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New Jersey Center for Teaching and Learning

Progressive Mathematics Initiative

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Algebra II

Working with Functions

Their Graphs, the Algebra Behind Them and Piecewise Functions

September 27, 2012

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The 12 Basic Functions (Parent Functions)

Many situations in the world that people study and collect data on follow one of the following 12 patterns. By recognizing a general pattern, or what we call the Parent Function, and then algebraically manipulating the function, you can almost come up with an exact match. Some people get paid a lot of money to do this!

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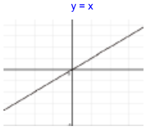
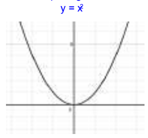
The 12 Basic Functions:

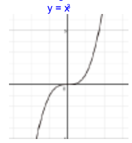
The Identity Function
$y = x$



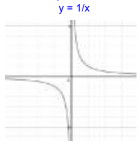
The Squaring Function
$y = x^2$



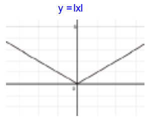
The Cubing Function
$y = x^3$



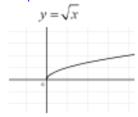
The Reciprocal Function
$y = 1/x$



The Absolute Value Function
$y = |x|$



The Square Root Function
$y = \sqrt{x}$



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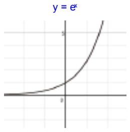
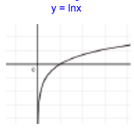
The 12 Basic Functions continued:

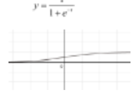
The Exponential Function
$y = e^x$



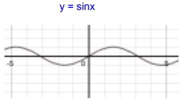
The Natural Log Function
$y = \ln x$



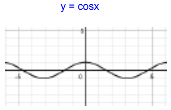
The Logistic Function
$y = \frac{1}{1+e^{-x}}$



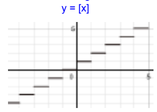
The Sine Function
$y = \sin x$



The Cosine Function
$y = \cos x$



The Greatest Integer Function
$y = [x]$



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Transforming Functions

(Or making parent functions match data.)

$$y = a\,f(\,bx \mp c) \pm d$$

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Goals and Objectives

Students will be able to transform any function, including parent functions, algebraically and graphically.

Why do we need this?

Many different factors in life affect data and information. These situations can be modeled by functions, but not all are the same. The 12 basic functions represent common graphs of information. Transformations of these functions get us closer to a result and even apply to situations that are not easily representable in a common form.

Transforming Functions - activity

$y = a\, f(bx \mp c) \pm d$

Teacher

Explore this on your own! Pull out a graphing calculator!

Choose one of the Basic Functions. Try adding and subtracting numbers inside and outside of the function. Then multiply and divide different functions by numbers in different places. What happens to the graph?
Make a list:

a makes the function ____________________

b makes the function ____________________

c makes the function ____________________

d makes the function ____________________

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Transforming Functions

Now, we are going to formalize your results and explore each part individually.

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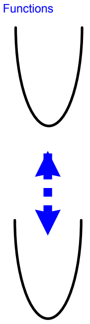
Transforming Functions



Vertical Shifts

$$y = f(x) \pm d$$

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Vertical Shifts occur when a constant is added to or subtracted from OUTSIDE of the function.

$$y = f(x) \pm d$$

Teacher

The function will be translated UP d if d is added.

+ d

The function will be translated DOWN d if d is subtracted.

- d

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Teacher

Try it on your calculator!

Use the function $y = x^2$. Write the new equation and then enter it in your calculator.

a) up 4

b) down 5

c) down 3

d) up 2

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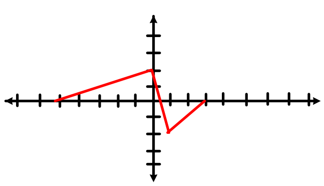
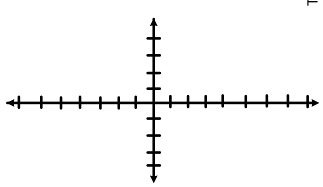
Teacher

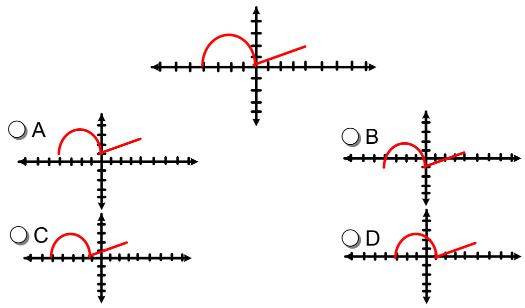
Let the graph of f(x) be:



Graph y = f(x) + 2:



1 Given the graph of h(x), which of the following graphs is y = h(x) - 1?



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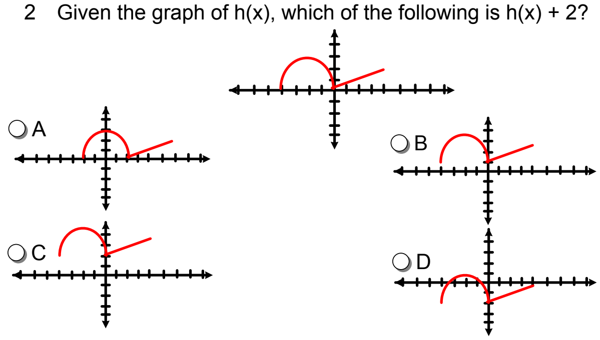
2 Given the graph of h(x), which of the following is h(x) + 2?



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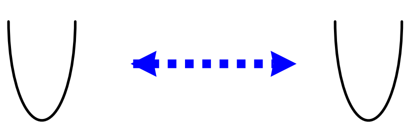
Horizontal Shifts

$$y = f(x \mp c)$$



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Horizontal Shifts occur when a constant is added to or subtracted INSIDE of the function.

$$y = f(x \mp c)$$

The function will be translated LEFT c if c is added INSIDE.

← + c

The function will be translated RIGHT c if c is subtracted INSIDE.

→ - c

Notice the direction is opposite the sign of c.

Teacher

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Try it on your calculator!

Use the function $y = x^3$. Write the new equation and then enter it in the calculator.

a) right 4

b) left 5

c) left 3

d) right 2

Teacher

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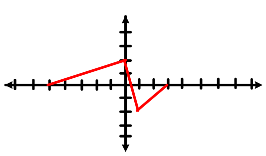
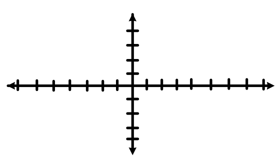
Let the graph of f(x) be:



Graph y = f(x + 2):



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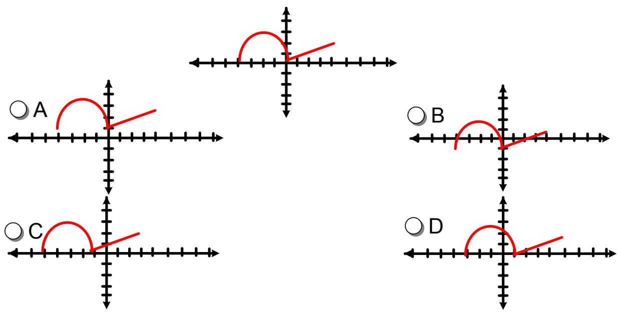
3 Given the graph of h(x), which of the following graphs is y = h(x - 1)?



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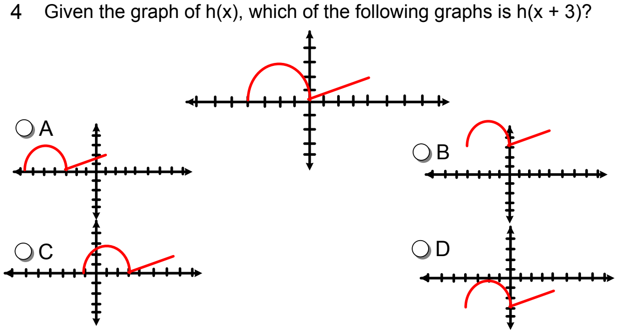
4 Given the graph of h(x), which of the following graphs is h(x + 3)?



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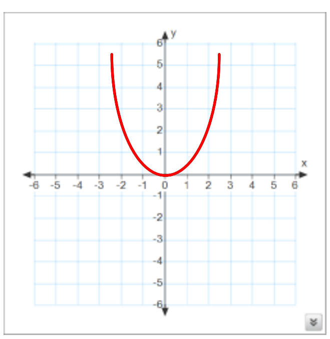
Transform g(x) as indicated:

1. h(x) = g(x) + 3
2. r(x) = g(x) - 5
3. p(x) = g(x - 4) + 2
4. m(x) = g(x + 3)
5. b(x) = g(x - 5) + 4



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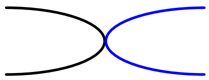
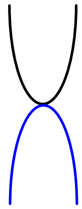
Reflections



$$y = -f(x)$$

$$y = f(-x)$$



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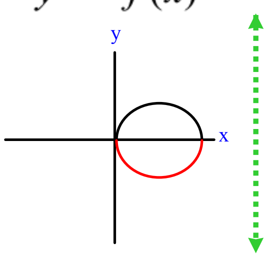
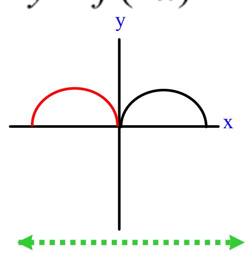
Reflection over the x-axis:

$$y = -f(x)$$



Reflection over the y-axis:

$$y = f(-x)$$



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Try it on your calculator!

What happens when you add in negatives? Look at the following...

a) $y = -x^2$

b) $y = (-x)^2$

c) $y = -x^3$

d) $y = (-x)^3$

e) $y = -|x|$

f) $y = |-x|$

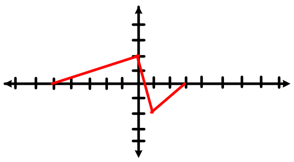
g) $y = -\sqrt{x}$

h) $y = \sqrt{-x}$

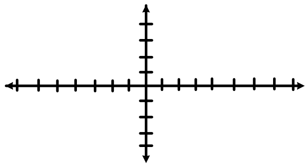
Teacher

Transforming Functions

Let the graph of f(x) be:



Graph y = f(-x):



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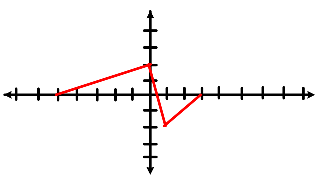
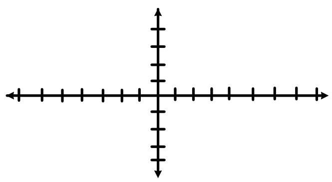
Transforming Functions

Let the graph of f(x) be:



Graph y = -f(x):



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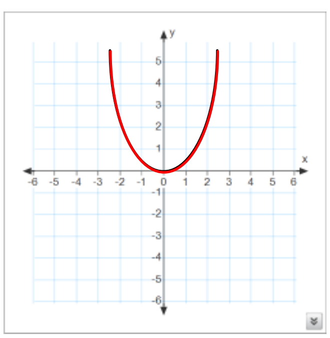
Transforming Functions

Transform g(x) as indicated:

1. h(x) = -g(x) + 4
2. r(x) = g(-x) - 2
3. p(x) = -g(x - 1) + 2
4. m(x) = -g(x + 2)
5. b(x) = g(-x) - 4



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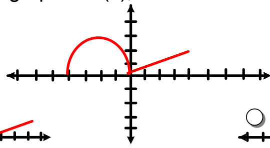
5 Given the graph of h(x), which of the following is -h(x)?

Teacher



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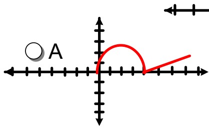

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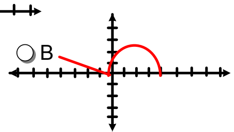

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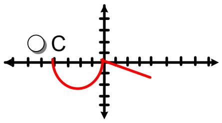

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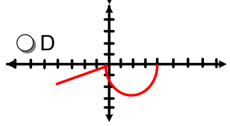
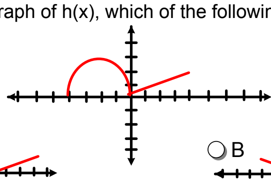
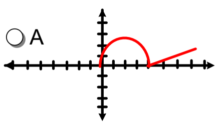


6 Given the graph of h(x), which of the following graphs is y = h(-x)?

Teacher



A


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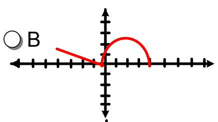

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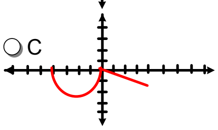

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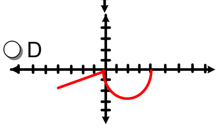
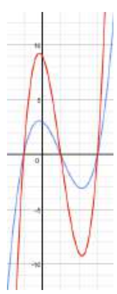


Vertical Stretch & Shrink

$$y = af(x)$$



Vertical Stretches and Shrinks occur when a constant is multiplied OUTSIDE of a function.

$$y = af(x)$$

The parent function y = f(x) is :
stretched if |a| > 1
shrunk if 0 < |a| < 1

Stretches and shrinks are the first transformation that do not yield congruent figures.

Note: Notice how the x-intercepts DO NOT change.

Teacher

Stretch or shrink the following functions as indicated. Graph the parent function first, and then the transformed function in the same window.

Try it on your calculator!

1) $y = 4\sin x$

2) $y = \frac{1}{4}\sin x$

3) $y = 3x^2$

4) $y = \frac{1}{3}x^2$

5) $y = 5|x|$

6) $y = \frac{1}{5}|x|$

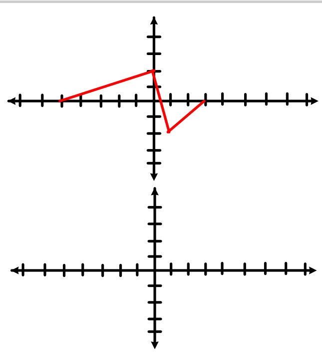
7) $y = 2\sqrt{x}$

8) $y = \frac{1}{2}\sqrt{x}$

Teacher

Let the graph of f(x) be:

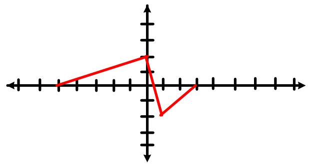
Graph y = 2f(x):



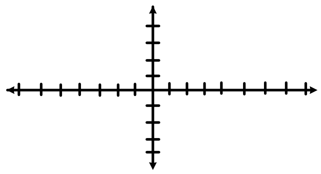
Teacher

Transforming Functions

Let the graph of f(x) be:



Graph $y = (^1/_3)f(x)$:



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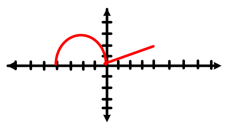
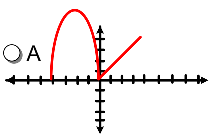
Transforming Functions

7 Given the graph of h(x), which of the following graphs is $y = 2h(x)$?



A



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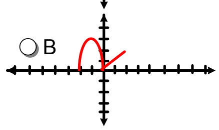

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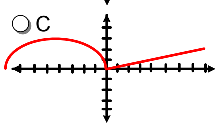

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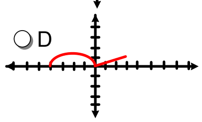

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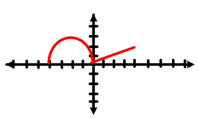
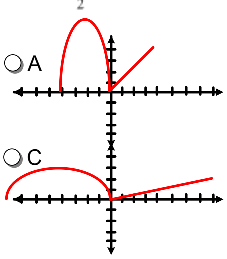
Transforming Functions

8 Given the graph of h(x), which of the following graphs is $y = \frac{1}{2}$ h(x)?



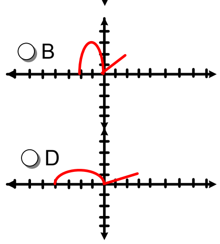
A

C



B

D



Teacher

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Horizontal Stretch & Shrink

$$y = f(bx)$$

Teacher

Horizontal Stretches and Shrinks occur when a constant is multiplied to x INSIDE a function.

Try it on your calculator!

$$y = f(bx)$$

The parent function y = f(x) is:
shrunk if |b| > 1
stretched if 0 < |b| < 1

Note: Notice how the y-intercepts DO NOT change.

Teacher

Stretch or shrink the following functions as indicated. Graph the parent function first, and then the transformed function in the same window.

Try it on your calculator!

1) $y = \sin(4x)$

2) $y = \sin\left(\frac{1}{4}\right)x$

3) $y = (3x)^2$

4) $y = \left(\frac{1}{3}x\right)^2$

5) $y = |5x|$

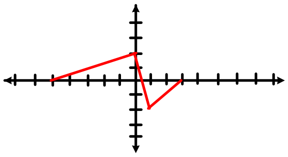
6) $y = \left|\frac{1}{5}x\right|$

7) $y = \sqrt{2x}$

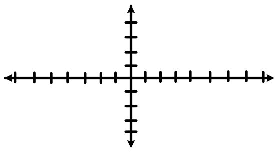
8) $y = \sqrt{\frac{1}{2}x}$

Transforming Functions

Let the graph of f(x) be:



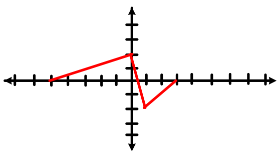
Graph y = f(2x):



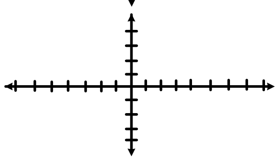
Teacher

Transforming Functions

Let the graph of f(x) be:



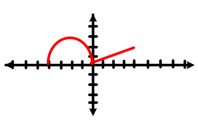
Graph y = f(.5x):

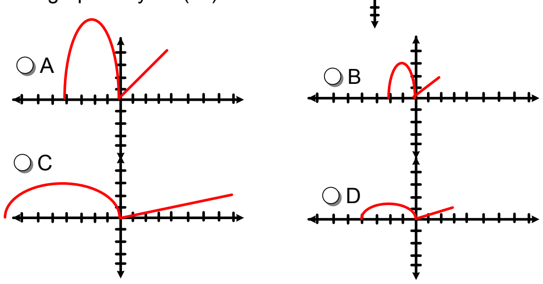


Teacher

Transforming Functions

9 Given the graph of h(x), which of the following graphs is y = h(2x)?

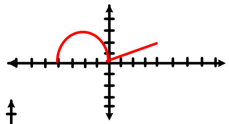


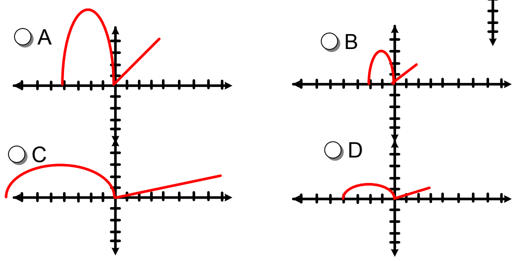


Teacher

Transforming Functions

Given the graph of h(x), which of the following graphs is $y = h(\frac{1}{2}x)$?





Teacher

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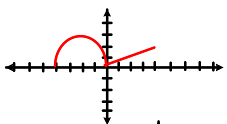
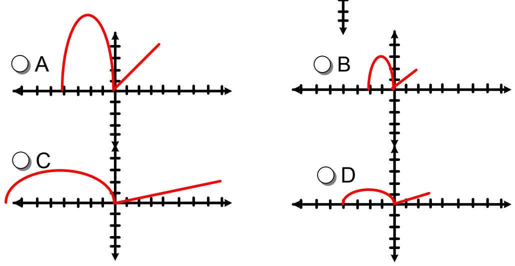
Transforming Functions

10 Given the graph of h(x), which of the following graphs is y = h(1/2x)?





Teacher

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Transforming Functions

Combining Transformations

What goes first? Any thoughts?

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Transforming Functions

$$y = a\,f(\,bx \mp c) \pm d$$

To combine transformations, follow order of operations:

Horizontal Stretch of b
↓
Horizontal Slide of c
↓
Vertical Stretch of a
↓
Vertical Shift of d

Teacher

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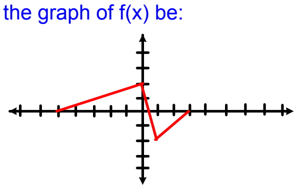
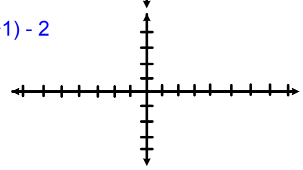
Transforming Functions

Let the graph of f(x) be:

Work slowly and use a different colored pencil for each transformation to help!



Graph y = 2f(.5x+1) - 2



Remember...

Horizontal Stretch of b
↓
Horizontal Slide of c
↓
Vertical Stretch of a
↓
Vertical Shift of d

Teacher

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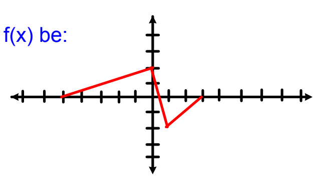
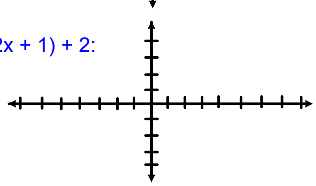
Transforming Functions

Let the graph of f(x) be:

Work slowly and use a different colored pencil for each transformation to help!



Graph $y = (-{}^{1}/_{3})f(2x + 1) + 2$:



Remember...

Horizontal Stretch of b
↓
Horizontal Slide of c
↓
Vertical Stretch of a
↓
Vertical Shift of d

Teacher

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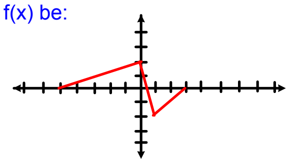
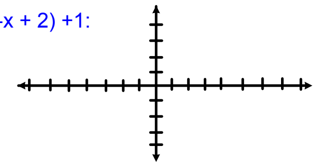
Transforming Functions

Let the graph of f(x) be:



Work slowly and use a different colored pencil for each transformation to help!

Graph $y = (-\frac{1}{2})f(-x + 2) + 1$:



Remember...

Horizontal Stretch of b
↓
Horizontal Slide of c
↓
Vertical Stretch of a
↓
Vertical Shift of d

Teacher

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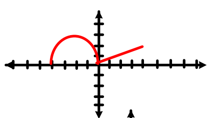
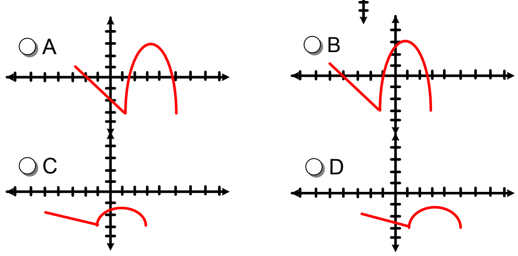
Transforming Functions

11 Given the graph of h(x), which of the following graphs is $y = 2h(-x + 1) - 3$?





- A
- B
- C
- D

Teacher

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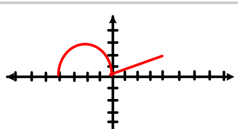
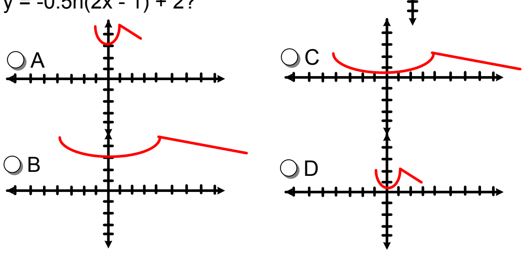
Transforming Functions

12 Given the graph of h(x), which of the following graphs is $y = -0.5h(2x - 1) + 2$?



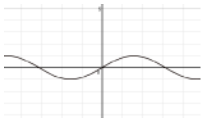


- A
- B
- C
- D

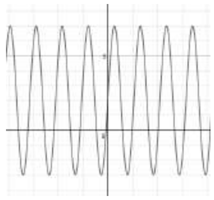
Teacher

Transforming Functions

Transforming Parent Functions



$y = \sin x$



$y = 5\sin(6x) + 2$

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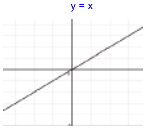
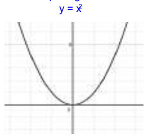
Transforming Functions

Remember the 12 basic functions...

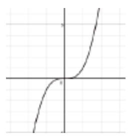
The Identity Function
$y = x$



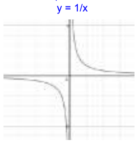
The Squaring Function
$y = x^2$



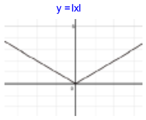
The Cubing Function
$y = x^3$



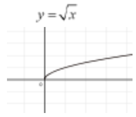
The Reciprocal Function
$y = 1/x$



The Absolute Value Function
$y = |x|$



The Square Root Function
$y = \sqrt{x}$



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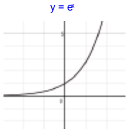
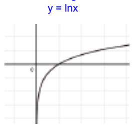
Transforming Functions

continued...

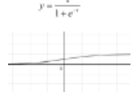
The Exponential Function
$y = e^x$



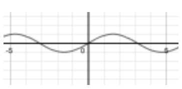
The Natural Log Function
$y = \ln x$



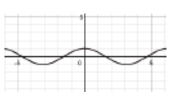
The Logistic Function
$y = \frac{1}{1+e^{-x}}$



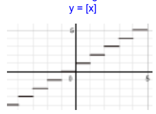
The Sine Function
$y = \sin x$



The Cosine Function
$y = \cos x$



The Greatest Integer Function
$y = [x]$



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Transforming Functions

$$y = a f(bx \mp c) \pm d$$

You have already transformed some of the 12 basic functions. Let's apply combinations of these functions to the first 4, the other 8 will be addressed later.

The Squaring Function $y = x^2$

The Cubing Function $y = x^3$

The Absolute Value Function $y = |x|$

The Square Root Function $y = \sqrt{x}$

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Transforming Functions

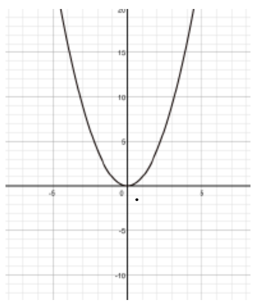
Transform $y = x^2$ into the following:
As the function changes, what happens to the domain and range?

a) $y = -2(x+3)^2$

b) $y = \frac{1}{3}(x-4)^2 + 2$

c) $y = -(x+1)^2 - 5$

d) $y = \frac{1}{2}(x-3)^2 - 3$



Teacher

Note: when studying parabolas, af(bx + c) + d becomes af(bx + h) + k, where (h, k) is the vertex of the parabola.

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Transforming Functions

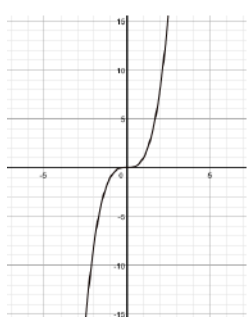
Transform $y = x^3$ into the following:
As the function changes, what happens to the domain and range?

a) $y = -(x-2)^3$

b) $y = \frac{1}{3}(x+5)^3 - 2$

c) $y = -3(x+1)^3 - 5$

d) $y = \frac{1}{2}(x+3)^3 + 2$



Teacher

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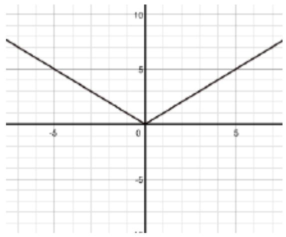
Transform y = lxl into the following:
As the function changes, what happens to the domain and range?

$a)\ y = -|x-3| + 2$

$b)\ y = \frac{1}{3}|x+2| - 5$

$c)\ y = -3|x-1| - 4$

$d)\ y = \frac{1}{2}|x+4| + 1$



Note: when studying the absolute value function, af(bx + c) + d becomes af(bx +h) + k, where (h, k) is the vertex of the absolute value function.

Teacher

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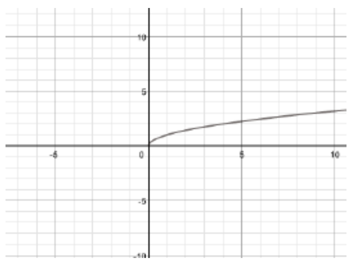
Transform $y = \sqrt{x}$ into the following:
As the function changes, what happens to the domain and range?

$a)\ y = -\sqrt{x-3} + 5$

$b)\ y = -3\sqrt{x-5} - 3$



Teacher

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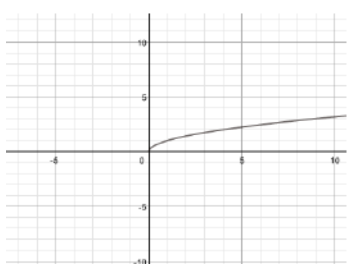
Transform $y = \sqrt{x}$ into the following:
Graph on the board and then check with your calculator! What happened?

$c)\ y = \frac{1}{3}\sqrt{-x+3} - 2$

$d)\ y = \frac{1}{2}\sqrt{-x+2} - 6$



Teacher

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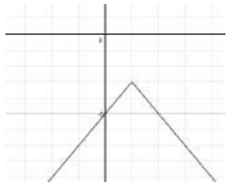
Transforming Functions

13 Which of the following is the equation for the graph shown?

Teacher



- A $y=-2|x-1|-3$
- B $y=-|x-1|-3$
- C $y=-|x+1|-3$
- D $y=-2|x+1|+3$

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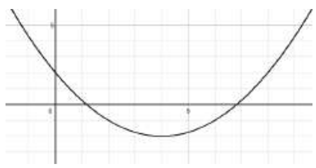
Transforming Functions

14 Which of the following is the equation for the graph shown?

Teacher



- A $y=4(x+4)^2-2$
- B $y=(x-4)^2+2$
- C $y=(x-4)^2-2$
- D $y=\frac{1}{4}(x-4)^2-2$

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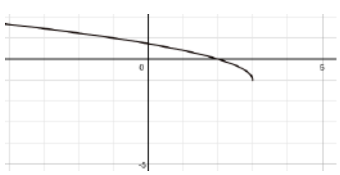
Transforming Functions

15 Which of the following is the equation for the graph shown?

Teacher



- A $y=-\sqrt{x+3}-1$
- B $y=\sqrt{-x+3}-1$
- C $y=-\sqrt{x-3}-1$
- D $y=\sqrt{-x-3}-1$

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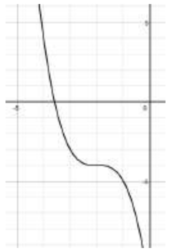
16 Which of the following is the eqution for the graph shown?

Teacher

- A $y=(x-2)^3-4$
- B $y=-(x+2)^3+4$
- C $y=(-x-2)^3+4$
- D $y=-(x+2)^3-4$



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Operations with Functions

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Goals and Objectives

Students will be able to manipulate multiple functions algebraically and simplify resulting functions.

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Why do we need this?

In this unit, we have graphically explored transformations of functions.
Sometimes, data is more complex and is combinations of different functions.
Algebraically manipulating functions allows us to combine different functions together and results in many more options for representing real life situations.

Functions can be combined to make other functions.

Here are the properties of combining functions:

Adding functions: $(f+g)(x) = f(x) + g(x)$

Subtracting functions: $(f-g)(x) = f(x) - g(x)$

Multiplying functions: $(fg)(x) = f(x) \bullet g(x)$

Dividing functions: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad g(x) \neq 0$

Teacher

Given: $f(x) = 3x - 1$ and $g(x) = 2x + 1$

Find: $a)\ (f+g)(x)$

$b)\ (f-g)(x)$

$c)\ (fg)(x)$

$d)\ \left(\frac{f}{g}\right)(x)$

$e)\ 4f(x) - 2g(x)$

Simplify your answers as much as possible.

What happens to the domain?

Teacher

Operations with Functions

17 Given f(x) = $3x^2$ + 2x and g(x) = x, find h(x) = (f + g)(x)

Teacher

- A $3x^2 + 2x + 1$
- B $4x^2 + 2x$
- C $3x^2 + 3x$
- D $3x^2 + x$

Operations with Functions

Given $f(x) = 3x^2 + 2x$ and $g(x) = x$, find h(x) if $h(x) = f(x) - g(x)$

Teacher

- A $3x^2 + 2x + 1$
- B $4x^2 + 2x$
- C $3x^2 + 3x$
- D $3x^2 + x$

Operations with Functions

18 Given f(x) = $3x^2$ + 2x and g(x) = x, find h(x) if h(x) = f(x) - g(x).

Teacher

- A $3x^2 + 2x + 1$
- B $4x^2 + 2x$
- C $3x^2 + 3x$
- D $3x^2 + x$

Operations with Functions

19 Given f(x) = $3x^2 + 2x$ and g(x) = x, find h(x) if $h(x) = \left(\frac{f}{g}\right)(x)$

Teacher

- A $3x + 2$
- B $\frac{3x^2 + 2x}{2}$
- C $\frac{1}{3x^2 + 2x}$
- D $\frac{1}{3x + 2}$

Operations with Functions

20 Given f(x) = $3x^2 + 2x$ and g(x) = x, find h(x) if h(x) = 2f(x) - xg(x).

Teacher

- A $6x^2 + 3x$
- B $4x^2 + 2x$
- C $6x^2 + x$
- D $5x^2 + 4x$

Operations with Functions

Given $f(x) = \sqrt{x+3}$ and $g(x) = x^3$; find:

Teacher

What is the domain of each?

a) (f + g)(x) b) (f - g)(x) c) (fg)(x) d) $\left(\frac{f}{g}\right)(x)$

Operations with Functions

21 Find the domain of (f + g)(x) if $f(x)=\frac{1}{x} \text{ and } g(x)=x^3$

Teacher

- A (-∞, ∞)
- B [0, ∞)
- C (-∞, 0) U (0, ∞)
- D (-∞, 0]

Operations with Functions

22 Find the domain of $\left(\frac{f}{g}\right)(x)$ if f(x) = x^2 and g(x) = x - 4.

Teacher

- A (-∞, 4) U (4, ∞)
- B (-∞, 0) U (0, ∞)
- C (4, ∞)
- D (-∞, 4)

Operations with Functions

23 Find the domain of (fg)(x) if f(x) = $\frac{1}{x}$ and g(x) = $\sqrt{x+3}$

Teacher

- A (-∞, 3) U (3, ∞)
- B [-3, 0) U (0, ∞)
- C (-∞, -3) U (3, ∞)
- D (-∞, ∞)

Operations with Functions

You may also be asked to find values of combined functions when given specific values for x.

Given $f(x)=\sqrt{2x-3}+2x$ and $g(x)=2x^2$, find:

a) (f + g)(2) b) (f - g)(3) c) (fg)(5) d) $\left(\frac{f}{g}\right)(0)$

Teacher

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Operations with Functions

24 Given f(x) = $\sqrt{4x}$ and g(x) = x - 12, find (f - g)(4).

- A -6
- B -4
- C 12
- D 10

Teacher

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Operations with Functions

25 Given f(x) = $(x-8)^2$ and g(x) = x^3, find (fg)(6).

- A 1728
- B -864
- C 864
- D 1288

Teacher

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26 Given f(x) = $|-x-8|$ and g(x) = $\sqrt{2x}$, find $\left(\frac{g}{f}\right)(8)$

Teacher

- A undefined
- B $\frac{1}{2}$
- C 4
- D $\frac{1}{4}$

Composite functions

$f \circ g(x)$ or $g \circ f(x)$

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Goals and Objectives

Students will be able to recognize function notation and correctly unite two or more functions together to create a new function.

Why do we need this?

Life is complicated at times. On many occasions, multiple situations happen to something before it obtains a final result. For example, you take extra food off of your plates before you put them in the dishwasher. These are two functions that go \together to obtain a result.

Def: Composite functions exist when one function is "nested" in the other function.

There are 2 ways of writing a composite function:

$$f(g(x)) \qquad f \circ g(x)$$

Each form is read "f of g of x" and both mean the same thing.

To simplify a composite of functions, substitute one function into the other in place of "x" and simplify.

Teacher

Given: $f(x) = 3x^2 + 2x$ *and* $g(x) = 4x$

Find: f(g(x)) and g(f(x))

$f(g(x)) = f(g(x))$

$= 3(g(x))^2 + 2(g(x))$

$= 3(4x)^2 + 2(4x)$

$= 3(16x^2) + 8x$

$= 48x^2 + 8x$

$g(f(x)) = g(f(x))$

$= 4(f(x))$

$= 4(3x^2 + 2x)$

$= 12x^2 + 8x$

Composite functions

Given: $f(x)=\sqrt{x-3}$ *and* $g(x)=2x^2+x-2$

Find: f(g(x)) and g(f(x))

f(g(x)) = f(g(x))

g(f(x)) = g(f(x))

Teacher

Composite functions

To simplify composite functions with numerical values, substitute the number into the "inner" function, simplify, and then substitute that value in for the variable in the "outer" function.

Given: $f(x)=3x^2+2x$ *and* $g(x)=4x$

Find: $a)\ f\circ g(3)$

$b)\ g(f(-2))$

Teacher

Composite functions

27 If f(x) = x^2 + 1 and g(x) = 3x - 1, find the value of f(g(2)).

- A 0
- B 5
- C 26
- D -4

Teacher

Composite functions

28 Find f(g(x)) if $f(x)=\dfrac{1}{|x|+3}$ and $g(x)=-x^3$

A $\dfrac{|x^3|}{3}$ B $\dfrac{1}{|x^3|+3}$ C $\dfrac{1}{x^3+3}$ D x^3+3

Teacher

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Composite functions

29 Find $g\circ f(x)$ if $f(x)=x^2$ and $g(x)=\sqrt{x^2+4}$

A x^2+4

B x^2+2

C $\sqrt{x^4+4}$

D $\sqrt{6x}$

Teacher

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Composite functions

30 Find $f\circ g(x)$ if $f(x)=x^2+x+4$ and $g(x)=\sqrt{x}$

A $x+\sqrt{x+4}$

B $\sqrt{x^2+x}+2$

C $x+\sqrt{x}+2$

D $x+\sqrt{x}+4$

Teacher

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Composite functions

31 Find g(f(-2)) if $f(x) = x + 2$ and $g(x) = \frac{1}{|x|}$

A *undefined*

B 0

C $\frac{5}{2}$

D $\frac{1}{2}$

Teacher

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Composite functions

32 Find f(g(-3)) if f(x) = $3x^2 + 2x - 3$ and g(x) = x - 2

A 62

B -88

C 82

D 19

Teacher

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Composite functions

33 Find the value of $f \circ h \circ g(0)$ *if*

$f(x) = x + 1 \quad g(x) = 3x - 1 \quad h(x) = |x|$

A 2

B 1

C 0

D -1

Teacher

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Inverse Functions

$$f^{-1}(x)$$

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Goals and Objectives

Students will be able to recognize and find an inverse function:

a) using coordinates,
b) graphically and
c) algebraically.

Why do we need this?

Sometimes, it is important to look at a problem from the inside out. Plus undoes minus. Multiply undoes divide. In order to look deeply into different problems, we must try to see things from the inside out. Inverse functions undo original functions.

Inverse Functions

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Teacher

Def: An <u>inverse function</u> is a function that undoes another function. The notation for the inverse of $f(x)$ is $f^{-1}(x)$.

Read, "the inverse of f of x."

You can prove that a function is an inverse of another using the following relationship:

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

Example: Prove that f(x) and g(x) are inverse functions.

$f(x) = 3x - 6$ $\qquad$ $g(x) = \frac{1}{3}x + 2$

Inverse Functions

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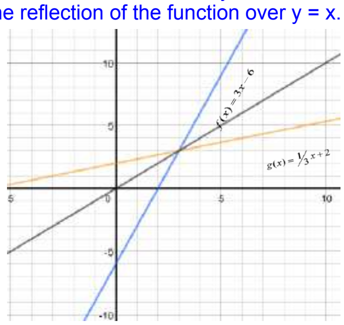
Teacher

You can also prove that two functions are inverses by GRAPHING. The graph of an inverse is the reflection of the function over y = x.

The following functions are inverses of each other. Look at their graphs and make a conjecture about the x and y values of inverse functions.

$f(x) = 3x - 6$

$g(x) = \frac{1}{3}x + 2$



Inverse Functions

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Teacher

The inverse of a function is the reflection over y = x. This will result in the switching of x and y values.

Examples:

a) Find the inverse of:

f(x)= {(1, 2), (3, 5), (-7, 6)}

b) Find the inverse of:

X	Y
3	2
4	4
5	-5
6	7

Inverse Functions

34 What is the inverse of {(1, 4), (5, 3), (2, -1)}?

Teacher

- A {(4,1), (3,5), (2,-1)}
- B {(-1,-4), (-5,-3), (-2,1)}
- C {(4,1), (3,5),(-1,2)}
- D {(-4,-1), (-3,-5), (1,-2)}

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Inverse Functions

35 If the inverse of a function is {(1, 0), (3, 3), (-4, -5)}, what was the original function?

Teacher

- A {(0,1), (3,3), (-5,-4)}
- B {(-1,-4), (-5,-3), (-2,1)}
- C {(0,-1), (-3,-3),(4,5)}
- D {(0,1), (3,3), (-4,-5)}

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Inverse Functions

36 What is the inverse of:

X	Y
3	4
1	0
-2	3
4	7

A

X	Y
4	3
1	0
2	3
4	7

B

X	Y
4	3
0	1
3	-2
7	4

C

X	Y
-3	-4
-1	0
-2	-3
-4	-7

D

X	Y
4	3
0	0
-2	3
7	4

Teacher

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Inverse Functions

37 Will the inverse of the following points be a function? Why or why not?

- Yes
- No

X	Y
1	2
2	3
3	4
5	2

Teacher

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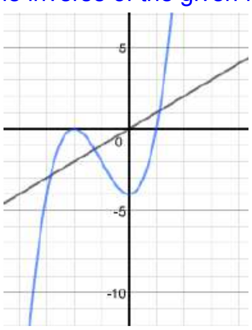
Inverse Functions

Draw the inverse of the given function.



Is the inverse a function? Why or why not?

Teacher

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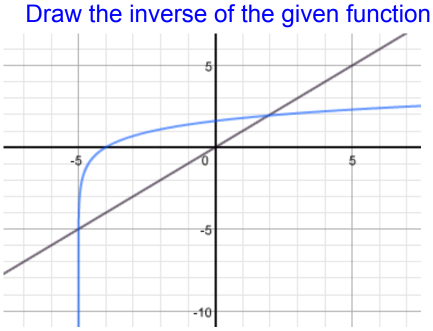
Inverse Functions

Draw the inverse of the given function.



Is the inverse a function? Why or why not?

Teacher

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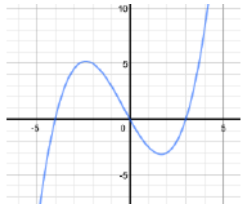
Inverse Functions

Just like the Vertical Line Test, there is a simple way to determine if a function has an inverse just from looking at its graph.

Def: The Horizontal Line Test is used to determine if a function has an inverse. Just like the Vertical Line Test, if a horizontal line crosses the function more than once, it WILL NOT have an inverse.

Move this line to check:



Teacher

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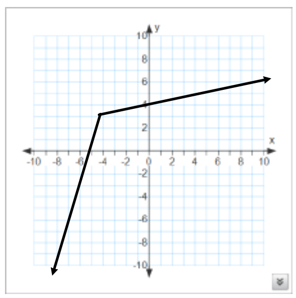
Inverse Functions

Will the inverse of the given function be a function?

- Yes
- No

Move this line to check:



Teacher

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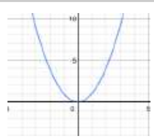
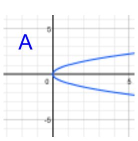
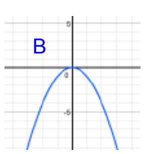
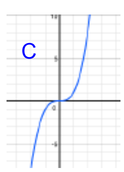
Inverse Functions

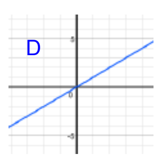
Which graph is the inverse of the given function?











Teacher

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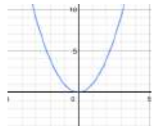
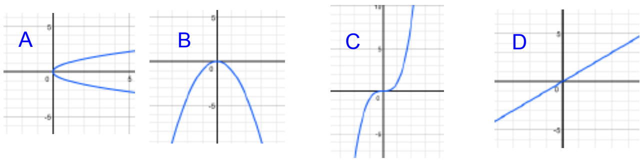
Inverse Functions

38 Which graph is the inverse of the given function?





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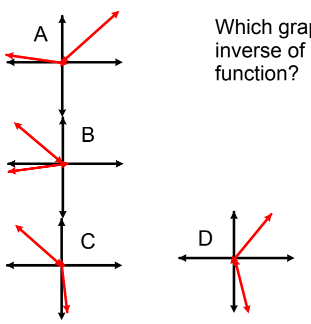
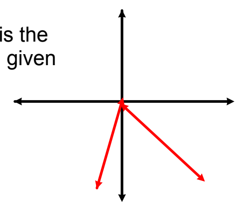
Inverse Functions



Which graph is the inverse of the given function?



Teacher

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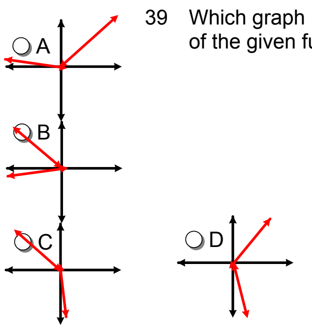
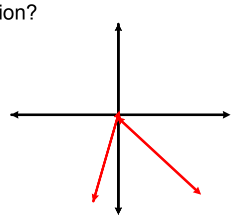
Inverse Functions

39 Which graph is the inverse of the given function?



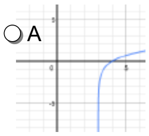


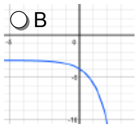
Teacher

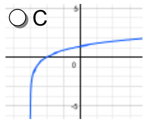
Inverse Functions

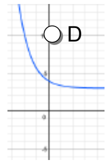
40 Which graph is the inverse of the given function?

Teacher

A 

B 

C 

D 

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Inverse Functions

41 Will the inverse of $f(x)= 2x^2 - 23x + 4$ be a function?

Teacher

Yes

No

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Inverse Functions

Finding the inverse of a function algebraically:

Knowing that the inverse of a function switches x and y values, we can take this concept further when given an equation.

Given: $y = 3x - 1$ → Switch x and y. $x = 3y - 1$ → Solve for y: $x + 1 = 3y$, $\frac{x+1}{3} = y$ → Inverse function: $y = \frac{x+1}{3}$

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Inverse Functions

Find the inverse of the following functions.

$a)\ y = 2x+1$ $b)\ f(x) = 4x+9$ $c)\ y = 3x^2 - 1$

Teacher

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Inverse Functions

42 Which of the following choices is the inverse of f(x)? $f(x) = 3x$

- A $f^{-1}(x) = \frac{x}{3}$
- B $f^{-1}(x) = \frac{3}{x}$
- C $f^{-1}(x) = -3x$
- D $f^{-1}(x) = 0$

Teacher

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Inverse Functions

43 Which of the following is the inverse of f(x)? $f(x) = 2 - 2x$

- A $f^{-1}(x) = 1 + \frac{x}{2}$
- B $f^{-1}(x) = 1 - \frac{x}{2}$
- C $f^{-1}(x) = -1 + \frac{x}{2}$
- D $f^{-1}(x) = -1 - \frac{x}{2}$

Teacher

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44 Find the inverse of $y = 2x^2 - 4$

Teacher

- A $y^{-1} = \sqrt{\frac{x}{2} - 2}$
- B $y^{-1} = \sqrt{2 - x}$
- C $y^{-1} = \pm\sqrt{2 - \frac{x}{2}}$
- D $y^{-1} = \pm\sqrt{\frac{x}{2} + 2}$

Piecewise Functions

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Goals and Objectives

Students will be able to recognize Piecewise Functions, graph them and correctly evaluate them at given points.

Why do we need this?

Recognize this problem?

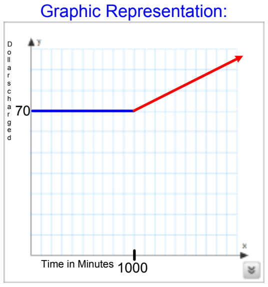
A cell phone carrier charge $70 for the first 1000 minutes and $.25 for each minute after the first 1000.

We all deal with piecewise functions and don't know it. Many common situations have limits, overages and maybe several parts. Knowing how to analyze these problems makes us smart consumers.

Let's examine the previous situation...

A cell phone carrier charge $70 for the first 1000 minutes and $.25 for each minute after the first 1000.

Graphic Representation:



In Mathematical Notation:

$$y = \begin{cases} 70 \text{ if } 0 \le x \le 1000 \\ .25(x-1000)+70 \text{ if } x > 1000 \end{cases}$$

In words:

The graph starts as a constant graph of y = 70 and after x = 1000 the graph becomes
y = .25(x - 1000) +70

Teacher

Def: A Piecewise Function is a combination of other functions.

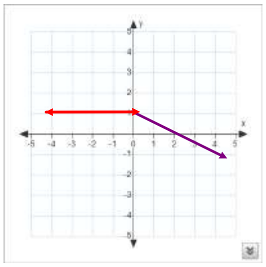
Notation:

$$y = \begin{cases} f(x) & \text{if parameters for } x \text{ in } f(x) \\ g(x) & \text{if parameters for } x \text{ in } g(x) \\ h(x) & \text{if parameters for } x \text{ in } f(x) \end{cases}$$

Note: There can be as many functions included as necessary.

Piecewise Functions

Create the piecewise notation for the following graph.



$$y = \begin{cases} & if \\ & if \end{cases}$$

Teacher

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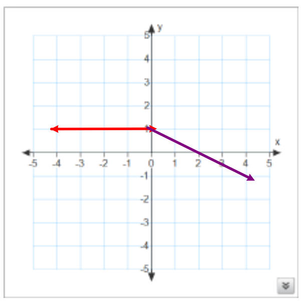
Piecewise Functions

What is the domain and range of the function?



Domain:

Range:

Teacher

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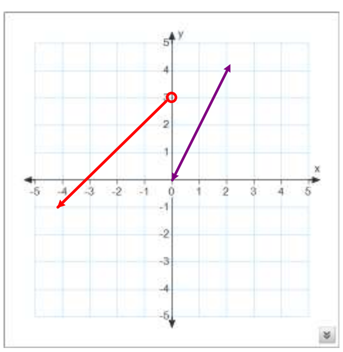
Piecewise Functions

Create the piecewise notation for the following graph.



$$y = \begin{cases} & if \\ & if \end{cases}$$

Teacher

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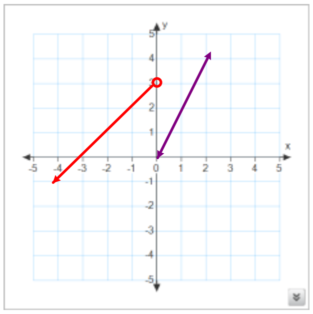
What are the domain and range of the function?

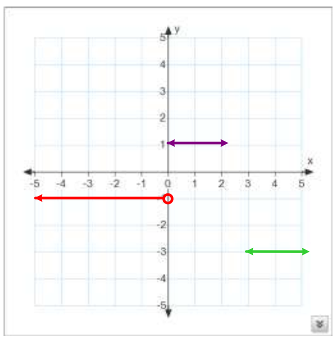


Domain:

Range:

Teacher

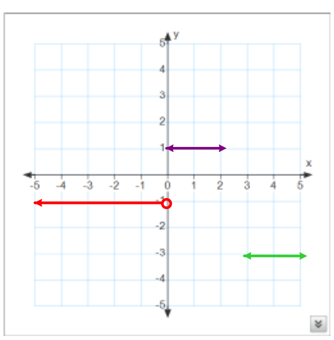
Create the piecewise notation for the following graph.



$$y = \begin{cases} & \text{if} \\ & \text{if} \\ & \text{if} \end{cases}$$

Teacher

State the domain and range of the function.



Domain:

Range:

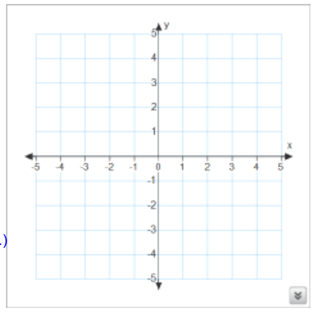
Teacher

Piecewise Functions

$$y=\begin{cases}2 & \text{if } x\le -1\\ 2x+3 & \text{if } -1<x\le 1\\ -4 & \text{if } x>1\end{cases}$$

Graphing a piecewise function:

- Visualize the entire graph of each individual piece
- Graph only the parts defined by x (Be aware of endpoints being either open or closed.)
- Repeat for each part.



Teacher

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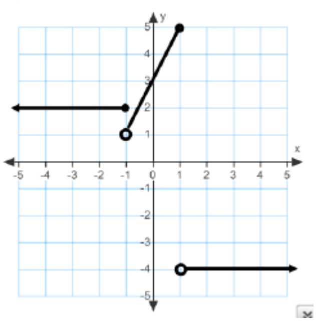
Piecewise Functions

What is the domain and range of the function?

$$y=\begin{cases}2 & \text{if } x\le -1\\ 2x+3 & \text{if } -1<x\le 1\\ -4 & \text{if } x>1\end{cases}$$

Domain:

Range:



Teacher

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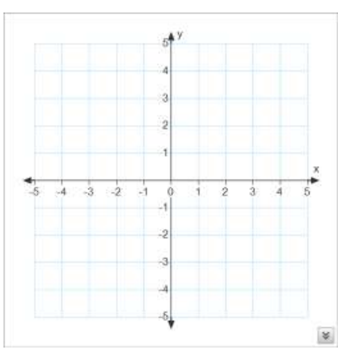
Piecewise Functions

Graph:

$$y=\begin{cases}\frac{1}{3}x^2 & \text{if } x<1\\ -2 & \text{if } x\ge 1\end{cases}$$



Teacher

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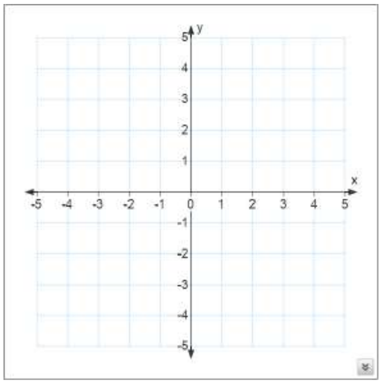
Piecewise Functions

Graph:

$$y=\begin{cases}|x-2| \text{ if } x>2\\ |x+1|-2 \text{ if } -3\le x\le 2\\ 3x+9 \text{ if } x<-3\end{cases}$$



Teacher

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Piecewise Functions

Evaluating Piecewise Functions

$$f(x)=\begin{cases}|x-2| \text{ if } x<1\\ x^2 \text{ if } x\ge 1\end{cases}$$

Find:

a) f(-3)
b) f(5)
c) f(1)
d) f(-10)

Piecewise Functions are evaluated exactly like we do regular functions. The only difference is that you will need to decide which part of the function to use depending on the value of x you are given.

Teacher

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Piecewise Functions

45 Find f(0) given:

$$f(x)=\begin{cases}-2x \text{ if } x\le 0\\ 3 \text{ if } 0<x\end{cases}$$

Teacher

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Piecewise Functions

46 Find f(1) given:

$$f(x)=\begin{cases}3x & \text{if } x<1\\ -2 & \text{if } x\geq 1\end{cases}$$

Teacher

Piecewise Functions

47 Find f(-11) given:

$$f(x)=\begin{cases}-2x^2 & \text{if } x<1\\ |-x+2| & \text{if } 1\leq x<4\\ x^3 & \text{if } x\geq 4\end{cases}$$

Teacher

Piecewise Functions

48 Find f(7) given:

$$f(x)=\begin{cases}-2x^2 & \text{if } x<1\\ |-x+2| & \text{if } 1\leq x<4\\ x^3 & \text{if } x\geq 4\end{cases}$$

Teacher

49 Find f(2) given:

$$f(x)=\begin{cases}-2x^2 & if \quad x<1\\ |-x+2| & if \quad 1\le x<4\\ x^3 & if \quad x\ge 4\end{cases}$$

Teacher

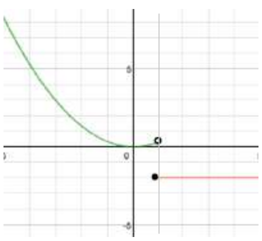
50 Find f(3) given:

$$f(x)=\begin{cases}-2x^2 & if \quad x<1\\ |-x+2| & if \quad 1\le x<4\\ x^3 & if \quad x\ge 4\end{cases}$$

Teacher

Making Piecewise Functions Continuous

Def: A continuous function is one that has no breaks, jumps, holes and will have a value for each point in the domain.



Is this function continuous?

Teacher

Piecewise Functions

We can make a piecewise function continuous by getting the open endpoint of the first equation to match the closed endpoint of the second.

$$f(x)=\begin{cases}3x+b & \text{if } x<1\\ -2b & \text{if } x\geq 1\end{cases}$$

The critical point is when x = 1.

1. Set the pieces equal to each other. → $3x+b=-2b$
2. Plug in x. → $3(1)+b=-2b$
3. Solve for b.
4. Write the new, continuous function.

$$3+b=-2b$$
$$3=-3b$$
$$-1=b$$

$$f(x)=\begin{cases}3x-1 & \text{if } x<1\\ 2 & \text{if } x\geq 1\end{cases}$$

Piecewise Functions

Find the value of b that makes the function continuous. Rewrite the function.

Teacher

$$f(x)=\begin{cases}b-3x & \text{if } x<1\\ x-2b & \text{if } x\geq 1\end{cases}$$

1) What is the critical point?

2) Find b.

3) Rewrite the function.

Piecewise Functions

Find the value of a that makes the function continuous. Rewrite the function.

Teacher

$$f(x)=\begin{cases}2a-x & \text{if } x<2\\ |x+a| & \text{if } 2\leq x\end{cases}$$

1) What is the critical point?

2) Find a.

3) Rewrite the function.

This is the end of Working with Functions.