New Jersey Center for Teaching and Learning

Progressive Science Initiative

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Uniform Circular Motion

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Uniform Circular Motion (UCM)

Click on the topic to go to that section

Period, Frequency, and Rotational Velocity

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Circular Motion

In this chapter we will study the motion of objects traveling in circular paths.

In previous chapters, we have considered objects at rest and objects moving in straight paths.

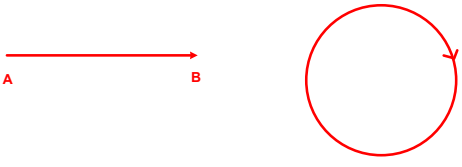
What do you think are some of the differences are between 'linear' and 'circular' motion?

Circular Motion

One of the main differences between linear and circular motion is that circular motion repeats!

This difference can change the way we think about motion a little:

For instance, instead of measuring the total time it takes for an object to get from point A to point B, we can measure the time it takes to go around a circle once.



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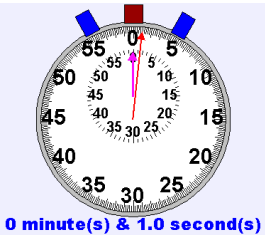
Period

The time it takes for an object to complete one trip around circular path is called its Period.

The symbol for Period is "T"

Periods are measured in units of time; we will use seconds (s).



http://njc.tl/ar

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Period

Often we are given the time (t) it takes for an object to make a number of trips (n) around a circular path. In that case,

$$T = \frac{t}{n} \qquad \frac{\text{The time}}{\text{The number of trips}}$$

http://njc.tl/ar

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1 What is the period of the second hand on a clock?

- A 60 seconds
- B 1 hour
- C 12 hours
- D it depends on the time

Answer

http://njc.tl/b5

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2 What is the period of the minute hand on a clock?

- A 60 seconds
- B 1 hour
- C 12 hours
- D it depends on the time

Answer

http://njc.tl/b6

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3 What is the period of the hour hand on a clock?

- A 60 seconds
- B 1 hour
- C 12 hours
- D it depends on the time

Answer

http://njc.tl/b7

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4 If it takes 50 seconds for an object to travel around a circle 5 times, what is the period of its motion?

Answer

http://njc.tl/9y

5 **If a runner can make a full trip around a small track in 90 seconds, how much time would it take to complete 6 loops of the track?**

Answer

http://njc.tl/b8

6 **If an object is traveling in circular motion and its period is 3.0s, how many revolutions will it complete in 1 minute?**

Answer

http://njc.tl/b9

Frequency

Another useful measurement in circular motion is *how often* a cycle repeats.

The number of revolutions that an object completes in a given amount of time is called the frequency of its motion.

The symbol for frequency is "f" and the units we measure with are "Hz" (hertz)


f = 0.5 Hz
T = 2.0 s


f = 1.0 Hz
T = 1.0 s


f = 2.0 Hz
T = 0.5 s

http://njc.tl/as

Frequency

Often we are given the time (t) it takes for an object to make a number of revolutions (n). In that case,

$$f = \frac{n}{t} \qquad \frac{\textit{number of revolutions}}{\textit{The time}}$$

A common example of frequency can be seen on the dashboard of a car, noting revolutions per minute (rpm) of the engine crankshaft.

http://njc.tl/as

7 **An object travels around a circle 50 times in ten seconds, what is the frequency (in Hz) of its motion?**

Answer

http://njc.tl/a0

8 **If a wheel spins with a frequency of 8 Hz, how much time will it take to spin around 16 times?**

Answer

http://njc.tl/ba

9 **If an object is traveling in circular motion with a frequency of 7.0 Hz, how many revolutions will it make in 20s?**

Answer

http://njc.tl/bb

Period and Frequency

Period	Frequency
$T = \frac{t}{n}$	$f = \frac{n}{t}$

These two equations look similar.
In fact, they are exactly opposite one another.

$\frac{t}{n}$ $\frac{n}{t}$

Another way to say this is that they are inverses.

http://njc.tl/ay

Period and Frequency

We can relate them mathematically:

Period is the inverse of Frequency	$T = \frac{1}{f}$
Frequency is the inverse of Period	$f = \frac{1}{T}$

http://njc.tl/ay

10 **An object has a period of 4.0s. What is the frequency of its motion?**

Answer

http://njc.tl/a2

11 **An object is revolving with a frequency of 8.0 Hz. What is its period?**

Answer

http://njc.tl/a3

Rotational Velocity

In kinematics, we defined the speed of an object as

$$s = \frac{d}{t}$$

For an object moving in a circle, instead of using t (time), we will measure the speed with T (period), the time it takes to travel around a circle.

To find the speed, then, we need to know the distance around the circle.

Another name for the distance around a circle is the circumference.

http://njc.tl/at

Rotational Velocity

Each trip around a circle, an object travels a length equal to the circle's circumference.

The circumference of a circle is given by $C = 2\pi r$

The time it takes to go around once is the period T

And the object's speed is given by $s = \frac{d}{t}$

So the magnitude of its velocity must be:

$$v = \frac{2\pi r}{T}$$

http://njc.tl/at

*Rotational Velocity

A velocity must have a magnitude and a direction.

The magnitude of an object's instantaneous velocity is its speed. So for an object in uniform circular motion, the magnitude of its velocity is:

$$v = \frac{2\pi r}{T}$$

If an object is in uniform circular motion, the direction of its velocity is always changing!

We say the velocity is tangent to its circular motion.

http://njc.tl/at

*** 12 An object is in circular motion. The radius of its motion is 2.0 m and its period is 5.0s. What is its velocity?**

Answer

http://njc.tl/a5

***13 An object is in circular motion. The radius of its motion is 2.0 m and its velocity is 20 m/s. What is its period?**

Answer

http://njc.tl/a6

***14 An object is in circular motion. The period of its motion is 2.0 s and its velocity is 20 m/s. What is the radius of its motion?**

Answer

http://njc.tl/a7

*Rotational Velocity

Since $f = \frac{1}{T}$, we can also determine the velocity of an object in uniform circular motion by the radius and frequency of its motion.

$v = \frac{2\pi r}{T}$ *and* $f = \frac{1}{T}$ *so* $v = 2\pi r f$

Of course the direction of its velocity is still tangent to its circular motion.

http://njc.tl/au

*15 **An object is in circular motion. The radius of its motion is 3.0 m and its frequency is 0.25Hz. What is its velocity?**

Answer

http://njc.tl/bc

*16 **An object is in circular motion. The radius of its motion is 2.0 m and its velocity is 30 m/s. What is its frequency?**

Answer

http://njc.tl/a8

*17 **An object is in circular motion. The frequency of its motion is 7.0 Hz and its velocity is 20 m/s. What is the radius of its motion?**

Answer

http://njc.tl/a9

Kinematics of UCM

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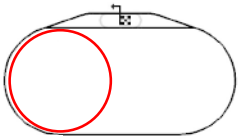
Kinematics of Uniform Circular Motion

We say that an object is in Uniform Circular Motion if its motion is in a circle of constant radius at constant speed.

Think about NASCAR for a moment. When those drivers race around a corner, its much like they are traveling on the arc of a circle.



Click here to see a video of the first corner at Texas Motor Speedway.

http://njc.tl/av

Kinematics of Uniform Circular Motion

At Texas Motor Speedway, the NASCAR drivers go around corner one traveling at speeds of 200mph (over 300km/hr)!

What is going on with their acceleration when they are entering the corner?

Is the car accelerating?

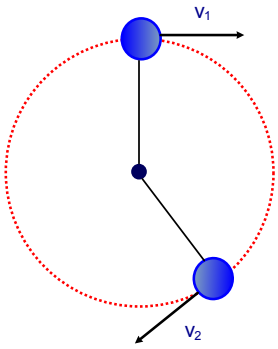
Click here for a video explanation of what is going on around the corner on that speedway

http://njc.tl/av

Kinematics of Uniform Circular Motion

As we said earlier, an object is in Uniform Circular Motion if its motion is in a circle of constant radius at constant speed.

The direction of velocity is always changing, but the distance from the center of the circle remains the same, and so does the time it takes to travel completely around the circle.



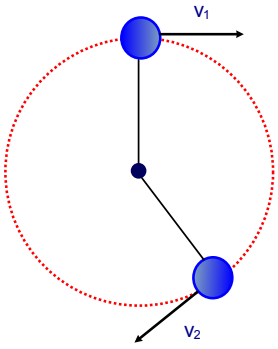
http://njc.tl/av

Kinematics of Uniform Circular Motion

Imagine the direction that a ball swinging in a circle on a string is actually moving at any point in time...

If you were to cut the string, the ball would keep moving straight out of the circular path.

This direction of the velocity is actually perpendicular to the radius of the circle (the string), and it is called tangent to the circle.



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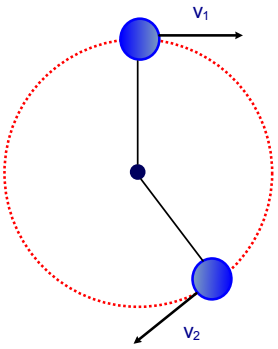
Kinematics of Uniform Circular Motion

If we think of a ball on a string, we know the ball is moving tangent to the string, but which direction is the string pulling?



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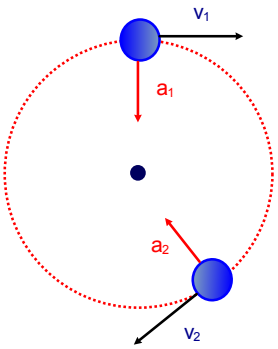
Kinematics of Uniform Circular Motion

The string is actually pulling toward the center of the circle... no matter what point you look at!

This string is applying a force toward the center, so there must be an acceleration toward the center of the circle.



http://njc.tl/av

Kinematics of Uniform Circular Motion

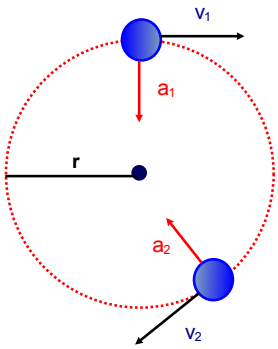
This acceleration is called the centripetal, or radial, acceleration.

It direction is always towards the center of the circle.

It's magnitude is given by

$$a = \frac{v^2}{r}$$

where r is the radius of the circle



http://njc.tl/av

18 Is it possible for an object moving with a constant speed to accelerate? Explain.

A No, if the speed is constant then the acceleration is equal to zero.

B No, an object can accelerate only if there is a net force acting on it.

C Yes, although the speed is constant, the direction of the velocity can be changing.

D Yes, if an object is moving it can experience acceleration

Answer

http://njc.tl/ab

19 **An object moves in a circular path at a constant speed. Compare the direction of the object's velocity and acceleration vectors.**

A Both vectors point in the same direction

B The vectors point in opposite directions

C The vectors are perpendicular

D The question is meaningless, since the acceleration is zero

Answer

http://njc.tl/ac

20 **What type of acceleration does an object moving with constant speed in a circular path experience?**

A free fall

B constant acceleration

C linear acceleration

D centripetal acceleration

Answer

http://njc.tl/ad

21 **Consider a particle moving with constant speed such that its acceleration of constant magnitude is always perpendicular to its velocity.**

A It is moving in a straight line

B It is moving in a circle

C It is moving in a parabola

D None of the above is definitely true all of the time

Answer

http://njc.tl/ae

22 **An object is traveling with a velocity of 6.0 m/s in a circular path whose radius is 4.0m. What is the magnitude of it centripetal acceleration?**

Answer

http://njc.tl/af

23 **An object is traveling with a velocity of 6.0 m/s in a circular path. It's acceleration is 3.0 m/s^2. What is the radius of its path?**

Answer

http://njc.tl/ag

24 **An object is traveling in a circular path whose radius is 65m. Its acceleration is 3.0 m/s^2. What is its velocity?**

Answer

http://njc.tl/ah

25 **An object is traveling in a circle with a radius of 2m at a period of 4s. Find its acceleration.**

Answer

http://njc.tl/bd

26 **An object is traveling in a circle with a radius of 2m at a frequency of 3 Hz. Find its acceleration.**

Answer

http://njc.tl/be

27 **An object is traveling in a circular path whose radius is 65m. Its acceleration is 3.0 m/s^2. What is the period of its motion?**

Answer

http://njc.tl/bf

Dynamics of UCM

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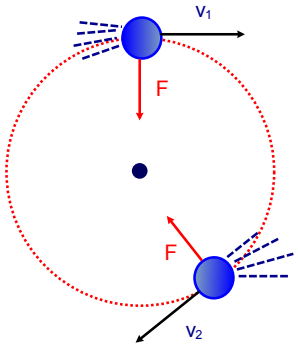
Dynamics of Uniform Circular Motion

For an object to be in uniform circular motion, there must be a net force acting on it.

We already know the centripetal acceleration, so we can similarly write the net force:

$$\Sigma F = ma = \frac{mv^2}{r}$$



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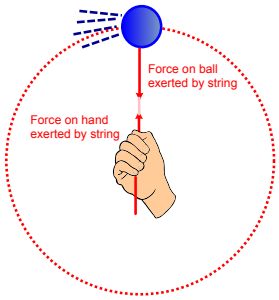
Dynamics of Uniform Circular Motion

We can see that the force must be inward by thinking about a ball on a string:



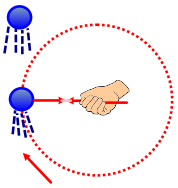
http://njc.tl/aw

Centrifugal Force Does Not Exist

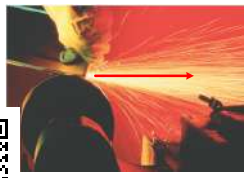
Despite what you may have heard, there is no "centrifugal" force pointing outward.

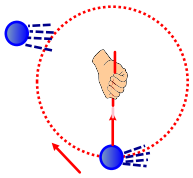
What happens is that the natural tendency of the object to move in a straight line must be overcome.

If the centripetal force vanishes, the object flies off tangent to the circle.



This happens.





This never happens.

http://njc.tl/aw

28 **What force is needed to make an object move in a circle?**

- **A** kinetic friction
- **B** static friction
- **C** centripetal force
- **D** weight

Answer

http://njc.tl/aj

29 **When an object experiences uniform circular motion, the direction of the net force is**

- **A** in the same direction as the motion of the object
- **B** in the opposite direction of the motion of the object
- **C** is directed toward the center of the circular path
- **D** is directed away from the center of the circular path

Answer

http://njc.tl/ak

30 **A car with a mass of 1800 kg goes around an 18 m radius turn at a speed of 35 m/s. What is the centripetal force on the car?**

Answer

http://njc.tl/6j

31 **A ball on a string spins in a horizontal circle of radius 2.5m with a speed of 3 m/s. The tension in the string is 5.4 N. What is the mass of the ball?**

Answer

32 **A 75 kg mass is attached to the end of a 5.0 m long metal rod string which rotates in a horizontal circular path. If the maximum force that the rod can withstand is 8500 N. What is the maximum speed that the mass can attain without breaking the rod?**

Answer

http://njc.tl/6k

33 **What is the net force on a 2kg mass spinning in a circle with radius 1.5 m with a frequency of 4 Hz?**

Answer

Applications: Vertical Uniform Circular Motion

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Car on a hilly road....

Have you ever been driving in a car or a bus that went over a bump, or into a dip? Do you remember feeling slightly more light or heavy than usual? You probably felt a funny feeling in your stomach...

This is not a trick of the mind... when an object rides over a hill or into a dip, the normal force from the surface is effected by centripetal motion, and it changes how heavy you feel!

http://njc.tl/ax

Or floating in Zero G....

We sometimes call this 'feeling' of heaviness or lightness an object's *apparent weight*.

Apparent weight is what we draw on our free body diagram as Normal Force.

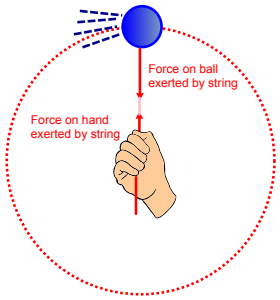
It is the same as the force of a surface pushing against an object.

NASA has a reduced gravity aircraft which can provide periods of weightlessness

http://njc.tl/ax

Dynamics of Uniform Circular Motion

We can see that the force must be inward by thinking about a ball on a string:

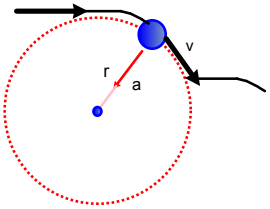


http://njc.tl/ax

Curved Paths

Why a hill or a dip?

The concept of circular motion can be used for an object moving along any curved path, as a small segment of the path will be approximately circular.



http://njc.tl/ax

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A car is traveling at a velocity of 20 m/s. The driver of the car has a mass of 60 kg.The car is located at the bottom of a dip in the road. The radius of the dip is 80m.

What is the apparent weight of the driver (the normal force supplied by the seat of the car to support him) at the bottom of the dip?

v = 20 m/s
m = 60 kg
r = 80 m

http://njc.tl/ax

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This is a sketch of the problem.

What next?

Draw a free body diagram and indicate the direction of acceleration.

v = 20 m/s
m = 60 kg
r = 80 m

Answer

http://njc.tl/ax

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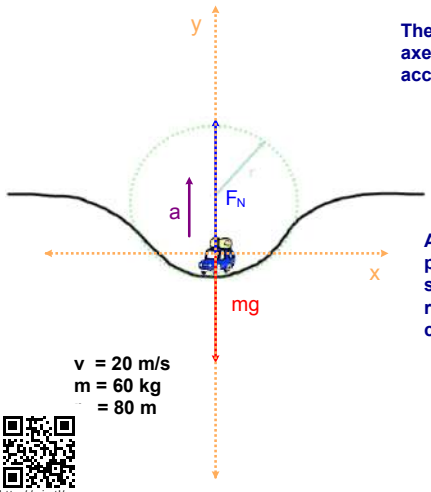

The dotted lines represent axes with one axis parallel to acceleration

All forces are parallel or perpendicular to the axes, so there is no need to resolve any vector into components

v = 20 m/s
m = 60 kg
= 80 m

http://njc.tl/ax

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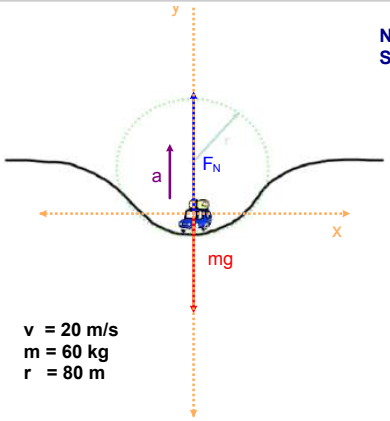

Next apply Newton's Second Law the y-axis.

v = 20 m/s
m = 60 kg
r = 80 m

Answer

http://njc.tl/ax

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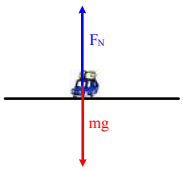

On the flat road the driver's weight (the normal force from the seat) is just mg.

$\mathbf{F_N}$ = mg

F_N = (60kg)(9.8 m/s^2) = 590 N

Apparent Weight

Flat road	Bottom of dip (r = 80m)
590 N	888 N

The gravitational force on the driver (mg) doesn't change, but her apparent weight (F_N) does.

Is there a situation where she will appear weightless?

v = 20 m/s
m = 60 kg
r = ∞

http://njc.tl/ax

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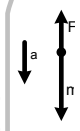
34 **A car is traveling at a velocity of 20 m/s. The driver of the car has a mass of 60 kg.**

The car is located at the top of a hill in the road. The radius of the hill is 80 m. What is the apparent weight of the driver (the normal force supplied by the seat of the car to support him) at the top of the hill?

Answer



Flat ro

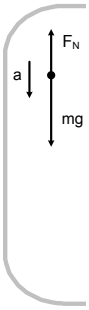
http://njc.tl/ax

35

At what velocity must a car drive over a hill if the driver (and the car for that matter) are to appear weightless?

The driver of the car has a mass of 60 kg. The radius of the hill is 80m.

Answer



http://njc.tl/ax

36 **The occupants of a car traveling at a speed of 25 m/s note that on a particular part of a road their apparent weight is 20% higher than their weight when driving on a flat road.**

Is that part of the road a hill, or a dip?

❍ **A** Hill

❍ **B** Dip

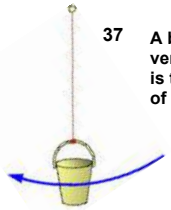
Answer

http://njc.tl/al

Buckets and Rollercoasters...

Outside of cars driving on hills, we can see applications of circular motion in other simple ways.

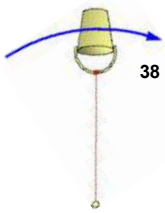
Have you ever been on an upside-down roller coaster and not fallen out? It turns out, the same properties of circular motion are responsible for your safety on a roller coaster!



37 **A bucket of water with a mass of 2.5kg is being spun is a vertical circle of radius 0.80m with a speed of 3.5m/s. What is the tension in the rope when the bucket is at the bottom of the circle?**

Answer

http://njc.tl/bg

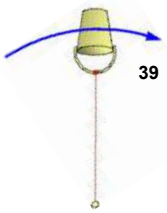


38 **A bucket of water with a mass of 2.5kg is being spun is a vertical circle of radius 0.80m with a speed of 3.5m/s. What is the tension in the rope when the bucket is at the top of the circle?**

Answer

[This object is a pull tab]

http://njc.tl/bh



39 **A bucket of water with a mass of 2.5kg is being spun in a vertical circle of radius 0.80m. What is the minimum velocity the bucket must have to make it around the top of the circle?**

Answer

http://njc.tl/bi

40 A bucket of water is being spun in a vertical circle of radius 1.2 m. What is the smallest velocity which will result in the water not leaving the bucket?

Answer

http://njc.tl/bj

41 Assuming a constant speed, and that the mass of the bucket is 1.3 kg, what will the tension in the string be at the bottom of the circle?

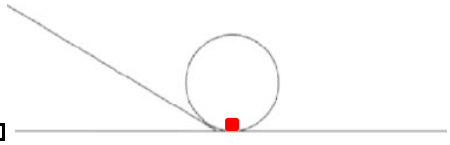
Answer

http://njc.tl/bk

Rollercoasters...

42 A roller coaster car has a mass of 500kg and is traveling through a vertical loop of radius 20m with a speed of 20 m/s.

What is the apparent weight at the bottom of the loop?



Answer

ΣF

F_N

F_N

F_N

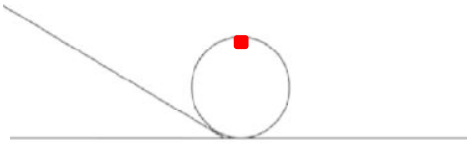
F_N

http://njc.tl/bl

Rollercoasters...

43 A roller coaster car has a mass of 500kg and is traveling through a vertical loop of radius 20m with a speed of 20 m/s.

What is the apparent weight at the top of the loop?



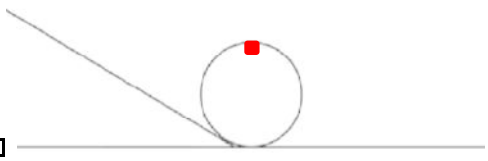
Answer

http://njc.tl/bm

Rollercoasters...

44 A roller coaster car has a mass of 500kg and is traveling through a vertical loop of radius 20m.

What is the minimum speed to just maintain contact with the track at the inside of the top of the loop?



Answer

http://njc.tl/bn

45 A roller coaster car is on a track that forms a circular loop in the vertical plane. If the car is to just maintain contact with track at the top of the loop, what is the minimum value for its centripetal acceleration at this point?

- A g downward
- B 0.5g downward
- C g upward
- D 2g upward

Answer

http://njc.tl/an

46 **A roller coaster car (mass = M) is on a track that forms a circular loop (radius = r) in the vertical plane. If the car is to just maintain contact with the track at the top of the loop, what is the minimum value for its speed at that point?**

- **A** rg
- **B** √(rg)
- **C** √(2rg)
- **D** √(0.5rg)

Answer

http://njc.tl/ao

47 **A pilot executes a vertical dive then follows a semi-circular arc until it is going straight up. Just as the plane is at its lowest point, the force of the seat on him is:**

- **A** less than mg and pointing up
- **B** less than mg and pointing down
- **C** more than mg and pointing up
- **D** more than mg and pointing down

Answer

http://njc.tl/ap

Applications: Horizontal Uniform Circular Motion

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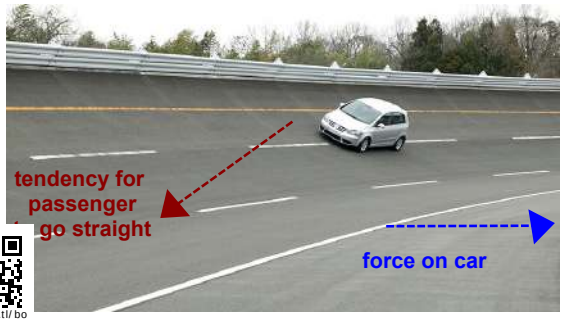
http://njc.tl/bo

Banked and Unbanked Curves

When a car goes around a curve, there must be a net force towards the center of the circle of which the curve is an arc. If the road is flat, that force is supplied by friction.



http://njc.tl/bo

Banked and Unbanked Curves

If the frictional force is insufficient, the car will tend to move more nearly in a straight line, as the skid marks below show.

http://njc.tl/bo

Unbanked Curves and Friction

As long as the tires do not slip, the friction is static.

If the tires do start to slip, the friction is kinetic, which is bad in two ways:

Bad #1) The kinetic frictional force is smaller than the static.

Bad #2) The static frictional force can point towards the center of the circle, but the kinetic frictional force opposes the direction of motion, making it very difficult to regain control of the car and continue around the curve.

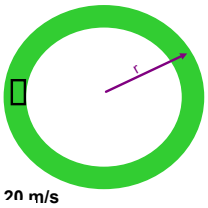
http://njc.tl/bo

A car is going around a track with a velocity of 20 m/s. The radius of the track is 150 m. What is the minimum coefficient of static friction that would make that possible?

v = 20 m/s
r = 150 m
μ = ?

A car is going around a track with a velocity of 20 m/s. The radius of the track is 150 m. What is the minimum coefficient of static friction that would make that possible?

Top View
(looking down on the track)



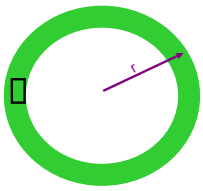
Front View
(the car heading towards you)

v = 20 m/s
m

A car is going around a track with a velocity of 20 m/s. The radius of the track is 150 m. What is the minimum coefficient of static friction that would make that possible?

STEP 1: Draw a Free Body Diagram

Top View
(looking down on the track)

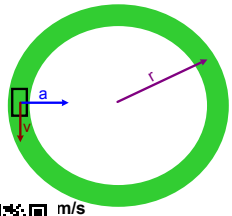


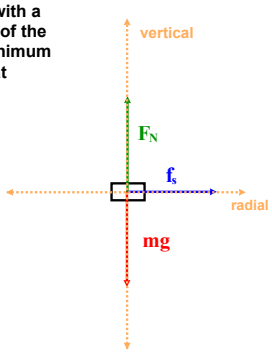
Front View
(the car heading towards you)

Answer

m/s
m

A car is going around a track with a velocity of 20 m/s. The radius of the track is 150 m. What is the minimum coefficient of static friction that would make that possible?

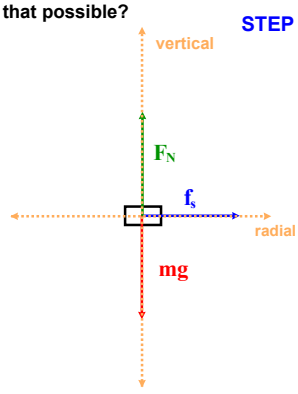




m/s
m

A car is going around a track with a velocity of 20 m/s. The radius of the track is 150 m. What is the minimum coefficient of static friction that would make that possible?

STEP 2: Substitute and Solve



Answer

v = 20 m/s
r = 150 m

48 A car goes around a curve of radius r at a constant speed v. Then it goes around the same curve at half of the original speed. What is the centripetal force on the car as it goes around the curve for the second time, compared to the first time?

- A twice as big
- B four times as big
- C half as big
- D one-fourth as big

Answer