Name: ____________________________ Unit 4 Topic 2

z Test for Means

- Many hypotheses are tested using a statistical test based on the following general formula:

$$Test\ Value = \frac{(observed\ value) - (expected\ value)}{standard\ error}$$

- This looks a lot like our z-formula…

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

  - z = test value
  - $\bar{X}$ = observed value
  - μ = expected value
  - $\sigma/\sqrt{n}$ = standard error

Example 1

- A researcher reports that the average salary of assistant professors is more than $42,000. A sample of 30 assistant professors has a mean salary of $43,260. At alpha=0.05, test the claim that the assistant professors each more than $42,000 per year. The standard deviation of the population is $5230.
  - Step 1: State the hypothesis and identify the claim
  - Step 2: Find the critical value(s).
  - Step 3: Compute the test value.
  - Step 4: Make the decision to reject or not reject the null hypothesis.
  - Step 5: Summarize results.

Example 2

- A researcher claims that the average cost of men's athletic shoes is less than $80. He selects a random sample of 36 pairs of shoes from a catalog and finds the average cost of the sample is 75. Assume the standard deviation is 19.2 for the population. Is there enough evidence to support the researcher's claim at alpha=0.10?

Example 3

- The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is $24,672. To see if the average cost of rehabilitation is different at a particular hospital, a researcher selects a random sample of 35 stroke victims at the hospital and finds that the average cost of their rehabilitation is $25,226. The standard deviation of the population is $3251. At alpha=0.01, can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from $24,672?

Outcomes of a Hypothesis-Testing Situation

P-value Method for Hypothesis Testing

- Statisticians usually test hypotheses at the common alpha levels of 0.05 or 0.01 and sometimes 0.10. Recall that the choice of the level depends on the seriousness of the type I error. Besides listing an alpha value, many computer statistical packages give a P-value for the hypothesis tests.
- The **P-value** (or probability value) is the probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true.
- In other words, the P-value is the actual area under the standard normal distribution curve (or other curve, depending on what test is being used) representing the probability of a particular sample statistic or a more extreme sample statistic occurring if the null hypothesis is true.
- For example, suppose that an alternative hypothesis is that the mean is greater than 50 and the mean of the sample is 52. If the computer printed a P-value of 0.0356 for a statistical test, then the probability of getting a sample mean of 52 or greater is 0.0356 if the true population mean is 50 (for a given sample and standard deviation).
    - The relationship between the P-value and alpha can be explained in this manner. For P=0.0356, the null hypothesis would be rejected at 0.05, but not at 0.01.
- For a two-tailed test, the area in onw tail must be doubled. If alpha is 0.05 and the area in one tail is 0.0356, the P-value will be 2(0.0356) = 0.0712. That is, the null hypothesis should not be rejected at 0.05, since 0.0712 is greater than 0.05.
- ***In summary, then,
    - If the P-value is less than alpha, reject the null hypothesis.
    - If the P-value is greater than alpha, do not reject the null hypothesis.

Procedure for Solving Hypothesis-Testing Problems (P-Value Method)

- **Step 1:** State the hypotheses and identify the claim.
- **Step 2:** Compute the test value.
- **Step 3:** Find the P-value.
- **Step 4:** Make the decision.
- **Step 5:** Summarize the results.

Example 1

- A researcher wishes to test the claim that the average cost of tuition and fees at a four-year public college is greater than $5700. She selects a random sample of 36 four-year public colleges and find the mean to be $5950. The population standard deviation is $659. Is there evidence to support the claim at alpha=0.05? Use the P-value method.

Example 2

- A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 mile per hour. At alpha=0.05, is there enough evidence to reject the claim? Use the P-value method.

Decision Rule When Using P-Value

- If $P - value \leq \alpha$, reject the null hypothesis.
- If $P - value > \alpha$, do not reject the null hypothesis.

Guidelines for P-Values

- If P-value ≤ 0.01, reject the null hypothesis. The difference is highly significant.
- If P-value > 0.01, but P-value ≤ 0.05, reject the null hypothesis. The difference is significant.
- If P-value > 0.05, but P-value ≤ 0.10, consider the consequences of type I error before rejecting the null hypothesis.
- If P-value > 0.10, do not reject the null hypothesis. The difference is not significant.