Unit 1: Equations and Polynomials

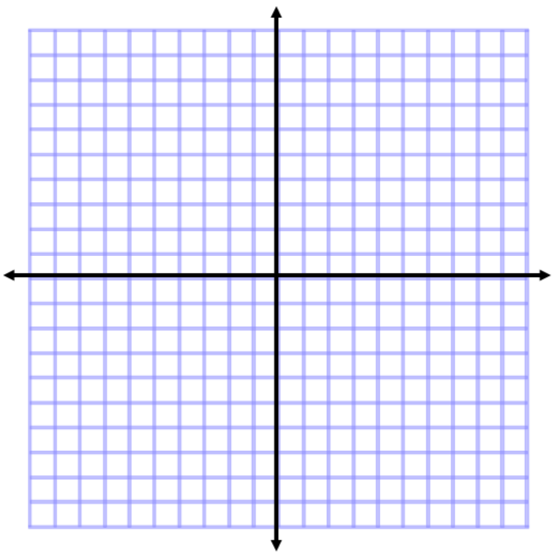
Lesson 7: Linear Programming

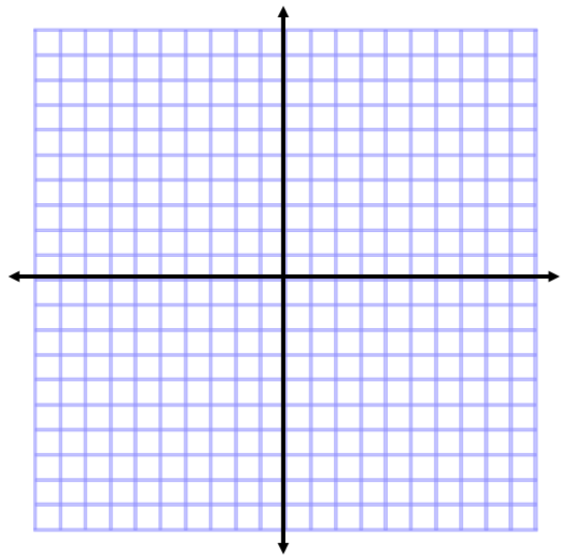
<u>Warm-Up</u>

Solve each system of inequalities by graphing

1. $\begin{cases} x \geq 5 \\ y > -3x + 6 \end{cases}$

2. $\begin{cases} x + 3y < -6 \\ 2x - 3y \leq 4 \end{cases}$





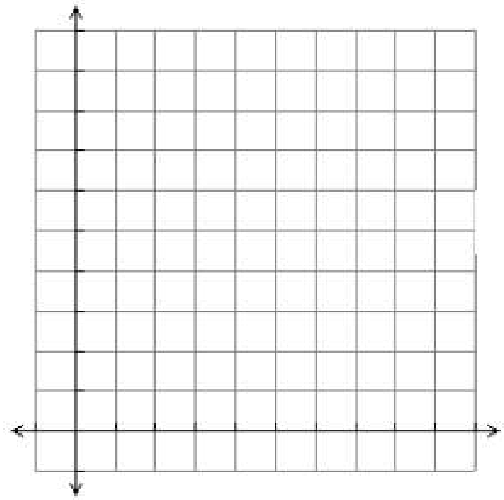
<u>LINEAR PROGRAMMING:</u>

<u>VERTEX PRINCIPAL OF LINEAR PROGRAMMING:</u>

Example # 1

Find the values of x and y that maximize and minimize P for the objective function $P = 3x + 2y$. What is the value of P at each vertex?

$$\text{Constraints}\begin{cases} y \geq \frac{3}{2}x - 3 \\ y \leq -x + 7 \\ x \geq 0, y \geq 0 \end{cases}$$

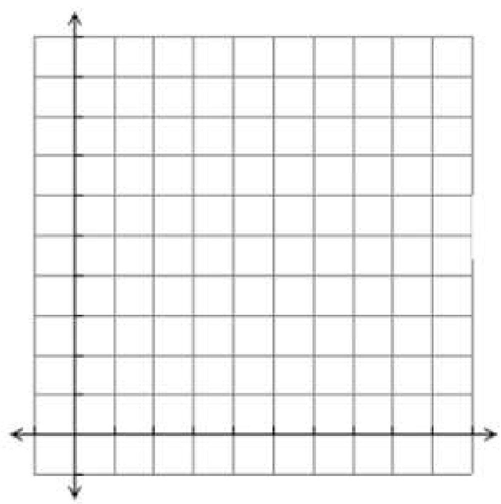


Example # 2

$$\begin{cases} x + y \leq 8 \\ 2x + y \leq 10 \\ x \geq 0, y \geq 0 \end{cases}$$

Maximum for

$N = 100x + 40y$

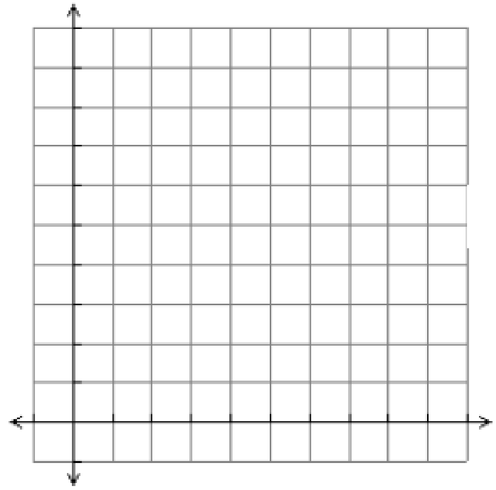


PRACTICE 1-6 LINEAR PROGRAMMING

1. $\begin{Bmatrix} x+y\geq 8 \\ y\geq 5 \\ x\geq 7 \end{Bmatrix}$

Minimum for

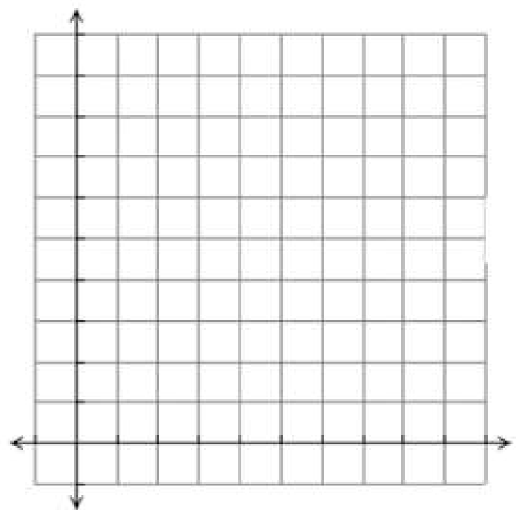
P = 3x + y



2. $\begin{Bmatrix} x+y\geq 6 \\ y\leq 5 \\ x\leq 8 \end{Bmatrix}$

Maximum for

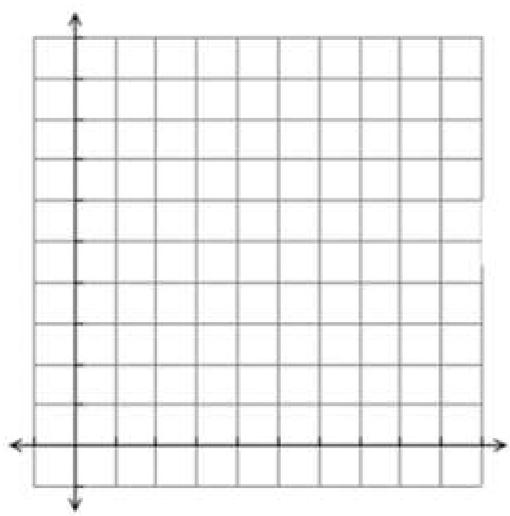
C = x + 3y



3. $\begin{Bmatrix} x+2y\geq 8 \\ x\geq 2 \\ y\geq 0 \end{Bmatrix}$

Maximum for

C = 2x + 3y



4. Superbats Inc. manufactures two different types of wood baseball bats, the Homer-Hitter and the Big Timber. The Homer-Hitter takes 8 hours to trim and turn on the lathe and 2 hours to finish. Each Homer-Hitter sold makes a profit of $17. The Big Timber takes 5 hours to trim and turn on the lathe and 5 hours to finish, and its profit is $29. The total time available for trimming and lathing is 80hours. The total available time for finishing is 50 hours.

(a) What are your variables and what do they represent?

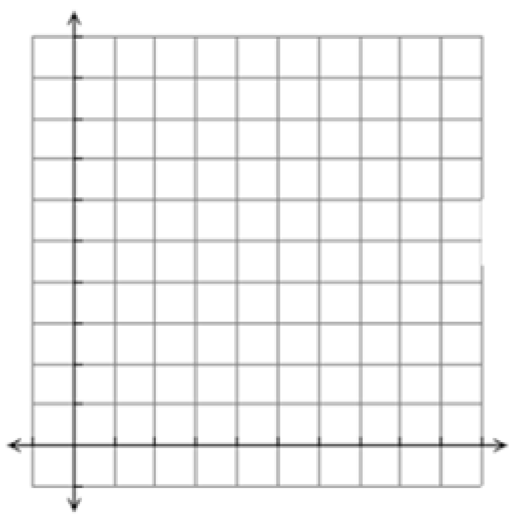
(b) Write the objective quantity equation.

(c) Write the system of inequalities that describes the constraints.

(d) Graph the system of inequalities and find the vertices.

(e) How many of each type should be produced in order to maximize their profit? What is the

maximum profit?



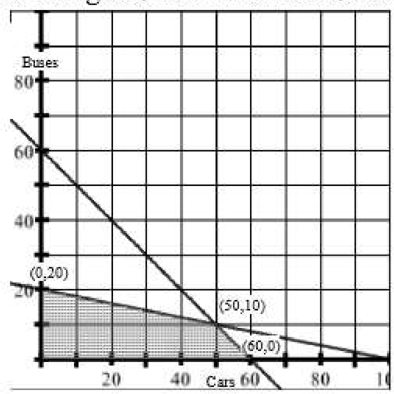
NAME ____ DATE ____________

Unit 1 Lesson 7 part 2 Graded Practice

1. The area of a parking lot is 600 square meters. A car requires 6 square meters. A bus requires 30 square meters. The attendant can handle only 60 vehicles. If a car is charged \$2.50 and a bus \$7.50, how many of each should be accepted to maximize income?

Constraints:
$c \geq 0;\ b \geq 0$
$c + b \leq 60$
$6c + 30b \leq 600$
Profit:
$P(c,b) = 2.5c + 7.5b$

	Car (c)	Bus (b)	Combined
Area:	6	30	600
Quantity:			60
\$:	\$2.50	\$7.50	

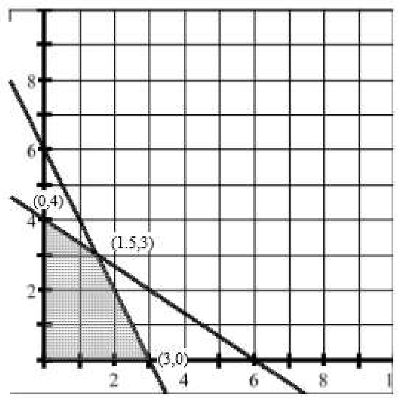


2. The B & W Leather Company wants to add handmade belts and wallets to its product line. Each belt nets the company \$18 in profit, and each wallet nets \$12. Both belts and wallets require cutting and sewing. Belts require 2 hours of cutting time and 6 hours of sewing time. Wallets require 3 hours of cutting time and 3 hours of sewing time. If the cutting machine is available 12 hours a week and the sewing machine is available 18 hours per week, what ratio of belts and wallets will produce the most profit within the constraints?

Constraints:
$b \geq 0;\ w \geq 0$
Cutting: $2b + 3w \leq 12$
Sewing: $6b + 3w \leq 18$

	Belts (b)	Wallets (w)	Combined
Cutting:	2	3	12
Sewing:	6	3	18
\$:	\$18	\$12	

Profit:
$P(b,w) = 18b + 12w$

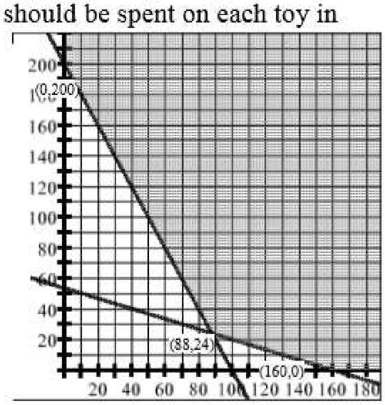


3. Toys-A-Go makes toys at Plant A and Plant B. Plant A needs to make a minimum of 1000 toy dump trucks and fire engines. Plant B needs to make a minimum of 800 toy dump trucks and fire engines. Plant A can make 10 toy dump trucks and 5 toy fire engines per hour. Plant B can produce 5 toy dump trucks and 15 toy fire engines per hour. It costs \$30 per hour to produce toy dump trucks and \$35 per hour to operate produce toy fire engines. How many hours should be spent on each toy in order to minimize cost? What is the minimum cost?

Constraints:
$d \geq 0; f \geq 0$
Plant A: $10d + 5f \geq 1000$
Plant B: $5d + 15f \geq 800$

Cost:
$C(x,y) = 30d + 35f$

	Dump hrs (d)	Fire hrs (f)	Combined
Plant A:	10	5	1000
Plant B:	5	15	800
\$:	\$30	\$35	



4. A diet is to include at least 140 milligrams of Vitamin A and at least 145 milligrams of Vitamin B. These requirements can be obtained from two types of food. Type X contains 10 milligrams of Vitamin A and 20 milligrams of Vitamin B per pound. Type Y contains 30 milligrams of Vitamin A and 15 milligrams of Vitamin B per pound. If type X food costs $12 per pound and type Y food costs $8 per pound how many pounds of each type of food should be purchased to satisfy the requirements at the minimum cost?

Constraints:

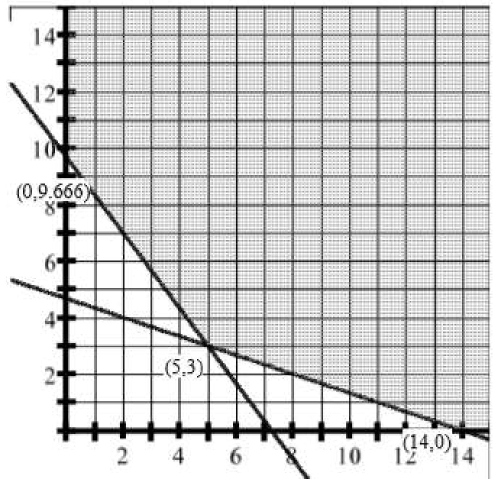
$x \geq 0;\ y \geq 0$

Vit A: $10x + 30y \geq 140$

Vit. B: $20x + 15y \geq 145$

	Food X	Food Y	Combined
Vit A:	10	30	140
Vit B:	20	15	145
$:	$12	$8	

Cost:
$C(x, y) = 12x + 8y$



5. The Cruiser Bicycle Company makes two styles of bicycles: the Traveler, which sells for $300, and the Tourister, which sells $600. Each bicycle has the same frame and tires, but the assembly and painting time required for the Traveler is only 1 hour, while it is 3 hours for the Tourister. There are 300 frames and 360 hours of labor available for production. How many bicycles of each model should be produced to maximize revenue?

Traveler $= x$
Tourister $= y$

	Traveler (x)	Tourister (y)	Combined
Frame:			300
Labor:	1	3	360
$:	$300	$600	

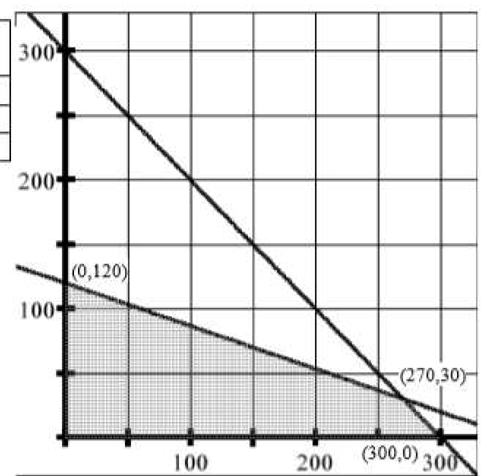
Constraints:

$x \geq 0;\ y \geq 0$

Frames: $x + y \leq 300$

Labor: $x + 3y \leq 360$

Revenue:
$R(x, y) = 300x + 600y$



6. One of the dolls that Dolls R Us manufactures is called the Talking Tommy doll. Another doll without the talking mechanism is called the Silent Sally doll. Eight Talking Tommy dolls can be produced in one hour. Twenty Silent Sally dolls can be made in the same time. Because of the demand, the company must produce at least twice as many Talking Tommy dolls as Silent Sally dolls. The company spends no more than 48 hours a week producing these two dolls. The profit on each Talking Tommy is $3.00, and the profit for each Silent Sally is $7.50.

(a) What are your variables and what do they represent?

(b) Write the objective quantity equation.

(c) Write the system of inequalities that describes the constraints.

(d) Graph the system of inequalities and find the vertices.

(e) How many of each type should be produced in order to maximize profit? What is the maximum profit?