Lesson 12-1 Find the *n*th term and arithmetic means of an arithmetic sequence.

Find the 35th term in the arithmetic sequence $-5, -1, 3, \ldots$.
Begin by finding the common difference d.
$d = -1 - (-5)$ or 4
Use the formula for the *n*th term.
$a_n = a_1 + (n - 1)d$
$a_{35} = -5 + (35 - 1)(4)$ or 131

11. Find the next four terms of the arithmetic sequence 3, 4.3, 5.6, ... **6.9, 8.2, 9.5, 10.8**

12. Find the 20th term of the arithmetic sequence for which $a_1 = 5$ and $d = -3$. **−52**

13. Form an arithemetic sequence that has three arithmetic means between 6 and −4. **6, 3.5, 1, −1.5, −4**

Lesson 12-1 Find the sum of *n* terms of an arithmetic series.

The sum S_n of the first n terms of an arithmetic series is given by
$S_n = \frac{n}{2}(a_1 + a_n)$.

14. What is the sum of the first 14 terms in the arithmetic series $-30 - 23 - 16 - \cdots$? **217**

15. Find n for the arithmetic series for which $a_1 = 2$, $d = 1.4$, and $S_n = 250.2$. **18**

Lesson 12-2 Find the *n*th term and geometric means of a geometric sequence.

Find an approximation for the 12th term of the sequence $-8, 4, -2, 1, \ldots$.
First, find the common ratio.
$a_2 \div a_1 = 4 \div (-8)$ or -0.5
Use the formula for the *n*th term.
$a_{12} = -8(-0.5)^{12-1}$ *$a_n = a_1r^{n-1}$*
$= -8(-0.5)^{11}$ or about 0.004

16. Find the next three terms of the geometric sequence 49, 7, 1, ... $\mathbf{\frac{1}{7}, \frac{1}{49}, \frac{1}{343}}$

17. Find the 15th term of the geometric sequence for which $a_1 = 2.2$ and $r = 2$.

18. If $r = 0.2$ and $a_7 = 8$, what is the first term of the geometric sequence? **125,000**

19. Write a geometric sequence that has three geometric means between 0.2 and 125.
0.2, ±1, 5, ±25, 125   **17. 36,044.8**

Lesson 12-2 Find the sum of *n* terms of a geometric series.

Find the sum of the first 12 terms of the geometric series $4 + 10 + 25 + 62.5 + \cdots$.
First find the common ratio.
$a_2 \div a_1 = 10 \div 4$ or 2.5
Now use the formula for the sum of a finite geometric series.

$$S_n = \frac{a_1 - a_1r^n}{1 - r}$$

$$S_{12} = \frac{4 - 4(2.5)^{12}}{1 - 2.5} \quad n = 12, a_1 = 4, r = 2.5$$

$S_{12} \approx 158{,}943.05$ *Use a calculator.*

20. What is the sum of the first nine terms of the geometric series $1.2 - 2.4 + 4.8 - \cdots$? **205.2**

21. Find the sum of the first eight terms of the geometric series $4 + 4\sqrt{2} + 8 + \cdots$.
$\mathbf{60(1 + \sqrt{2})}$

OBJECTIVES AND EXAMPLES

Lesson 12-3 Find the limit of the terms and the sum of an infinite geometric series.

Find $\lim_{n\to\infty} \frac{2n^2+5}{3n^2}$.

$$\lim_{n\to\infty} \frac{2n^2+5}{3n^2} = \lim_{n\to\infty}\left(\frac{2}{3}+\frac{5}{3n^2}\right)$$
$$= \lim_{n\to\infty}\frac{2}{3} + \lim_{n\to\infty}\frac{5}{3}\cdot\lim_{n\to\infty}\frac{1}{n^2}$$
$$= \frac{2}{3}+\frac{5}{3}\cdot 0$$

Thus, the limit is $\frac{2}{3}$.

REVIEW EXERCISES

Find each limit, or state that the limit does not exist and explain your reasoning.

22. $\lim_{n\to\infty}\frac{3n}{4n+1}$ $\frac{3}{4}$

23. $\lim_{n\to\infty}\frac{6n-3}{n}$ 6

24. $\lim_{n\to\infty}\frac{2^n n^3}{3n^3}$ See margin.

25. $\lim_{n\to\infty}\frac{4n^3-3n}{n^4-4n^3}$ 0

26. Write $5.\overline{123}$ as a fraction. $5\frac{41}{333}$

27. Find the sum of the infinite series $1260 + 504 + 201.6 + 80.64 + \cdots$, or state that the sum does not exist and explain your reasoning. 2100

Lesson 12-4 Determine whether a series is convergent or divergent.

Use the ratio test to determine whether the series $3 + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!}$ is convergent or divergent.

The nth term a_n of this series has a general form of $\frac{3^n}{n!}$ and $a_{n+1} = \frac{3^{n+1}}{(n+1)!}$. Find $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}$.

$$r = \lim_{n\to\infty}\frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}}$$
$$r = \lim_{n\to\infty}\left[\frac{3^{n+1}}{(n+1)!}\cdot\frac{n!}{3^n}\right]$$
$$r = \lim_{n\to\infty}\frac{3}{n+1} \text{ or } 0$$

Since $r < 0$, the series is convergent.

28. Use the ratio test to determine whether the series $\frac{1}{5} + \frac{2^2}{5^2} + \frac{3^2}{5^3} + \frac{4^2}{5^4} + \cdots$ is *convergent* or *divergent*. convergent

29. Use the comparison test determine whether the series $\frac{6}{1} + \frac{7}{2} + \frac{8}{3} + \frac{9}{4} + \cdots$ is *convergent* or *divergent*. divergent

30. Determine whether the series $2 + 1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3} + \frac{2}{7} + \cdots$ is *convergent* or *divergent*. divergent

Lesson 12-5 Use sigma notation.

Write $\sum_{n=1}^{3}(n^2-1)$ in expanded form and then find the sum.

$$\sum_{n=1}^{3}(n^2-1) = (1^2-1)+(2^2-1)+(3^2-1)$$
$$= 0 + 3 + 8 \text{ or } 11$$

Write each expression in expanded form and then find the sum.

31. $\sum_{a=5}^{9}(3a-3)$ 90

32. $\sum_{k=1}^{\infty}(0.4)^k$ $\frac{2}{3}$

31–32. See margin for expanded form.

Express each series using sigma notation.

33. $-1 + 1 + 3 + 5 + \cdots$ $\sum_{a=0}^{\infty}(2n-1)$

34. $2 + 5 + 10 + 17 + \cdots + 82$ $\sum_{a=1}^{9}(n^2+1)$

Additional Answ

24. Does not exist; $\lim \frac{2^n n^3}{3n^3} = \lim \frac{2^n}{3}$; $\lim \frac{2^n}{3}$ becomes i[...] large as n approa[...] the sequence ha[...]

31. $(3\cdot5-3)+(3\cdot6$ $(3\cdot7-3)+(3\cdot8$ $(3\cdot9-3)$

32. $(0.4)^1 + (0.4)^2 + (0$

Find the fourth term of $(2x - y)^6$.

$$(2x - y)^6 = \sum_{r=0}^{6} \frac{6!}{r!(6 - r)!}(2x)^{6 - r}(-y)^r$$

To find the fourth term, evaluate the general term for $r = 3$.

$$\frac{6!}{3!(6 - 3)!}(2x)^{6 - 3}(-y)^3$$

$$= \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 3!}(2x)^3(-y^3) \text{ or } -160x^3y^3$$

35. $(a - 4)^6$

36. $(2r + 3$

Find the designated term of each b
expansion.

37. 5th term of $(x - 2)^{10}$ **3360x^6**

38. 3rd term of $(4m + 1)^8$ **114,688**

39. 8th term of $(x + 3y)^{10}$ **262,440**

40. 6th term of $(2c - d)^{12}$ **−101,3**

Lesson 12-7 Use Euler's Formula to write the exponential form of a complex number.

Write $\sqrt{3} - i$ in exponential form.

Write the polar form of $\sqrt{3} - i$.

$r = \sqrt{(\sqrt{3})^2 + (-1)^2}$ or 2, and

$\theta = \text{Arctan } \frac{-1}{\sqrt{3}}$ or $\frac{5\pi}{6}$

$$\sqrt{3} - i = 2\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right) = 2e^{i\frac{5\pi}{6}}$$

Write each expression or complex
exponential form.

41. $2\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$ **$2e^{i\frac{3\pi}{4}}$**

42. $4i$ **$4e^{i\frac{\pi}{2}}$**

43. $2 - 2i$ **$2\sqrt{2}e^{i\frac{7\pi}{4}}$**

44. $3\sqrt{3} + 3i$ **$6e^{i\frac{\pi}{6}}$**

Lesson 12-8 Iterate functions using real and complex numbers.

Find the first three iterates of the function $f(z) = 2z$ if the initial value is $3 - i$.

$z_0 = 3 - i$

$z_1 = 2(3 - i)$ or $6 - 2i$

$z_2 = 2(6 - 2i)$ or $12 - 4i$

$z_3 = 2(12 - 4i)$ or $24 - 8i$

Find the first four iterates of each f
using the given initial value. If neces
your answers to the nearest hundre

45. $f(x) = 6 - 3x, x_0 = 2$ **0, 6, −12,**

46. $f(x) = x^2 + 4, x_0 = -3$ **13; 173; 2**
895,984,493

Find the first three iterates of the fu
$f(z) = 0.5z + (4 - 2i)$ for each initia

47. $z_0 = 4i$

48. $z_0 = -8$

49. $z_0 = -4 + 6i$

50. $z_0 = 12$ -

47–50. See margin.

Lesson 12-9 Use mathematical induction to prove the validity of mathematical statements.

Proof by mathematical induction:

1. First, verify that the conjecture S_n is valid for the first possible case, usually $n = 1$.
2. Then, assume that S_n is valid for $n = k$ and use this assumption to prove that it is also valid for $n = k + 1$.

Use mathematical induction to prove
proposition is valid for all positive int
values of n. **51–53. See Answer Appe**

51. $1 + 2 + 3 + \cdots + n = \frac{n(n + 1)}{2}$

52. $3 + 8 + 15 + \cdots + n(n - 2) = \frac{n(n +}{}$

53. $9^n - 4^n$ is divisible by 5.