Unit 4b Topic 1: Introduction to Hypothesis Testing

- **Hypothesis Testing** – a decision making process for evaluating claims about a population.
- Researcher must:
  - Define population under study
  - State the particular hypotheses that will be investigated
  - Give the significance level
  - Select a sample from the population
  - Collect the data
  - Perform the calculations required for the statistical test
  - Reach conclusion.
- Hypotheses concerning means, proportions, variance and standard deviation can be investigated.
- For the mean
  - Z-test
  - T-test
  - (just like from finding confidence intervals)
- For proportions
  - Z-test
- For variance and standard deviation
  - Chi-square-test

Three Methods Used to Test Hypotheses

- Traditional Method – we will discuss this first
- P-value Method – has become popular with the advent of modern computers and high-powered statistical calculators
- Confidence Interval Method – illustrates the relationship between hypothesis testing and confidence intervals

Steps in Hypothesis Testing – Traditional Method

- Every hypothesis testing situation must begin with the statement of a hypothesis
- **Statistical Hypothesis** – a conjecture about a population parameter; may or may not be true.
  - **Null Hypothesis (H_0)** – a statistical hypothesis that states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters.
  - **Alternative Hypothesis (H_1)** – a statistical hypothesis that states the existence of a difference between a parameter and a specific value, or states that there is a difference between two parameters.

Situation A

- A medical researcher is interested in finding out whether a new medication will have any undesirable side effects. The researcher is particularly concerned with the pulse rate of the patients who take the medication. Will the pulse rate increase, decrease, or remain unchanged after a patient takes the medication?
- Since the researcher knows that the mean pulse rate for the population under study is 82 beats per minute, the hypotheses for this situation are
    - **H_0**: $\mu = 82$
    - **H_1**: $\mu \neq 82$
- The null hypothesis specifies that the mean will remain unchanged, and the alternative hypothesis states that it will be different.
- This test is called a **two-tailed** test, since the possible side effects of the medication could be to raise or lower the pulse rate.

Situation B

- A chemist invents an additive to increase the life of an automobile battery. If the mean lifetime of the automobile battery without the additive is 36 months, then her hypotheses are
    - **H_0**: $\mu = 36$
    - **H_1**: $\mu > 36$
- In this situation, the chemist is interested only in increasing the lifetime of the batteries, so her alternative hypothesis is that the mean is greater than 36 months. The null hypothesis is that the mean is equal to 36 months.
- This test is called **right-tailed**, since the interest is in an increase only.

Situation C

- A contractor wishes to lower heating bills by using a special type of insulation in houses. If the average of the monthly heating bills is $78, her hypotheses are
    - **H_0**: $\mu = \$78$
    - **H_1**: $\mu < \$78$
- This test is **left-tailed**, since the contractor is interested only in lowering heating costs.

Stating Hypotheses

- To state hypotheses correctly, researchers must translate the *conjecture* or *claim* from words to mathematical symbols.

Equal to =	Greater than >
Not equal to ≠	Less than <

- The null and alternative hypotheses are stated together, and the null hypothesis contains the equals sign. The claim also goes as the alternative, or research, hypothesis.

Two-tailed	Right-tailed	Left-tailed
H$_0$: $\mu = k$	**H$_0$**: $\mu = k$	**H$_0$**: $\mu = k$
H$_1$: $\mu \neq k$	**H$_1$**: $\mu > k$	**H$_1$**: $\mu < k$

Practice Stating Hypotheses

1. A researcher thinks that if expectant mothers use vitamin pills, the birth weight of the babies will increase. The average birth weight of the population is 8.6 pounds.

2. An engineer hypothesizes that the mean number of defects can be decreased in a manufacturing process of compact disks by using robots instead of humans for certain tasks. The mean number of defective disks per 1000 is 18.

3. A psychologist feels that playing soft music during a test will change the results of the test. The psychologist is not sure whether the grades will be higher or lower. In the past, the mean of the scores was 73.

What's Next?

- After stating the hypothesis, the researcher designed the study.
    - Selects correct statistical test
    - Chooses an appropriate level of significance
    - Formulates a plan for conducting the study
- Recall: when samples of a specific size are selected from a population, the means of these samples will vary about the population mean, and the distribution of the sample means will be approximately normal when the sample size is greater than 30.
    - So even if the null hypothesis is true, the mean of a sample may not be exactly equal to the population mean.
    - Two outcomes:
        - Null hypothesis true, and the difference between the sample mean and population mean is due to chance
        - Null hypothesis false, and the sample came from a population whose mean is not the same as the sample mean, but is some other unknown value.

How do you decide whether to reject the null?

- The farther away the sample mean is from the population mean, the more evidence there would be for rejecting the null hypothesis
- If the sample mean is close enough to the population mean, the researcher may conclude that the difference in means was due to chance, and would not reject the null hypothesis.
    - But how close is "close enough"? Where does the researcher draw the line? – Here's where we use statistical tests and level of significance.

Statistical Tests

- **Statistical Test** – uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.
- The numerical value obtained from a statistical test is called the **test value**.
- In this type of statistical test, the mean is computed for the data obtained from the sample and it is compared to the population mean. Then a decision is made to reject or not reject the null hypothesis on the basis of the value obtained from the statistical test. If the difference is significant, the null hypothesis is rejected. If it is not, then the null hypothesis is not rejected.

Possible Outcomes in Hypothesis Testing

	H_0 TRUE	**H_0 FALSE**
REJECT	**ERROR** Type I	Correct decision
DO	Correct decision	**ERROR** Type II

Type I Error occurs if you reject the null hypothesis when it is true.

Type II Error occurs if you do not reject the null hypothesis when it is false.

Example

- The hypothesis testing situation can be likened to a jury trial. In a jury trial, there are four possible outcomes
    - Defendant is either guilty or innocent
    - Defendant is either convicted or acquitted
- Hypotheses are
    - **H_0**: Defendant is innocent
    - **H_1**: Defendant is guilty
- Next, the evidence is presented in court by the prosecutor, and based on the evidence, the jury decides the verdict, innocent or guilty.
    - Type I Error: Defendant is not guilty, and is convicted.
    - Type II Error: Defendant is guilty, and not convicted

How large a difference is necessary to reject the null?

- The **<u>level of significance</u>** is the maximum probability of committing a type I error. This probability is symbolized by the Greek letter α. That is,

$$P(\text{Type I Error}) = \alpha$$

- The probability of a type II error is symbolized by β, the Greek letter beta. That is,

$$P(\text{Type II Error}) = \beta$$

  - In most hypothesis-testing cases, β cannot be easily computed; however, α and β are related in that decreasing one increases the other.

Significance Levels

- Statisticians generally agree on using three arbitrary significance levels. These represent α, the probability of a type I error.
  - 0.10, 10%
  - 0.05, 5%
  - 0.01, 1%
- Next, using α, we choose a critical value from a table for the appropriate test.
- Critical value determines critical and noncritical regions.

Critical Values

- **<u>Critical Value</u>** – separates the critical region from the noncritical region. The symbol used for critical values is C.V.
- **<u>Critical/Rejection Region</u>** – the range of values of the test value that indicates that there is a significant difference and that the null hypothesis should be rejected
- **<u>Noncritical/Nonrejection Region</u>** – range of values of the test value that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected
- The critical value can be on the right side or the left side of the mean for a one-tailed test. Its location depends on the inequality sign of the alternative hypothesis.
  - $>$ → C.V. to the right of the mean
  - $<$ → C.V. to the left of the mean
- A **one-tailed test** indicates that the null hypothesis should be rejected when the test value is in the critical region on one side of the mean.
  - A one-tailed test is either a **right-tailed test** or **left-tailed test**, depending on the direction of the inequality of the alternative hypothesis.

Obtaining Critical Values

- To obtain the critical value, the researcher must choose an alpha level.
  - Back to Situation B…
  - Suppose α = 0.01.

- Researcher must find a z-value such that 1% of the area falls to the right of the z-value and 99% falls to the left.
- Then the researcher must find the area value in the Z-Table closest to 0.9900. The critical z-value is 2.33.
- Try finding the critical values for Situation C and Situation A, for α = 0.01.

Procedure for Finding the Critical Values for Specific α Values, Using Z-Table

- **Step 1:** Draw the figure and indicate the appropriate area.
  a) If the test is left-tailed, the critical region, with an area equal to α, will be on the left side of the mean.
  b) If the test is right-tailed, the critical region, with an area equal to α, will be on the right side of the mean.
  c) If the test is two-tailed, α must be divided by 2; one-half of the area will be to the right of the mean, and one-half will be to the left of the mean.
- **Step 2:**
  a) For a left-tailed test, use the z value that corresponds to the area equivalent to α in the table.
  b) For a right-tailed test, use the z value that corresponds to the area equivalent to 1 - α.
  c) For a two-tailed test, use the z value that corresponds to $\alpha/2$ for the left value. It will be negative. For the right value, use the z value that corresponds to the area equivalent to 1 - $\alpha/2$. It will be positive.

Example

- Using the Z-Table, find the critical value(s) for each situation and draw the appropriate figure, showing the critical region.
  a) A left-tailed test with α = 0.10
  b) A two-tailed test with α = 0.02
  c) A right-tailed test with α = 0.005

Procedure for Solving Hypothesis-Testing Problems (Traditional Method)

- **Step 1:** State the hypothesis and identify the claim
- **Step 2:** Find the critical value(s) from the appropriate table
- **Step 3:** Compute the test value
- **Step 4:** Make the decision to reject or not reject the null hypothesis
- **Step 5:** Summarize the results