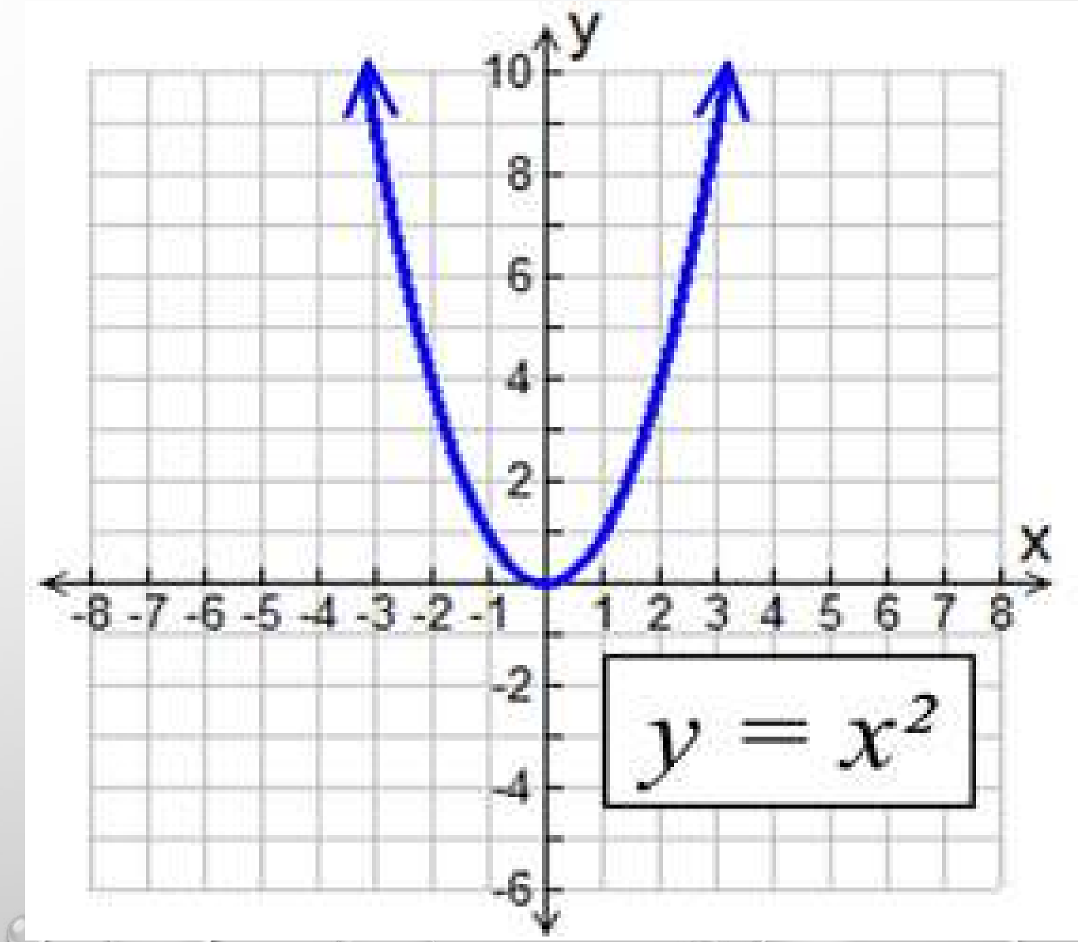
# QUADRATIC EQUATIONS

ALGEBRA 2 UNIT 3

# GENERAL EQUATION

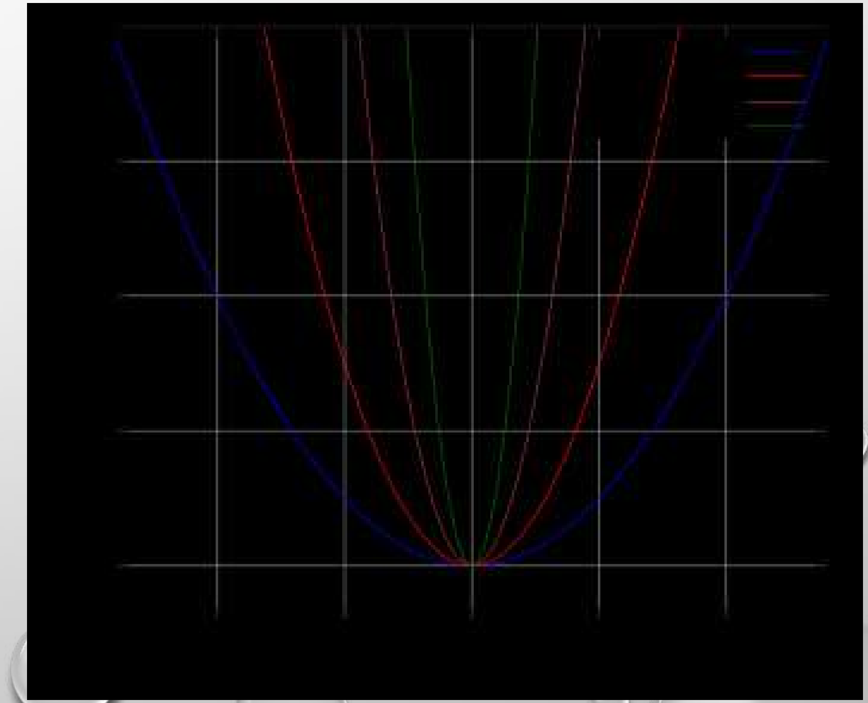
# $Y = AX^2$

- WHAT IF A WAS POSITIVE?
  - TEST IN YOUR CALCULATOR
- WHAT IF A WAS NEGATIVE?
  - TEST IN YOUR CALCULATOR.



# $Y = AX^2$

- WHAT IF A WAS GREATER THAN 1?
  - TEST IN YOUR CALCULATOR
- WHAT IF A WAS LESS THAN 1?
  - TEST IN YOUR CALCULATOR.



# GENERAL GRAPH OF THE EQUATION

# DOMAIN AND RANGE

- DOMAIN: SET OF ALL X VALUES OF A FUNCTION
  - USUALLY WILL BE ALL REAL NUMBERS

- RANGE: SET OF ALL Y VALUES OF A FUNCTION
  - DEPENDS ON YOUR MAXIMUM OR MINIMUM VALUE

# X-INTERCEPT AND Y-INTERCEPT

- X-INTERCEPT: WHERE THE FUNCTION TOUCHES OR INTERSECTS THE X- AXIS
- Y-INTERCEPT: WHERE THE FUNCTION TOUCHES OR INTERSECTS THE Y- AXIS

# INTERVALS OF INCREASING OR DECREASING

- INCREASING: WHERE THE FUNCTIONS SLOPE IS INCREASING

- DECREASING: WHERE THE FUNCTIONS SLOPE IS DECREASING

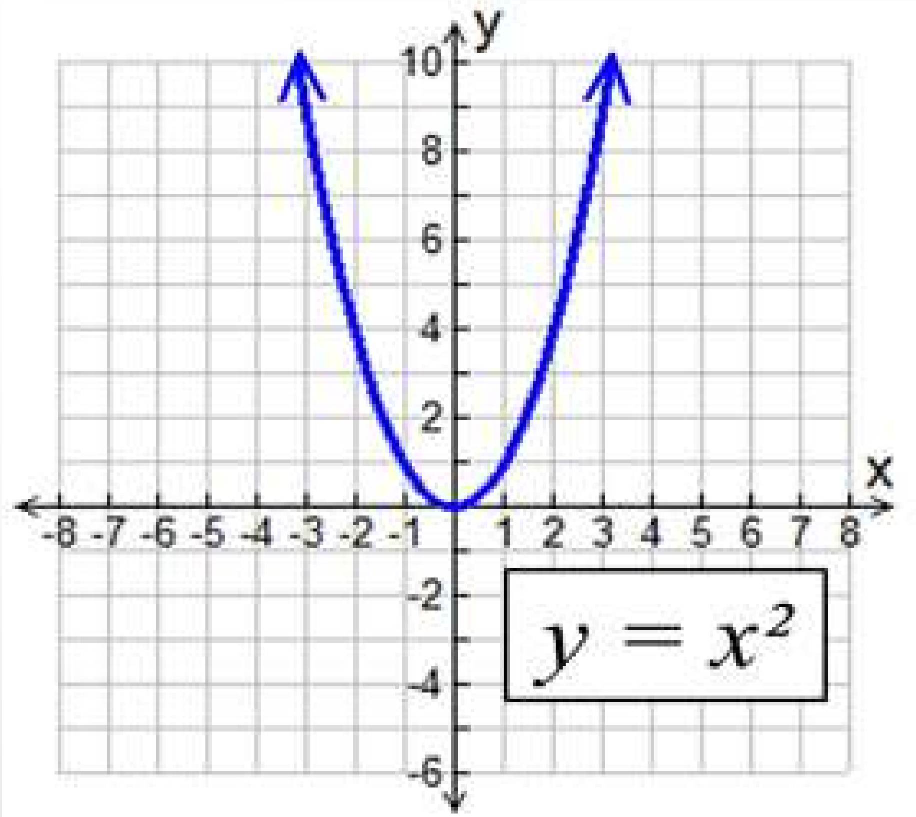
- LABELED AS [X,X]– INCREASING/DECREASING FROM WHAT X VALUE TO WHAT X VALUE

# MAXIMUM OR MINIMUM VALUES

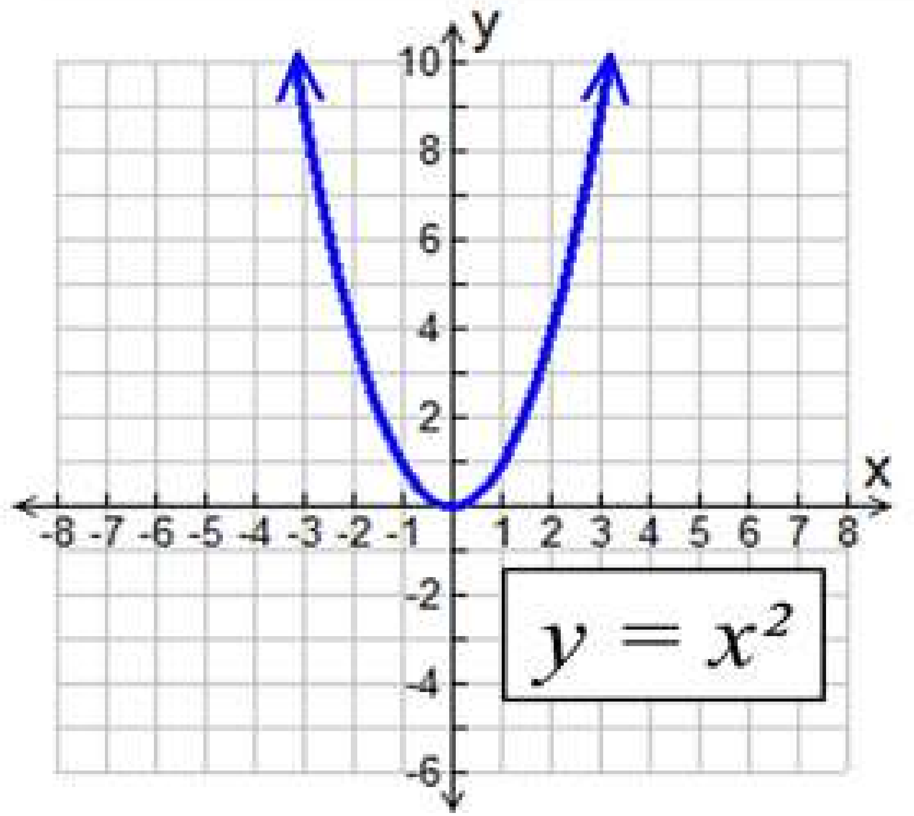
- MAXIMUM: THE HIGHEST POINT OF THE FUNCTION
- MINIMUM: THE LOWEST POINT OF THE FUNCTION
- *BOTH LABELED AS A POINT (X, Y)

# $Y = X^2$

- WHAT DO YOU NOTICE ABOUT THE GRAPH?
  - ANY SYMMETRY?
  - ANY HIGH/LOW POINTS?
  - ANY RESTRICTIONS ON VALUES?



# $Y = X^2$




- WHAT IS DOMAIN ?
- WHAT IS RANGE?
- WHAT IS X-INTERCEPT?
- WHAT IS Y-INTERCEPT?
- WHAT IS INTERVAL OF INCREASING?
- WHAT IS INTERVAL OF DECREASING?
- WHAT IS MAXIMUM OR MINIMUM?

# REAL-LIFE IMPORTANCE

- WHAT IS THE IMPORTANCE OF EACH OF THESE IN REAL-LIFE?
  - DOMAIN AND RANGE?
  - X- AND Y-INTERCEPTS?
  - INTERVALS OF INCREASING AND DECREASING?
  - MAXIMUM OR MINIMUM VALUES?

# QUADRATIC EQUATIONS IN STANDARD FORM

# STANDARD FORM

- $Y = AX^2 + BX + C$
  - A, B, AND C ARE CONSTANTS AND CAN BE NEGATIVE OR POSITIVE

# STANDARD FORM GRAPHS

- TRY GRAPHING THESE EQUATIONS:
  - $Y = X^2 - 6X + 8$
  - $Y = -X^2 + 8X + 15$
    - FIND DOMAIN, RANGE, X-INTERCEPTS, Y-INTERCEPTS, INTERVALS OF INCREASING AND DECREASING, AND MAXIMUM OR MINIMUM VALUES.

# STANDARD FORM GRAPHS

- CAN FIND THE VALUES IN YOUR CALCULATOR.

# QUADRATIC EQUATIONS IN INTERCEPT FORM

# INTERCEPT FORM

- Y = A(X – Q)(X – P)
  - WHERE A, Q, P ARE CONSTANTS.
    - A TELLS IF IT OPENS UP/DOWN
    - Q AND P WILL BE INTERCEPTS

# INTERCEPT FORM

- TRY GRAPHING THESE EQUATIONS:
  - $Y = 2(X - 3)(X + 4)$
  - $Y = -.5(X + 1)(X - 2)$
    - FIND DOMAIN, RANGE, X-INTERCEPTS, Y-INTERCEPTS, INTERVALS OF INCREASING AND DECREASING, AND MAXIMUM OR MINIMUM VALUES.

# THE VERTEX OF WHICH PARABOLA IS HIGHER?

- $Y = X^2$ OR $Y = 4X^2$
- $Y = -2X^2$ OR $Y = -2X^2 - 2$
- $Y = 3X^2 - 3$ OR $Y = 3X^2 - 6$

# SOLVE QUADRATICS IN STANDARD FORM
# (WITHOUT CALCULATOR)

# FACTORING QUADRATICS

- WHEN YOU FACTOR A QUADRATIC YOU ARE REALLY PUTTING IT INTO INTERCEPT FORM SO YOU CAN EASILY FIND THE ZEROS (X-INTERCEPTS) OF THE FUNCTION.
- ONCE YOU HAVE THE FUNCTION IN INTERCEPT FORM YOU SET EACH OF THE PARENTHESES EQUAL TO ZERO AND SOLVE.

# FACTORING QUADRATICS IN STANDARD FORM

- FACTOR: $Y = X^2 + 2X + 1$
  - 1ST STEP: CREATE A FACTOR/SUM CHART
  - 2ND STEP: BREAK DOWN MIDDLE TERM INTO TWO TERMS
  - 3RD STEP: GROUP 1ST TWO TERMS AND LAST TWO TERMS TOGETHER
  - 4TH STEP: FACTOR OUT ALL SIMILAR TERMS
  - 5TH STEP: SIMPLIFY

# FACTORING QUADRATICS IN STANDARD FORM

- FACTOR: $Y = X^2 + 2X + 1$
  - $1^{ST}$ STEP: CREATE A FACTOR/SUM CHART

| FACTORS (A *C) | SUM (B) |
|---|---|
| **1*1** | **2** |
| -1*-1 | -2 |
| | |
| | |

# FACTORING QUADRATICS IN STANDARD FORM

- FACTOR: $Y = X^2 + 2X + 1$
  - 2ND STEP: BREAK DOWN MIDDLE TERM INTO TWO TERMS
    - $Y = X^2 + 1X + 1X + 1$

# FACTORING QUADRATICS IN STANDARD FORM

- FACTOR: $Y = X^2 + 2X + 1$
  - 3RD STEP: GROUP 1ST TWO TERMS AND LAST TWO TERMS TOGETHER
    - $Y = (X^2 + 1X) + (1X + 1)$

# FACTORING QUADRATICS IN STANDARD FORM

- FACTOR: $Y = X^2 + 2X + 1$
  - 4TH STEP: FACTOR OUT ALL SIMILAR TERMS
    - $Y = (X^2 + 1X) + (1X + 1)$
    - $Y = X(X + 1) + 1(X + 1)$

# FACTORING QUADRATICS IN STANDARD FORM

- FACTOR: $Y = X^2 + 2X + 1$
  - $5^{TH}$ STEP: SIMPLIFY
    - $Y = X(X + 1) + 1(X + 1)$
    - $Y = (X + 1)(X + 1)$
- WHAT DOES THIS MEAN? – THE TWO ZEROS OF THE FUNCTION ARE $(X + 1) = 0$ AND $(X + 1) = 0$ OR THE GRAPH PASSES THROUGH THE X VALUE OF -1.

# FACTORING QUADRATICS IN STANDARD FORM

- FACTOR: $Y = X^2 + 2X - 8$
  - 1ST STEP: CREATE A FACTOR/SUM CHART
  - 2ND STEP: BREAK DOWN MIDDLE TERM INTO TWO TERMS
  - 3RD STEP: GROUP 1ST TWO TERMS AND LAST TWO TERMS TOGETHER
  - 4TH STEP: FACTOR OUT ALL SIMILAR TERMS
  - 5TH STEP: SIMPLIFY

# FACTORING QUADRATICS IN STANDARD FORM

- FACTOR: $Y = X^2 + 2X - 8$
  - $Y = X^2 - 2X + 4X - 8$
  - $Y = (X^2 - 2X) + (4X - 8)$
  - $Y = X(X - 2) + 4(X - 2)$
  - $Y = (X - 2)(X + 4)$

| FACTOR (1*-8) | SUM (2) |
|---|---|
| 1*-8 | -7 |
| -1*8 | 7 |
| **-2*4** | **2** |
| 2*-4 | -8 |

# FACTORING QUADRATICS IN STANDARD FORM

- FACTOR: $Y = X^2 - 25$
- FACTOR: $Y = X^2 - 4X – 21$
- FACTOR: $Y = X^2 - 4X – 12$
- FACTOR: $Y = 2X^2 - 7X - 4$

| FACTOR (A*C) | SUM (B) |
| --- | --- |
| | |
| | |
| | |
| | |

# FACTORING WORKSHEET

# FINDING MAXIMUMS AND MINIMUMS

- TO FIND THE MAXIMUM OR MINIMUM VALUE OF A QUADRATIC YOU CAN USE THE FORMULA:
  - $(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right))$

# QUADRATIC FORMULA

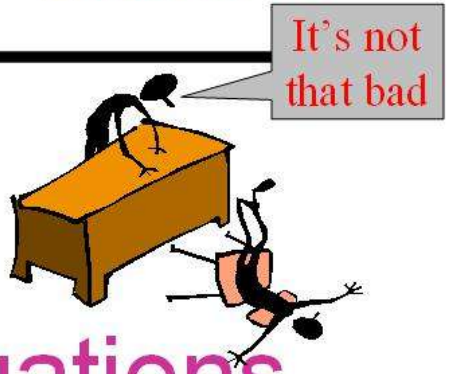
# QUADRATIC FORMULA

- ALSO USED TO FIND THE FACTORS OF A QUADRATIC EQUATION
  - WHEN YOU USE THE QUADRATIC FORMULA THE ANSWERS ARE THE ZERO'S OF THE FUNCTION. YOU THEN NEED TO TAKE THE ZERO'S AND PUT THEM INTO INTERCEPT FORM TO RE-WRITE THE EQUATION.

# QUADRATIC FORMULA

The Quadratic Formula ...

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$




For Quadratic Equations

$ax^2 + bx + c = 0$

# QUADRATIC FORMULA SONG

- HTTP://WWW.YOUTUBE.COM/WATCH?V=O8EZDEK3QCG

# USING THE QUADRATIC FORMULA

- 1st STEP: PLUG A, B, AND C INTO THE FORMULA
- 2nd STEP: SIMPLIFY THE EXPRESSION
- 3rd STEP: FIND THE TWO INTERCEPTS
- 4th STEP: WRITE THE FINAL ANSWER IN FACTORED FORM

# USING THE QUADRATIC FORMULA

- $Y = 2X^2 + 17X + 21$
  - 1ST STEP: PLUG A, B, AND C INTO THE FORMULA

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-17 \pm \sqrt{(-17)^2 - 4 * 2 * 21}}{2 * 2}$$

# USING THE QUADRATIC FORMULA

- Y = 2X² + 17X + 21
  - 2ND STEP: SIMPLIFY THE EXPRESSION

- $\frac{-17 \pm \sqrt{(-17)^2 - 4 * 2 * 21}}{2 * 2} \Rightarrow \frac{-17 \pm \sqrt{121}}{4}$

# USING THE QUADRATIC FORMULA

- Y = 2X² + 17X + 21
  - 3RD STEP: FIND THE TWO INTERCEPTS
  - $\frac{-17 \pm \sqrt{(-17)^2 - 4 * 2 * 21}}{2 * 2} \Rightarrow \frac{-17 \pm \sqrt{121}}{4} \Rightarrow \frac{-17 + 11}{4}\ and\ \frac{-17 -}{4}$
  - FINAL ANSWERS OF -3/2 AND -7

# USING THE QUADRATIC FORMULA

- Y = $2X^2$ + 17X + 21
  - 4TH STEP: WRITE THE FINAL ANSWER IN FACTORED FORM
  - ANSWERS: -3/2 AND 7
  - Y = (2X +3) AND (X – 7)

# THE DISCRIMINANT

- THE DISCRIMINANT = $B^2 - 4AC$
  - IF $B^2 - 4AC > 0$ THEN TWO REAL SOLUTIONS
  - IF $B^2 - 4AC < 0$ THEN NO REAL SOLUTIONS
  - IF $B^2 - 4AC = 0$ THEN ONE REAL SOLUTION

# THE DISCRIMINANT

- TEST:
  - $Y = 2X^2 - 5X + 10$
  - $Y = -9X^2 + 12X - 4$

# IMPORTANCE OF INFORMATION RECAP

# IF EQUATION IS IN STANDARD FORM

- WHEN YOU FACTOR YOU ARE WRITING IT IN INTERCEPT FORM
  - TO FIND THE ZEROS (X-INTERCEPTS) IN INTERCEPT FORM, SET EACH PARENTHESES EQUAL TO ZERO AND SOLVE

# IF EQUATION IS IN INTERCEPT FORM

- TO FIND THE ZEROS (X- INTERCEPTS) SET EACH PARENTHESES EQUAL TO ZERO AND SOLVE
- TO FIND THE Y- INTERCEPT SET X EQUAL TO ZERO AND SOLVE

# QUADRATIC FORMULA

- THE QUADRATIC FORMULA WILL GIVE YOU THE ZEROS (X-INTERCEPTS) OF THE FUNCTION.
  - TO WRITE THE EQUATION IN INTERCEPT FORM YOU NEED TO CHANGE THE SIGN OF THE ZERO AND PUT IT INTO THE EQUATION.– IF A FRACTION PUT DENOMINATOR WITH X AND NUMERATOR BY ITSELF.

# QUADRATIC FORMULA AND THE VERTEX

- YOU CAN ALSO USE THE QUADRATIC FORMULA TO FIND THE VERTEX OF THE PARABOLA:
  - THE MAX OR MIN POINT WILL BE:
    - $(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right))$

# APPLICATIONS OF QUADRATICS

# OVERVIEW OF APPLICATIONS

- THROWING A BALL
- HEIGHT VERSUS TIME FUNCTION
- DROPPING ANYTHING
- PROFIT VERSUS COST
- MAXIMIZATION OR MINIMIZATION OF SOMETHING

# QUADRATIC APPLICATIONS

- ALTHOUGH A STADIUM FIELD OF SYNTHETIC TURF APPEARS TO BE FLAT, ITS SURFACE IS ACTUALLY SHAPED LIKE A PARABOLA. THIS IS SO THE RAINWATER RUNS OFF TO THE SIDES. IF WE TAKE A CROSS SECTION OF THE TURF, IT CAN BE MODELED BY $Y = -.000234(X-80)^2 + 1.5$, WHERE X IS THE DISTANCE FROM THE LEFT END OF THE FIELD AND Y IS THE HEIGHT OF THE FIELD. WHAT IS THE HEIGHT OF THE FIELD 40 FEET IN?

# QUADRATIC APPLICATIONS

- A SHOT-PUT THROWER CAN BE MODELED USING THE FOLLOWING EQUATION: $Y = -.0241X^2 + X + 5.5$ WHERE X IS THE DISTANCE TRAVELED IN FEET AND Y IS THE HEIGHT OF THE SHOT PUT IN FEET. HOW HIGH IS THE SHOT PUT AFTER IT TRAVELS 4 FEET. HOW FAR AWAY WILL THE SHOT PUT HIT THE GROUND?

# QUADRATIC APPLICATIONS

- MARCUS KICKS A FOOTBALL IN ORDER TO SCORE A FIELD GOAL. THE HEIGHT OF THE BALL IS GIVEN BY THE EQUATION $Y = (-1/200)X^2 + X$ WHERE Y IS THE HEIGHT OF THE FOOTBALL AND X IS THE HORIZONTAL DISTANCE THE BALL TRAVELS. WE WANT TO KNOW IF HE KICKED THE BALL HARD ENOUGH TO GO OVER THE GOAL POST WHICH IS 10 FEET HIGH.