Pre-Calculus: Exponential & Logarithmic Functions

Stage 1 Desired Results

ESTABLISHED GOALS:

Competencies:

- ***Students will demonstrate the ability to solve equations, inequalities, and systems by analyzing structure and applying the properties of equality and rational expressions.***
- ***Students will demonstrate the ability to apply functions to solve problems by interpreting and analyzing multiple representations of functions.***
- ***Students will demonstrate the ability to graph equations, functions, and figures by using tables and analyzing equations.***
- ***Students will demonstrate the ability to model real world problems by building and analyzing the appropriate expression, equation, or function.***
- ***Students will demonstrate the ability to analyze and summarize text and integrate knowledge to make meaning of discipline-specific materials.***
- ***Students will demonstrate the ability to produce coherent and supported writing in order to communicate effectively for a range of discipline-specific tasks, purposes, and audiences.***
- ***Students will demonstrate the ability to speak purposefully and effectively by strategically making decisions about content, language use, and discourse style.***

Content Standards:

- HSA.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- HSF.IF.C.7.E Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- HSF.IF.C.8.B Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing

Transfer

Students will be able to independently use their learning to ***make informed decisions based on their understanding of exponential growth and decay.***

Meaning

ENDURING UNDERSTANDINGS

Students will understand that…

- a constant percent increase or decrease at regular intervals always results in an exponential relationship
- an inverse function reverses the effect of the original function.

ESSENTIAL QUESTIONS

- How do different factors alter an outcome?

Acquisition

Students will know…

- *that the domain of a function is the range of its inverse, and the range of a function is the domain of its inverse.*
- *that composing a function with its inverse results in x.*
- *that a graph and its inverse are symmetric about the line* y=x.
- *that a function has an inverse if and only if that function is one-to-one.*
- *that the horizontal line test can be used to determine if a function is one-to-one.*
- *the domain, range, and graph of* $y=2^x$.
- *the domain, range, and graph of* $y=e^x$.
- *that log function and exponential functions are inverses.*
- *the domain, range, and graph of* $y=\log_b x$.
- *properties of logarithms.*
- *that using the properties of logarithms to solve logarithmic equations may introduce extraneous solutions that do not check in the original*

Students will be skilled at…

- *adding composing functions and determining the resulting domain.*
- *computing iterates of a function.*
- *finding the inverse of a function.*
- *determining if two functions are inverses of each other.*
- *using symmetry to sketch the graph of an inverse function.*
- *simplifying expressions containing irrational exponents.*
- *solving equations with the unknown as an exponent*
- *sketching the graph of* $y=a^x$.
- *sketching transformations of* $y=a^x$.
- *sketching the graph of* $y=e^x$.
- *sketching transformations of* $y=e^x$.
- *using both the logarithmic and exponential forms of equations .*
- *sketching transformations of* $y=\log_b x$.
- *sketching transformations of y=ln x.*
- *solving equations and inequalities involving*

exponential growth or decay. • HSF.BF.A.1.C (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. • HSF.BF.B.4.B (+) Verify by composition that one function is the inverse of another. • .HSF.BF.B.4.C (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. • HSF.BF.B.4.D (+) Produce an invertible function from a non-invertible function by restricting the domain. • HSF.BF.B.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. • HSF.LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. • HSF.LE.A.4 For exponential models, express as a logarithm the solution to $ab_{ct} = d$ where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. • MP2 Reason abstractly and quantitatively. • MP4 Model with mathematics. • MP7 Look for and make use of structure. • MP8 Look for and express regularity in repeated reasoning.	*equation.* • *properties of inequalities involving exponential and logarithmic functions.* • *logarithm function require positive inputs, and this may affect the solution set for a function involving log functions.* <u>vocabulary:</u> *iterate, inverse function, one-to-one, exponential, asymptote, e, logarithmic, natural log*	*exponential, logarithmic, and natural logarithmic functions.* • *simplifying and expanding expressions using log properties.* • *converting between two bases.*
Content Area Literacy Standards		***21st Century Skills***
• RST.11-12.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text. • RST.11-12.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11-12 texts and topics*. • RST.11-12.5 Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas. • RST.11-12.7 Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem. • RST.11-12.8 Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information. • RST.11-12.9 Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.		• *reason effectively* • *use systems thinking* • *make judgments and decisions* • *solve problems* • *assess and evaluate information* • *access technology effectively* • *communicate clearly*

Stage 2 - Evidence

Evaluative Criteria	***Assessment Evidence***
	PERFORMANCE TASK(S):

	OTHER EVIDENCE:

Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

Language Arts Integration	***Mathematics Integration***
• 1.OA.1 Use	• 1.OA.1 Use
Technology Integration	***District Materials***
• 1.OA.1 Use	