Pre-Calculus: Polynomial & Rational Functions

Stage 1 Desired Results

ESTABLISHED GOALS:

Competencies:

- ***Students will demonstrate the ability to simplify algebraic expressions by applying the properties of operations and arithmetic of complex numbers.***
- ***Students will demonstrate the ability to solve equations, inequalities and systems by analyzing structure and applying the properties of equality, inequality, and rational expressions.***
- ***Students will demonstrate the ability to apply functions to solve problems by interpreting and analyzing multiple representations of functions.***
- ***Students will demonstrate the ability to graph equations, functions, and figures by using tables and analyzing equations.***
- ***Students will demonstrate the ability to model real world problems by building and analyzing the appropriate expression, equation, or function.***
- ***Students will demonstrate the ability to analyze and summarize text and integrate knowledge to make meaning of discipline-specific materials.***
- ***Students will demonstrate the ability to produce coherent and supported writing in order to communicate effectively for a range of discipline-specific tasks, purposes, and audiences.***
- ***Students will demonstrate the ability to speak purposefully and effectively by strategically making decisions about content, language use, and discourse style.***

Content Standards:

- HSN.CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- HSN.CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- HSN.CN.A.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- HSN.CN.B.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- HSN.CN.B.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
- HSN.CN.B.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Transfer

*Students will be able to independently use their learning to **represent relationships between quantities, manipulate and analyze the representation, and interpret its meaning.***

Meaning

ENDURING UNDERSTANDINGS

Students will understand that…

- equivalent expressions can be created by using multiple strategies and the guiding rules of mathematics.
- the graph and equation of a function communicate information that can be used to answer questions about real world problems.
- it is possible to get closer and closer to something but to never be able to touch it.

ESSENTIAL QUESTIONS

- How can the multiple representations of a function be used to best analyze the relationships between the two quantities it models?

Acquisition

Students will know…

- *the average rate of change of a function f on the interval [a,b] is the slope of the line joining two points (a, f(a)) and (b, f(b)).*
- *that a graph of a function can be sketched by identifying the parent function and then analyzing and interpreting the changes made to f(x), for example –f(x) or f(x+c).*
- *that given $f(x)=x^n$, when n is even, the graph of the function is symmetric with respect to the y-axis, and when n is odd, it is symmetric with respect to the origin.*
- *that, if the multiplicity of a polynomial's zero is odd, the graph crosses the x-axis at that value. If it is even, then it touches, but does not cross, the x-axis at that value.*
- *that the graph of a polynomial function of degree 2 or greater is a continuous smooth*

Students will be skilled at…

- *calculating the instantaneous rate of change using the average rate of change.*
- *computing , interpreting, and comparing average rate of change for a function.*
- *simplifying difference quotients of the form $f(x)-f(a) \div x-a$.*
- *determining the domain and range of a function from both an equation and from a graph.*
- *graphing the parent function and transformations for linear, quadratic, square root, and absolute value functions.*
- *graphing transformations by viewing them as changes made to the parent function.*
- *setting up equations that define functions for problem-solving, including maximum and minimum value problems.*

• HSN.CN.C.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite x2 + 4 as (x + 2i)(x - 2i)*. • HSN.CN.C.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. • HSA.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. • HSA.APR.B.2 Know and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x - a) is a factor of p(x). • HSA.APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. • HSA.APR.C.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial *identity (x2 + y2)² = (x2 - y2)² + (2xy)²* can be used to generate Pythagorean triples. • HSA.APR.C.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.1 • HSA.APR.D.6 Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) +r(x)/b(x)*, where *a(x), b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x),* using inspection, long division, or, for the more complicated examples, a computer algebra system • HSA.APR.D.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. • HSA.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)*; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. • HSF.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. • HSF.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* • HSF.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. • HSF.IF.C.7.C Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. • HSF.IF.C.7.D (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing	*curve.* • *leading Coefficient Test for polynomial functions.* • *that, for a polynomial function of degree, n has at most, n real zeros.* • *that the graph of a polynomial function of degree n has at most n-1 turning points.* • *the relationships between roots and coefficients for polynomials equations of any degree.* • *that complex zeros occur in conjugate pairs.* • *Descartes's Rule of Sign.* • *that, if the degree of the numerator is exactly one more than the degree of the denominator, then the graph of the function has a slant asymptote.* • *that, within an interval of the number line created by the x-intercepts, the sign of the value of the function stays the same.* • *the following algorithms and theorems:* o *Division Algorithm* o *Remainder Theorem* o *Intermediate Value Theorem* o *Factor Theorem* o *Fundamental Theorem of Algebra* o *Linear Factorization Theorem* o *Rational Roots Theorem* o *Upper and Lower Bound Theorem for* o *Upper and Lower Bound Theorem for Real Roots.* *<u>vocabulary:</u> rational function, key numbers, vertical/horizontal/slant asymptote, complex conjugate, double root, extraneous roots, persistence of sign, , synthetic division, even and odd functions*	• *sketching polynomial functions, using the leading coefficient test, the real zeros, and test intervals.* • *using the Intermediate Value Theorem to approximate a real zero.* • *sketching rational functions.* • *using long and synthetic division to find quotients and remainders and to show that a value of x is a solution to the equation.* • *factoring polynomials.* • *finding solutions to polynomial equations using the Factor Theorem.* • *simplifying rational expressions by factoring and by using long/synthetic division.* • *adding, subtracting, and multiplying complex numbers.* • *determining the quotient of complex numbers in standard form.* • *writing complex numbers in standard form* • *expressing polynomial equations in the form* $a_n(x-r_1)(x-r_2)...(x-r_n)$ • *determining a polynomial equation with prescribed roots and their multiplicities.* • *finding the rational roots of a polynomial equation.* • *solving an equation using the rational roots theorem and the upper and lower bound theorem.* • *solving an equation using the conjugate roots theorem.* • *determining remaining roots when provided one root.* • *using Descartes's Rule to verify the number of positive/negative roots.* • *determining the vertical and horizontal asymptotes of a rational function.* • *sketching the graph of rational functions.* • *finding the equation of a slant asymptote.* • *solving polynomial inequalities using factoring, key numbers, and the persistence of sign.*

end behavior. • MP1 Make sense of problems and persevere in solving them. • MP2 Reason abstractly and quantitatively. • MP4 Model with mathematics. • MP5 Use appropriate tools strategically. • MP7 Look for and make use of structure. • MP8 Look for and express regularity in repeated reasoning.		
Content Area Literacy Standards		***21st Century Skills***
• RST.11-12.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text. • RST.11-12.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11-12 texts and topics*. • RST.11-12.5 Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas. • RST.11-12.8 Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.		• *reason effectively* • *use systems thinking* • *solve problems* • *apply technology effetively*

Stage 2 - Evidence

Evaluative Criteria	***Assessment Evidence***
	PERFORMANCE TASK(S):
	OTHER EVIDENCE:

Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

Language Arts Integration	***Mathematics Integration***
• 1.OA.1 Use	• 1.OA.1 Use

Technology Integration	***District Materials***
• 1.OA.1 Use	