a) If they continue selling trees at this rate without replenishing trees, find the number of pine trees owned by the nursery after 10 years.

$a_n = 7000(0.88)^{n-1}$

$a_{10} = 7000(0.88)^{10-1}$

$a_{10} = 2215.35$ about 2,215 trees

b) Explain what happens to the number of pine trees owned by the nursery as time goes on forever.

The amount of pine trees approach 0.

c) For this situation, write a recursive and an explicit rule.

recursive: $a_0 = 7000$

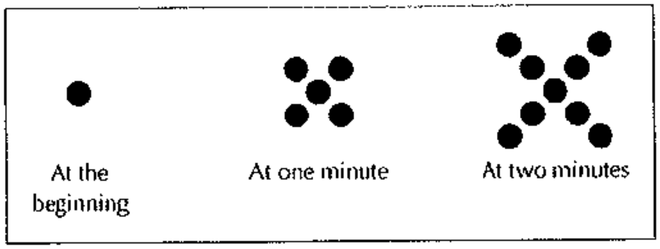
$a_n = 0.88 \cdot a_{n-1}$

explicit: $a_n = 7000(0.88)^{n-1}$

KEY

9-C *What's an Arithmetic Sequence?*

#1 – 4: Use the pattern below to answer the following questions.



Use the picture shown to answer the questions. Your solutions should indicate how many dots will be in the pattern at 3 minutes, 100 minutes, and *t* minutes. Be sure to show how your solution relates to the picture and how you arrived at your solution.

1. Describe the pattern that you see in the sequence of figures above.

   1, 5, 9 Adding 4 dots total to the next figure

2. Assuming the sequence continues in the same way, how many dots are there at 3 minutes?

   13 (9 + 4)

3. How many dots are there at 100 minutes?

   $a_n = 1 + (100-1)(4)$

   $a_n = 397$

4. How many dots are there at *t* minutes?

   $a_t = 1 + (t-1)4$

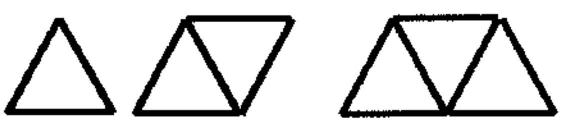
   $a_t = 1 + 4t - 4$

   $\boxed{a_t = 4t - 3}$

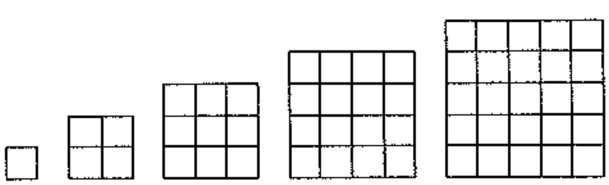
#5-6: Determine if the following diagrams represent arithmetic relationships. If the diagram is arithmetic, draw the next 2 pictures in the sequence. If it is not arithmetic, explain why not.

needs clarification in directions

5.



6.



1 2 3 4 5

#7-9: Find the missing terms for each arithmetic sequence and state the common difference, *d*.

7. 5, 11, <u>17</u>, 23, 29, <u>35</u> common difference: 6

8. 7, 3, −1, <u>−5</u>, <u>−9</u>, −13 common difference: −4

9. 8, <u>21</u>, <u>34</u>, 47, 60 common difference: 13

10. Describe your method for find the missing terms in **#7-9**. Will the method always work? How do you know? Find two consecutive terms and find the difference. Yes, because we are given that the sequence is arithmetic

11. In Riley Theatre, the front row has 30 seats ~~and 2 seats are added to each row after the first row~~. There are 42 rows in the theatre. Use this information to answer the questions.

and the next row has 2 more seats than the previous row.

a. Could this situation be modeled by an arithmetic sequence? If yes, write a recursive and explicit rule. If it is not arithmetic, explain why not.

Can you write recursive + explicit rules for the number of seats that a row would have?

recursive: $a_1 = 30$
$a_n = a_{n-1} + 2$

explicit: $a_n = 2n + 28$

b. Which row of the theatre has 50 seats?

$50 = 2n + 28$
$22 = 2n$
$11 = n$

$\boxed{11^{th} \text{ row}}$

c. Challenge: How many seats are in the theatre? Is it possible to write a model for this situation?

Last row: $a_{42} = 2(42) + 28$
$a_{42} = 112$

$30 + 32 + 34 + \cdots + 112$

$\boxed{S_{42} = 2982 \text{ seats}}$

$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$

#12 – 16: Identify whether the formula given is recursive or explicit.
Then, list the first five terms of each arithmetic sequence ($n \geq 1$).

12. $a_n = 5n - 2$

explicit

3, 8, 13, 18, 23

13. $a_n = 5 - 3(n - 1)$

explicit

5, 2, -1, -4, -7

14. $a_n = -15 + \frac{1}{2}n$

explicit

-14.5, -14, -13.5, -13, -12.5

15. $a_n = -3, a_n = 2a_{n-1} + 4$ not arithmetic, why here?

$a_1 = -3$ ✓ recursive

-3, $\cancel{-2}$,

Change to: $\boxed{a_1 = -3, \; a_n = a_{n-1} + 4}$

-3, 1, 5, 9, 13

16. $a_1 = \frac{1}{3}, a_{n+1} = a_n + \frac{1}{2}$

#17 – 20: Find the 20th term of each arithmetic sequence.

17. $a_n = 5n - 2$

$a_{20} = 5(20) - 2$

$\boxed{a_{20} = 98}$

18. $a_n = -15 + \frac{1}{2}n$

$a_{20} = -15 + \frac{1}{2}(20)$

$\boxed{a_{20} = -5}$

19. $a_{31} = 53, d = 5$

$a_1 = -97$ $\boxed{a_{20} = -2}$

20. $a_1 = 25, a_{n+1} = a_n - 3$

25, 22, 19, ...

$a_n = -3n + 28$

$a_{20} = -3(20) + 28$

$\boxed{a_{20} = -32}$

#21-22: Write the arithmetic sequence in both recursive and explicit form.

21. $-2, 5, 12, 19, 26, \ldots$

recursive: $a_1 = -2$

$a_n = a_{n-1} + 7$

explicit: $a_n = 7n - 9$

22. $27, 15, 3, -9, -21, \ldots$

recursive: $a_1 = 27$

$a_n = a_{n-1} - 12$

explicit: $a_n = -12n + 39$

#23-26: Given the explicit formula ($n \geq 1$) for an arithmetic sequence find the first five terms and the term named in the problem.

23. $a_n = -11 + 7n$

Find a_{34} = 227

-4, 3, 10, 17, 24

24. $a_n = 65 - 100n$

Find a_{39} = -3835

-35, -135, -235, -335, -435

25. $a_n = -7.1 - 2.1n$

Find a_{27} = -63.8

-9.2, -11.3, -13.4, -15.5, -17.6

26. $a_n = \dfrac{11}{8} + \dfrac{1}{2}n$

Find a_{23} = $\frac{103}{8}$

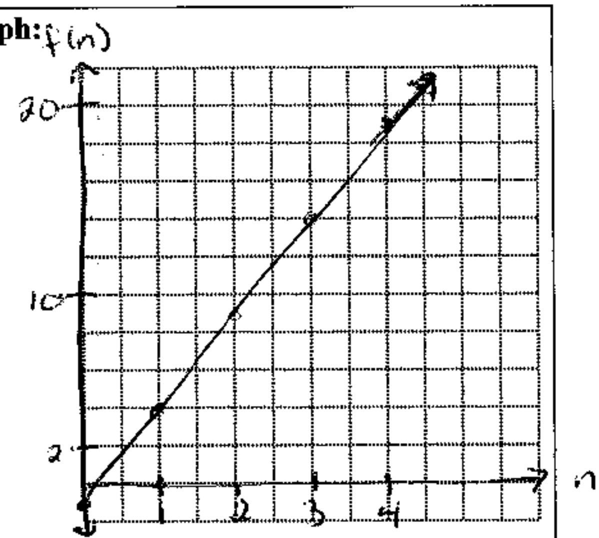
$\frac{15}{8}, \frac{19}{8}, \frac{23}{8}, \frac{27}{8}, \frac{31}{8}$

#27: **Use the given information to complete the other representations for each arithmetic sequence.**

Table:

0	1	2	3	4
-1	4	9	14	19

Graph:



Recursive Equation:

$a_0 = -1$

$a_n = a_{n-1} + 5$

Explicit Equation:

$$f(n) = 4 + 5(n-1)$$

Create a context:

#28-29: Given the first term and the common difference of an arithmetic sequence, find the <u>first five terms</u> and the <u>explicit formula</u>.

28. $a_1 = 28,\ d = 10$

28, 38, 48, 58, 68

~~$a_1 = 28$~~ $\boxed{a_n = 10n + 18}$

~~$a_n = a_{n-1} + 10$~~

29. $a_1 = -34,\ d = -10$

-34, -44, -54, -64, -74

~~$a_1 = -34$~~ $\boxed{a_n = -10n - 24}$

~~$a_n = a_{n-1} - 10$~~

#30-31: Given the first term and the common difference of an arithmetic sequence, find the <u>first five terms</u> and the <u>recursive formula</u>.

30. $a_1 = \frac{3}{5}, d = -\frac{1}{3}$

$\frac{3}{5}, \frac{4}{15}, \frac{-1}{15}, \frac{-2}{5}, \frac{-11}{15}$

$\boxed{a_1 = \frac{3}{5} \quad a_n = a_{n-1} - 1/3}$

31. $a_1 = -26, d = 200$

-26, 174, 374, 574, 774

$\boxed{a_1 = -26 \quad a_n = a_{n-1} + 200}$

#32-33: Given a term (not necessarily the first term) and the common difference of an arithmetic sequence, find the <u>first five terms</u> and the <u>explicit formula</u>.

32. $a_{37} = 249, d = 8$

$a_1 = -39$

-39, -31, -23, -15, -7

$a_1 = -39 \quad a_n = a_{n-1} + 8$

$a_1 = -31$

33. $a_{36} = -276, d = -7$

-31, -38, -45, -52, -59

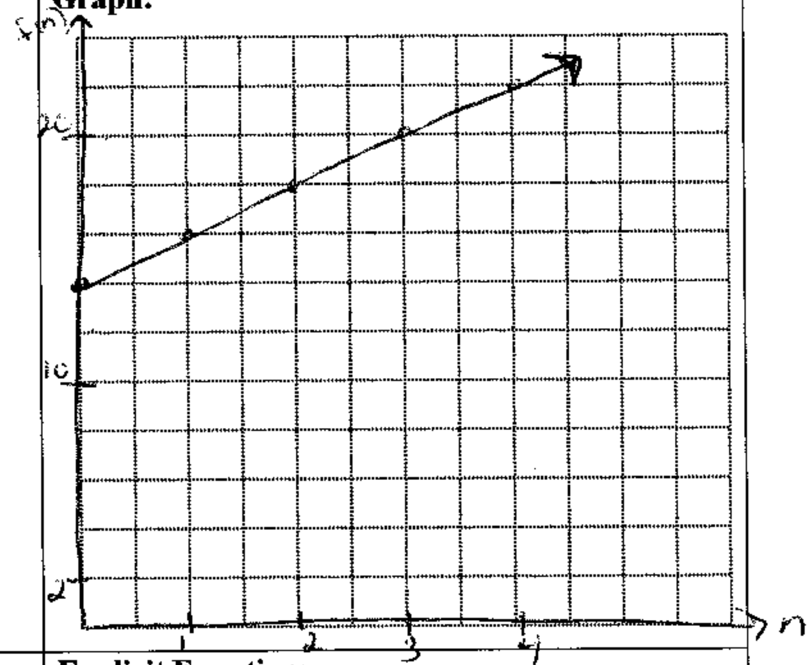
$a_1 = -31 \quad a_n = a_{n-1} - 7$

#34: Use the given information to complete the other representations for each arithmetic sequence.

Table:

0	1	2	3	4
14	16	18	20	22

Graph:



Recursive Equation:

$a_0 = 14$

$a_n = a_{n-1} + 2$

Explicit Equation:

$a_n = 2n + 14$

Create a context:

Janet wants to know how many seats are in each row of the theater. Jamal lets her know that each row has 2 seats more than the row in front of it. The first row has 14 seats.

#35-38: Complete the tables below so that the tables represent an arithmetic sequence.

35.

x	1	2	3
y	3	7.5	12

$d = 4.5$

36.

x	y
1	2
2	~~Ha~~ $19\frac{1}{3}$
3	$36\frac{2}{3}$
4	54

$54 = 2 + (4-1)d$

$54 = 2 + 3d$

$52 = 3d$

~~14=d~~ $d = 17\frac{1}{3}$

37.

x	y
1	5
2	12.5
3	20
4	27.5

$20 = 5 + (3-1)d$

$15 = 2d$

$7.5 = d$

38.

x	y
1	4
2	84
3	164
4	244
5	324

$324 = 4 + (5-1)d$

$320 = 4d$

$80 = d$

#39-40: Given two terms in an arithmetic sequence find the <u>recursive</u> formula.

39. $a_{18} = 3362$ and $a_{38} = 7362$

$d = 200 \quad a_1 = -38$

$\boxed{\begin{array}{l} a_1 = -38 \\ a_n = a_{n-1} + 200 \end{array}}$

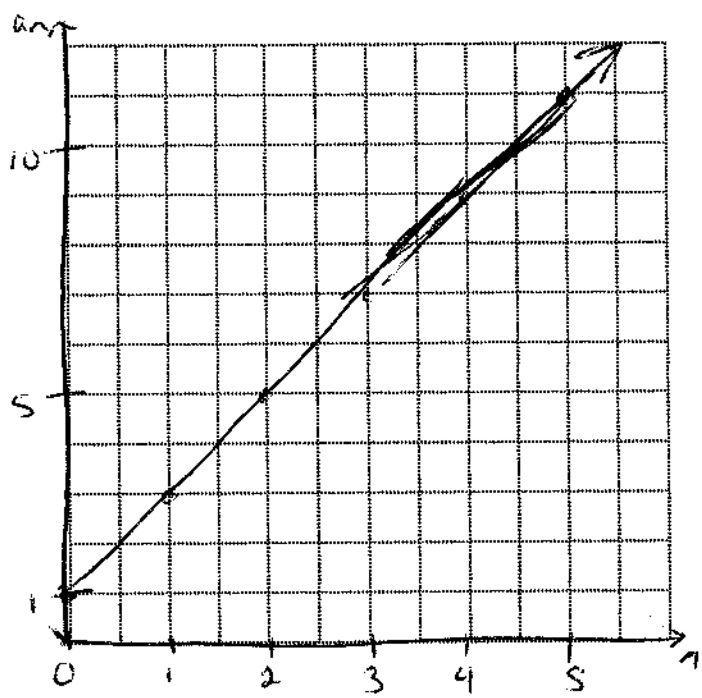
40. $a_{18} = 44.3$ and $a_{33} = 84.8$

$d = 2.7 \quad -1.6 = a_1$

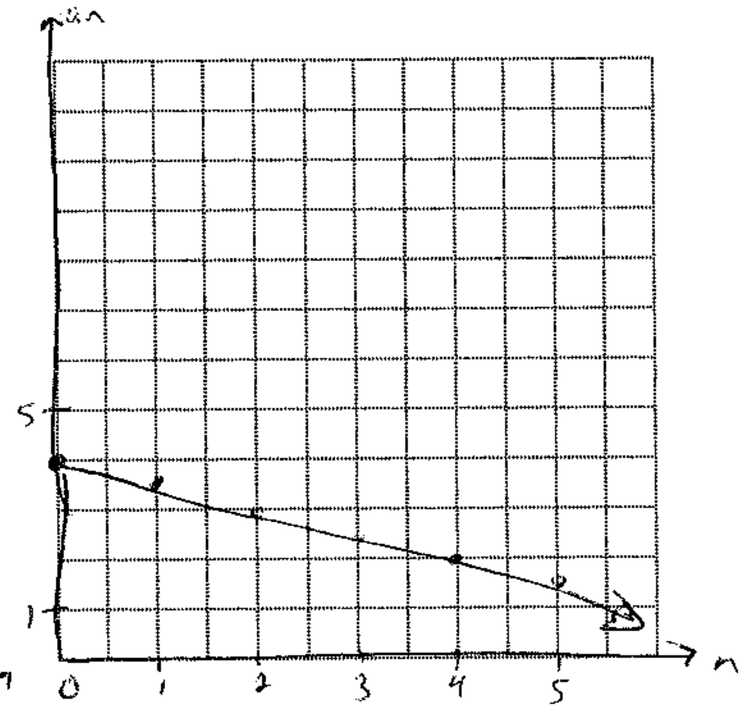
$\boxed{\begin{array}{l} a_1 = -1.6 \\ a_n = a_{n-1} + 2.7 \end{array}}$

#41-42: Graph the arithmetic sequence.

41. $a_n = 1 + 2n$



42. $a_n = 4 - \frac{1}{2}n$



43. The sum of the interior angles of a triangle is 180°, of a quadrilateral is 360° and a pentagon is 540°. As this pattern continues, find the sum of the interior angles of a dodecagon (12 sides).

# of sides	3	4	5	6	7	8	9	10	11	12
Sum of interior angles	180	360	540	720	900	1080	1260	1440	1620	1800

$\boxed{1800^\circ}$

Write an equation in <u>recursive</u> form (*note $n \geq 3$):

$a_3 = 180$

$a_n = a_{n-1} + 180$

Answer:

1800°

44. After knee surgery, your fitness instructor tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he tells you to increase that time by 6 minutes per week. Use this information to answer the questions.

a. Identify a_1 and d. Write a recursive form of the relationship.

$a_1 = 12$ $\quad$ $a_1 = 12$

$d = 6$ $\quad$ $a_n = a_{n-1} + 6$

b. What do a_1 and d represent in the problem?

$a_1 = 12$ means 12 minutes each day to start

$d = 6$ means increase of 6 min each week

c. How many weeks will it be before you are up to jogging 60 minutes per day?

$60 = 12 + (n-1)(6)$

$60 = 12 + 6n - 6$ $\quad$ $n = 9$ $\quad$ $\boxed{9 \text{ weeks}}$

$60 = 6 + 6n$

$54 = 6n$

45. Beginning May 1, a bookstore will give away 500 books as a promotion over two weeks. Each day, 20 books will be given away. The store is open 7 days a week.

Days of promotion	1	2	3	4	....	14
Posters remaining	500	480	460	440	....	?

a. How many books will the store have left at the end of 14 days?
$n = 14$

$a_{14} = 500 + (14-1)(-20)$

$a_{14} = 240$

$\boxed{240 \text{ Books}}$

b. Explain whether this sequence is arithmetic.

Yes, $d = -20$

KEY

9-D	*What's a Geometric Sequence?*

#1 – 2: Determine if the sequence is geometric. If it is, find the next __three terms__ in the sequence.

1. –3, –9, –27, –81, …

   Yes

   Identify the common ratio: 3

2. 4, 6, 10, 16, …

   no

   Identify the common ratio: X

#3 – 4: Given the explicit formula for a geometric sequence, find the __common ratio__ and the __first five terms__.

3. $a_n = 2 \cdot 4^{n-1}$

   r = 4

   2, 8, 32, 128, 512

4. $a_n = -5^{n-1}$

   r = 5

   –1, –5, –25, –125, –625

#5 – 6: Given the explicit formula for a geometric sequence, find the __common ratio__ and the __term named__ in the problem ($n \geq 1$)

5. $a_n = 3^{n-1}$

   Find a_{12} = 177147

   r = 3

6. $a_n = -2 \cdot 2^{n-1}$

   Find a_{10} = –1024

   r = 2

#7 – 8: Determine if the sequence is geometric. If it is, find the __explicit formula__ and the __recursive formula__ for the n^th term.

7. 144, 1446, 14466, 144666, …

   not Geometric

8. 3, 0, –3, –6, …

   not Geometric

Change

#9 – 10: Given the recursive formula for a geometric sequence, find the __common ratio__, the __8^th term__ and __the explicit formula__.

9. $a_n = a_{n-1} \cdot 3$

   $a_1 = -1$

   $\boxed{r = 3}$

   $\boxed{a_n = -1 \cdot 3^{n-1}}$

   $\boxed{a_8 = -2187}$

10. $a_{n+1} = a_n \cdot (-2)$

    $a_1 = 2$

    $\boxed{r = -2}$

    $\boxed{a_n = 2(-2)^{n-1}}$

    $\boxed{a_8 = -256}$

11. A ball is dropped from a height of 32m, and on each bounce it rises to seven-eighths of its previous height. How high does the ball rise on its fifth bounce?

$a_1 = 32$ $\quad a_5 = 32(7/8)^{5-1}$

$r = 7/8$

$\boxed{a_5 = \frac{2401}{128} \text{ or} \approx 18.758 \text{ m}}$

#12 – 13: Use the information given to answer each question.

12. Which of the two jobs described below will pay you the higher salary during the fifth year of employment? Explain your reasoning.

Job A: Make $20,000 the first year with annual raises of $1500. Arithmetic

Job B: Make $18,000 the first year with annual raises of 10% Geometric

Job B ; while starting off w/ $2,000 less, you get larger increases each year (exponential growth)

13. You borrow $100 from a relative and he offers two payment plans. Which plan will pay off the debt the fastest? Explain your reasoning.

Plan I – pay back $10 each month OR

Plan II – pay back 10% each previous month

this would never pay off the debt

Is this the purpose of the problem?

14. Looking back at problems **#12** and **#13**, which of these plans is geometric and why?

? rephrase wording

15. Looking back at problems **#12** and **#13**, which of these plans is arithmetic and why?

? rephrase wording

16. The men's college basketball tournament has 64 teams. In the first round, each team plays a game and half of the teams are eliminated.

a. Write a rule (choose recursive or explicit) for the number of games played in the n^{th} round.

$a_n = 64(1/2)^{n-1}$

b. For what values of *n* does the rule make sense?

$1 \le n \le 6$

#17-20: Use the following scenario to answer the given questions.

Bob Cooper was born in 1900. By 1930 he had 3 sons, all with the Copper last name. By 1960 each of Bob's 3 boys had exactly 3 sons of their own. By the end of each 30 year time period, the pattern of each Cooper boy having exactly 3 sons of their own continued.

17. How many Cooper sons were ~~born in the~~ 30 year period between 1960 and 1990?

This is a Geometric Series

maybe put this in 9-6

18. Create a diagram that would illustrate the Cooper family.

19. Predict how many Copper sons will be born between 1990 and 2020, if the pattern continues.

20. Write an equation that would help you predict the number of Cooper sons that would be born between 2020 and 2050. If you can't find the equation, explain it in words.

#21 – 24: Find the recursive and explicit (closed) equations for each geometric sequence.

21. 2, 4, 8, 16, …

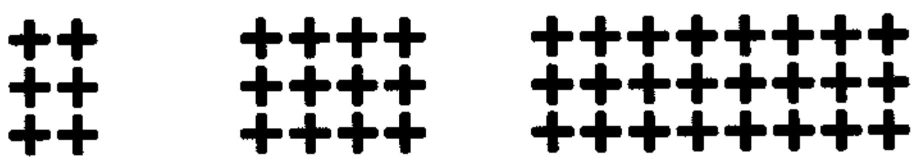
22.

Time (days)	Number of cells
1	3
2	6
3	12
4	24

23. Tania creates a chain letter and sends it to four friends. Each friend is then instructed to send it to four of their friends, and so forth.

	Recursive equation	Closed Equation
21.	$a_1 = 2$ $a_n = 2 \cdot a_{n-1}$	$a_n = 2(2)^{n-1}$
22.	$a_1 = 3$ $a_n = 2 \cdot a_{n-1}$	$a_n = 3(2)^{n-1}$
23.	$a_1 = 1$ $a_n = 4 \cdot a_{n-1}$	$a_n = 1 \cdot (4)^{n-1}$
24.	$a_1 = 6$ $a_n = 2 \cdot a_{n-1}$	$a_n = 6(2)^{n-1}$

24. Day 1 Day 2 Day 3



*Clarity: number of + symbols

#25 – 26: Find the missing values for each arithmetic or geometric sequence. Then answer the questions.

25. 5, 10, 15, 20, 25, 30, …

Does this sequence have a common difference or a common ratio? common difference

What is the value? 5

26. 20, 10, 5, 2.5, 1.25, …

Does this sequence have a common difference or a common ratio? common ratio

What is the value? 1/2

#27 – 32: Determine whether each situation represents an <u>arithmetic or geometric sequence</u> and then find the <u>recursive and explicit equation</u> for each.

27. 6, 12, 18, 24, …

arithmetic

recursive: $a_1 = 6$, $a_n = a_{n-1} + 6$

explicit: $a_n = 6n$

28.

Time (days)	Number of cells
1	5
2	8
3	12.8
4	20.48

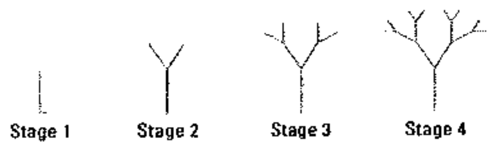
Geometric

recursive: $a_1 = 5$

$a_n = 1.6 \cdot a_{n-1}$

explicit: $a_n = 5(1.6)^{n-1}$

29.



? What are we counting here?
number of branches?

30. Michelle likes chocolate but she finds she has a mild allergy. She chooses to only eat 2 pieces every 3 days.

?? How is this a sequence? What ~~are we~~ is the input/output?

31. Scott decides to add running to his exercise routine and runs a total of one mile his first week. He plans to double the number of miles he runs each week.

Geometric recursive: $a_1 = 1$, $a_n = 2 \cdot a_{n-1}$ explicit: $a_n = 1 \cdot (2)^{n-1}$

32. Vanessa has \$60 to spend on rides at the State Fair. Each ride cost \$4. Reword

33. Adella bought a car for \$10,000. One year later, the car was worth \$8,000. One year later, the car was worth \$6,400. The pattern continues, and the next year the car was worth \$5,120.

? What are we trying to find?

#34 – 38: For each of the following tables:

- describe how to find the next term in the sequence.
- write a recursive rule for the function.
- describe how the features identified in the recursive rule can be used to write an explicit rule for the function, and
- write an explicit rule for the function.
- identify if the function is arithmetic, geometric or neither.

Example:

x	y
0	5
1	8
2	11
3	14
4	?
…	…
n	?

a) To find the next term: **add 3 to the previous term**

b) Recursive rule: $\boldsymbol{a_1 = 5,\ a_n = a_{n-1} + 3}$

c) To find the n^{th} term: **start with 5 and add 3 *n* times**

d) Explicit rule: $\boldsymbol{a_n = 5 + 3n}$

e) Arithmetic, geometric, or neither? **Arithmetic**

34.

x	y
1	5
2	10
3	20
4	40
5	?
…	…
n	?

a) To find the next term: multiply 2 to previous term

b) Recursive rule: $a_1 = 5,\ a_n = 2 \cdot a_{n-1}$

c) To find the nth term: start w/ 5, multiply by 2^{n-1}

d) Explicit rule: $a_n = 5(2)^{n-1}$

e) Arithmetic, geometric, or neither? Geometric

35.

x	y
1	–8
2	–17
3	–26
4	–35
5	–44
6	–53
7	?
…	…
n	?

a) To find the next term: Subtract 9 to previous term

b) Recursive rule: $a_1 = -8,\ a_n = a_{n-1} - 9$

c) To find the n^{th} term: start w/ 1 and add $-9n$

d) Explicit rule: $a_n = 1 - 9n$

e) Arithmetic, geometric, or neither? arithmetic

36.

supposed to be 11?

x	y
0	3
1	4
2	7
3	12
4	19
5	?
…	…
n	?

a) To find the next term: ______

b) Recursive rule: ______

c) To find the nth term: ______

d) Explicit rule: ______

e) Arithmetic, geometric, or neither? ______

37.

x	y
1	2
2	–6
3	18
4	–54
5	162
6	–486
…	…
n	?

a) To find the next term: multiply -3 to previous term

b) Recursive rule: $a_1 = 2$, $a_n = -3 \cdot a_{n-1}$

c) To find the nth term: Start w/ 2, multiply by $(-3)^{n-1}$

d) Explicit rule: $a_n = 2(-3)^{n-1}$

e) Arithmetic, geometric, or neither? Geometric

38.

x	y
1	10
2	2
3	$\frac{2}{5}$
4	$\frac{2}{25}$
5	$\frac{2}{125}$
6	$\frac{2}{625}$
…	…
n	?

a) To find the next term: multiply by 1/5 to previous term

b) Recursive rule: $a_1 = 10$, $a_n = \frac{1}{5} \cdot a_{n-1}$

c) To find the nth term: start w/ 10, multiply by $(1/5)^{n-1}$

d) Explicit rule: $a_n = 10(1/5)^{n-1}$

e) Arithmetic, geometric, or neither? Geometric

39. How are arithmetic and geometric sequences similar?

?

40. How are arithmetic and geometric sequences different?

Add vs. multiply

common difference

common ratio

9-E *What is a Series?*

1. How is a sequence different than a series?

list of terms

sum of n amount of term

2. Write an example of a sequence. Write an example of a series.

Sequence: 1, 2, 3, 4, ...

Series: 1+2+3+4

★ All Sigma Notation here is Geometric need some Arithmetic

#3-6: Determine the number of terms in the series:

ok ?

3. $\sum_{k=1}^{7} 2(3)^{k-1}$

$\boxed{7}$

4. $\sum_{k=4}^{20} 4\left(\frac{1}{2}\right)^{k-1}$

$\boxed{17}$

5. $\sum_{k=-3}^{10} 2^{k-5}$ $\boxed{13}$

6. $\sum_{k=a}^{b} 2(3)^{k-2}$

$(b-a)+1$

#7-9: Determine the given term of the series:

7. $\sum_{k=3}^{5} 2(3)^{k-1}$ find t_1 ~~first term~~ ~~Set~~ ~~first term~~

$\boxed{18}$

8. $\sum_{k=-2}^{7} 18\left(\frac{1}{2}\right)^{k}$ find t_2

ok ?

$\boxed{18}$

9. $\sum_{k=a}^{b} 2\left(\frac{1}{2}\right)^{k-1}$ find t_3

$2\left(\frac{1}{2}\right)^{a+3-1} = \boxed{2\left(\frac{1}{2}\right)^{a+2}}$

#10-11: Expand the following:

10. $\sum_{k=1}^{5} 5(-2)^{k-1}$

$5 + (-10) + 20 + (-40) + 80$

11. $\sum_{k=4}^{7} 3\left(\frac{1}{2}\right)^{k-2}$

$\frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{32}$

#12-15: Expand and evaluate the following:

12. $\sum_{i=1}^{6} 5\left(\frac{1}{3}\right)^{i-1}$

$\frac{1820}{243}$

13. $\sum_{m=8}^{20} \frac{1}{16}(2)^{m}$

131056

14. $\sum_{k=-2}^{5} 4(3)^{k-1}$

$\approx 1,185$

15. $\sum_{b=5}^{12} 5(2)^{1-b}$

≈ 0.623

#16-19: Write the following using summation notation:

16. $-2+4-8+16...+1024$

How do we expect students to solve for n before learning how to solve exponential equations?

17. $5-\frac{5}{4}+\frac{5}{16}-\frac{5}{64}+\frac{5}{256}-\frac{5}{1024}$

18. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\cdots$

Have not learned infinite series

19. $1+2+3+4+\cdots$

cannot be evaluated

no such thing as infinite arithmetic series

20. Solve the following equation: $\sum_{k=0}^{2} x^{k} = 36$

$x^0 + x + x^2 = 36$

$1 + x + x^2 = 36$

$x^2 + x - 35 = 0$

irrational solutions, do we want that?

21. A clock chimes once when the clock strikes 1, twice when the clock strikes 2, and so on. How many times does the clock chime in 12 hours? In a 24-hour day? Express both situations with summation notation.

12 hours:

$\sum_{n=1}^{12} n = \boxed{78}$

24 hours:

$\sum_{n=1}^{24} n = \boxed{300}$

9-F *What's a Partial Sum of an Arithmetic Series?*

#1 – 3: Write the expanded form of the arithmetic series. Show the first 3 and last 3 terms.

1. $\sum_{n=1}^{20} 3n - 2$

$1+4+7+\dots+52+55+58$

2. $\sum_{n=1}^{45} -n+8$

$7+6+5+\dots+(-35)+(-36)+(-37)$

3. $\sum_{n=1}^{70} \frac{1}{2}n+10$

$10.5+11+11.5+\dots+44+44.5+45$

#4 – 9: Find the sum of each finite series.

4. $2+3+4+\dots+23+24+25$

$25=2+(n-1)(1)$
$25=2+n-1$
$25=n+1$
$24=n$

$S_{24}=\frac{24}{2}(2+25)$

$\boxed{S_{24}=324}$

5. $9+10+11+\dots+47+48+49$

$49=9+(n-1)(1)$
$49=9+n-1$
$49=n+8$
$41=n$

$S_{41}=\frac{41}{2}(9+49)$

$\boxed{S_{41}=1189}$

6. $32+35+38+\dots+65+68+71$

$71=32+(n-1)(3)$
$71=32+3n-3$
$71=3n+29$
$42=3n$
$14=n$

$S_{14}=\frac{14}{2}(32+71)$

$\boxed{S_{14}=721}$

7. $13+14+15+\dots+50+51+52$

$52=13+(n-1)(1)$
$52=13+n-1$
$52=n+12$
$40=n$

$S_{40}=\frac{40}{2}(13+52)$

$\boxed{S_{40}=1300}$

8. $-2+0+2+\dots+28+30+32$

$32=-2+(n-1)(2)$
$32=-2+2n-2$
$32=2n-4$
$36=2n$
$18=n$

$S_{18}=\frac{18}{2}(-2+30)$

$\boxed{S_{18}=270}$

9. $58+55+52+\dots+19+16+13$

$13=58+(n-1)(-3)$
$13=58-3n+3$
$13=-3n+61$
$-48=-3n$
$16=n$

$S_{16}=\frac{16}{2}(58+13)$

$\boxed{S_{16}=568}$

#10 – 12: Write each series in summation notation.

10. $8+9+10+\dots+20+21+22$

$22=8+(n-1)(1)$
$22=8+n-1$
$22=n+7$
$15=n$

$\boxed{\sum_{n=1}^{15}(n+7)}$

11. $1+3+5+\dots+23+25+27$

$27=1+(n-1)(2)$
$27=1+2n-2$
$27=2n-1$
$28=2n$
$14=n$

$\boxed{\sum_{n=1}^{14}(2n-1)}$

12. $\frac{1}{2}+\frac{3}{2}+\frac{5}{2}+\dots+\frac{11}{2}+\frac{13}{2}+\frac{15}{2}$

$\frac{15}{2}=\frac{1}{2}+(n-1)(1)$
$\frac{15}{2}=\frac{1}{2}+n-1$
$\frac{15}{2}=n-\frac{1}{2}$
$8=n$

$\boxed{\sum_{n=1}^{8}(n-\frac{1}{2})}$

#13 – 15: For each situation, write an equation in sigma notation that models each problem. Solve each problem.

13. Andrew wants to add all the digits 1-100 quickly. Write the summation notation for how Andrew can add these numbers. Find the sum.

$$\sum_{n=1}^{100} n \qquad 5050$$

14. The front row of a theater has 25 seats. Each of the other rows has 3 more seats than the row before it. What is the total number of seats in the first 15 rows?

$25 + 28 + \cdots + \underset{n=15}{\boxed{67}}$

$a_{15} = 25 + (15-1)(3)$

$a_{15} = 67$

$S_{15} = \frac{15}{2}(25+67)$

$\boxed{S_{15} = 690 \text{ seats}}$

15. Robert's father pays him to do lawn work. He pays Robert 40 cents for the first hour of work and then increases his pay by $1 every hour. At the end of 20 hours, how much did Robert earn?

$a_{20} = 0.4 + (20-1)(1)$

$a_{20} = 0.4 + 19$

$a_{20} = 19.4$

$S_{20} = \frac{20}{2}(0.4 + 19.4)$

$\boxed{S_{20} = \$198}$

KEY

9-G *What's a Partial Sum of a Geometric Series?*

#1 – 4: For each of the following:

a. Expand each Geometric Series below.

b. Then find the sum.

1. $\sum_{i=1}^{5} 1(4)^{i-1}$

   a. expand: $1+4+16+64+256$

   b. sum: 341

2. $\sum_{m=1}^{7} 4(3)^{m+2}$

   a. expand: $108+324+972+2916+8748+26244+78732$

   b. sum: 118044

3. $\sum_{n=1}^{3} 2\left(\frac{1}{2}\right)^{2n}$

   a. expand: $\frac{1}{2}+\frac{1}{8}+\frac{1}{32}$

   b. sum: $\frac{21}{32}$

4. $\sum_{i=1}^{12} 8(8)^{\frac{i}{2}}$

   a. expand:

   b. sum:

Change i too messy

#5 – 8: For each of the following:

a. Classify each of the series below as arithmetic, geometric, or neither.

b. Plot (graph) $S_1, S_2, S_3, S_4,$ and S_5 for each series below.

c. Are the graphs most similar to a linear, quadratic, or exponential equation?

5. $S_n = 1+(-2)+(-5)+(-8)+\cdots (-11)$

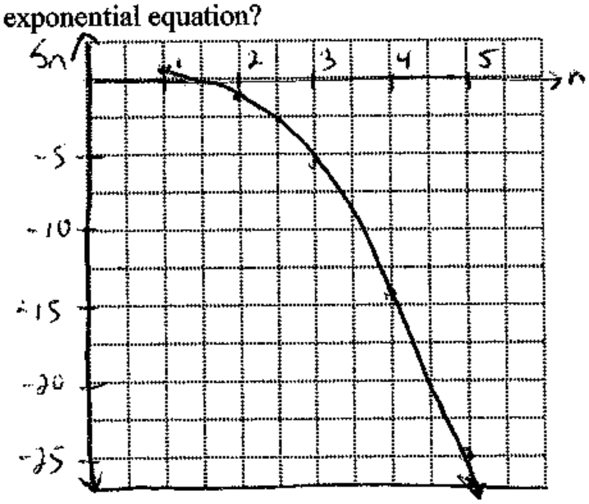
$S_1 = 1$

$S_2 = -1$

$S_3 = -6$

$S_4 = -14$

$S_5 = -25$



6. $S_n = 4 + 8 + 16 + 32 + 64 + \cdots$

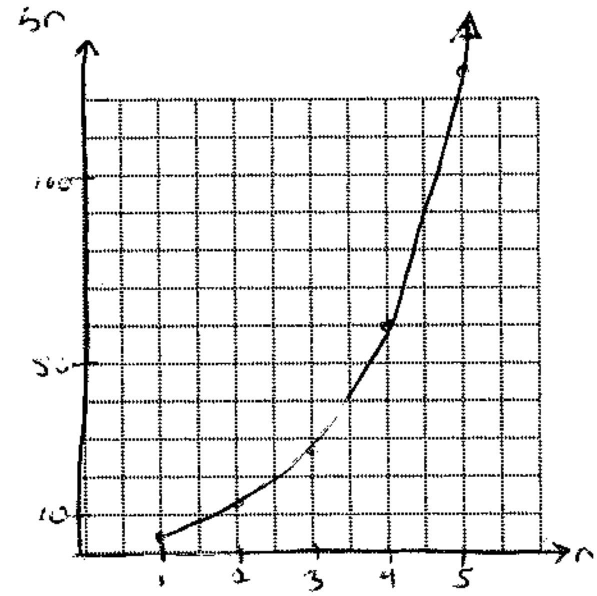
$S_1 = 4$

$S_2 = 12$

$S_3 = 28$

$S_4 = 60$

$S_5 = 128$



7. $S_n = 5 + 7 + 4 + 8 + 5 + \cdots$? Is this right?

$S_1 = 5$

$S_2 = 12$

$S_3 =$

8. $S_n = 216 + 36 + 6 + \frac{1}{6} + \cdots$

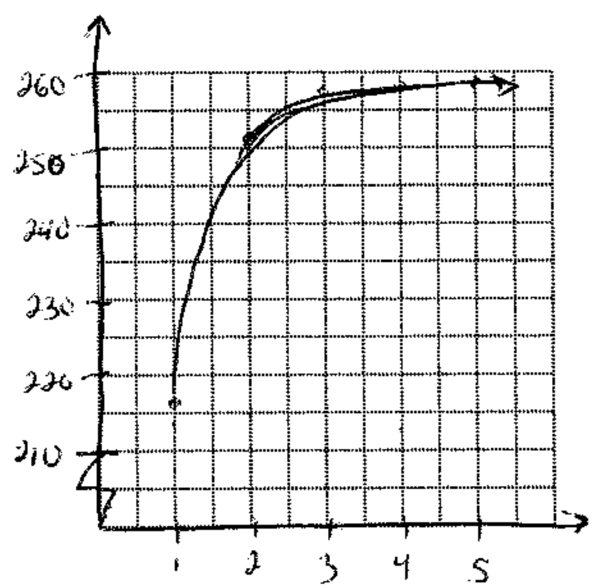
$S_1 = 216$

$S_2 = 252$

$S_3 = 258$

$S_4 = 258.16$

$S_5 = 258.194$



9. **Complete the table and then answer the question at the bottom of the page.**

	$\sum_{i=1}^{5} 3(r)^i$	$\sum_{i=1}^{10} 3(r)^i$	$\sum_{i=1}^{50} 3(r)^i$	$\sum_{i=1}^{100} 3(r)^i$
$r = -2$	-66	2046	2.25×10^{15}	2.53×10^{30}
$r = -1$	-3	0	0	0
$r = -\frac{1}{2}$	-1.03125	-0.999	-1	-1
$r = 0$	0	0	0	0
$r = \frac{1}{2}$	2.90625	2.997	3	3
$r = 1$	15	30	150	300
$r = 2$	186	6138	6.75×10^{15}	7.6×10^{30}

Identify patterns you see within the results of this table.

when

$-1 < r < 1$ the sum converges to a constant

Complete the table below. Find the ending balances for each row in the table.

Investment	Time	Percent Interest	Compounding Frequency	Ending Balance (write down your thinking of how to get to the total)
$7,300	5 years	7%	annually	$10,238.63
$7,300	5 years	7%	semi-annually	$10,297.37
$7,300	5 years	7%	quarterly	$10,327.88
$7,300	5 years	7%	monthly	$10,348.66
$7,300	5 years	7%	weekly	$10,356.76
$7,300	5 years	7%	daily	$10,358.85
$7,300	5 years	7%	Choose a value larger than 500	1000 $10,359.06

10. What do you notice about these ending balances?

The amount is leveling off, approaching a max amount
the larger the frequency, more money

11. Where do you want to go to college?

a. The average cost of a bachelor's degree in Minnesota is $14, 000 per year. Use this information to answer part B.

b. If you deposited money in a bank account today, receiving 8% interest annually, how much would you need to deposit to pay for college expenses based on the cost in part A?

~~14000~~

For one year
of college?
Be clear for time

KEY

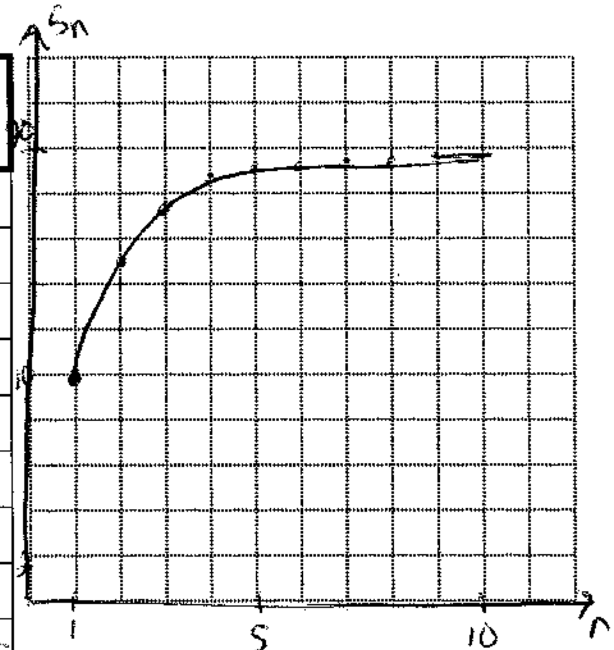
9-H *Return to Sequences... What's an Infinite Sum?*

#1 – 7: Complete each table – find the value of each term, find the partial sum, graph each partial sum and answer the question.

1.

$$a_n = 10\left(\frac{1}{2}\right)^{n-1}$$

a_1	10	S_1	10
a_2	5	S_2	15
a_3	2.5	S_3	17.5
a_4	1.25	S_4	18.75
a_5	0.625	S_5	19.375
a_6	0.3125	S_6	19.6875
a_7	0.15625	S_7	19.84375
a_8	0.078125	S_8	19.921875
a_9	0.0390625	S_9	19.9609375
a_{10}	0.01953125	S_{10}	19.98046875

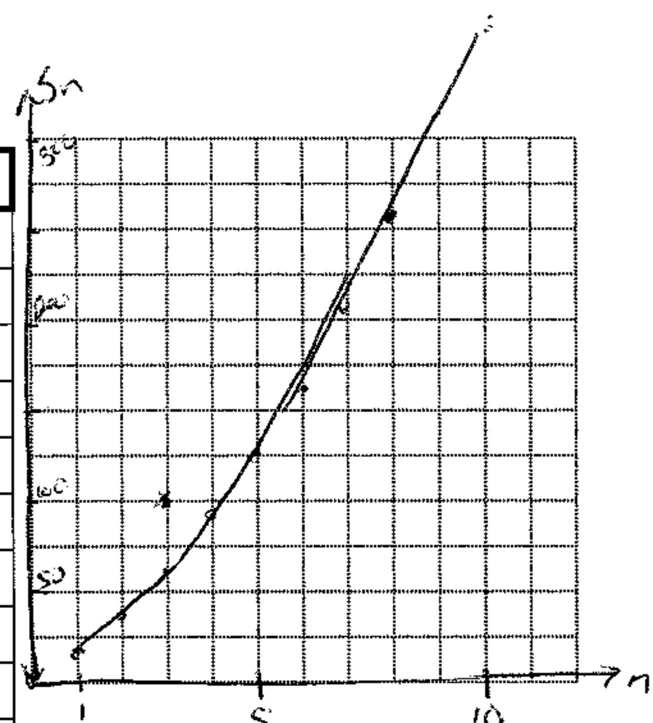


Does the series above diverge or converge?

2.

$$a_n = 10 + 5n$$

a_1	15	S_1	15
a_2	20	S_2	35
a_3	25	S_3	60
a_4	30	S_4	90
a_5	35	S_5	125
a_6	40	S_6	165
a_7	45	S_7	210
a_8	50	S_8	260
a_9	55	S_9	315
a_{10}	60	S_{10}	375



Does the series above diverge or converge?

3.

$a_n = 3(5)^{n-1}$			
a_1		S_1	
a_2		S_2	
a_3		S_3	
a_4		S_4	
a_5		S_5	
a_6		S_6	
a_7		S_7	
a_8		S_8	
a_9		S_9	
a_{10}		S_{10}	

Does the series above diverge or converge?

4.

$a_n = n^2$			
a_1		S_1	
a_2		S_2	
a_3		S_3	
a_4		S_4	
a_5		S_5	
a_6		S_6	
a_7		S_7	
a_8		S_8	
a_9		S_9	
a_{10}		S_{10}	

Does this series diverge or converge as n gets larger and larger $(n \to \infty)$?

5.

$a_n = 9(2)^{n-1}$			
a_1		S_1	
a_2		S_2	
a_3		S_3	
a_4		S_4	
a_5		S_5	
a_6		S_6	
a_7		S_7	
a_8		S_8	
a_9		S_9	
a_{10}		S_{10}	

Does this series diverge or converge as n gets larger and larger $(n \to \infty)$?

$a_n = 2n + 5$

6.

a_1		S_1	
a_2		S_2	
a_3		S_3	
a_4		S_4	
a_5		S_5	
a_6		S_6	
a_7		S_7	
a_8		S_8	
a_9		S_9	
a_{10}		S_{10}	

Does this series diverge or converge as n gets larger and larger $(n \to \infty)$?

7.

$$a_n = 12\left(\frac{1}{3}\right)^{n-1}$$

a_1		S_1	
a_2		S_2	
a_3		S_3	
a_4		S_4	
a_5		S_5	
a_6		S_6	
a_7		S_7	
a_8		S_8	
a_9		S_9	
a_{10}		S_{10}	

Does this series diverge or converge as n gets larger and larger $(n \to \infty)$?

8.

$$a_n = \frac{1}{n(n+1)}$$

a_1		S_1	
a_2		S_2	
a_3		S_3	
a_4		S_4	
a_5		S_5	
a_6		S_6	
a_7		S_7	
a_8		S_8	
a_9		S_9	
a_{10}		S_{10}	

Does this series diverge or converge as n gets larger and larger $(n \to \infty)$?