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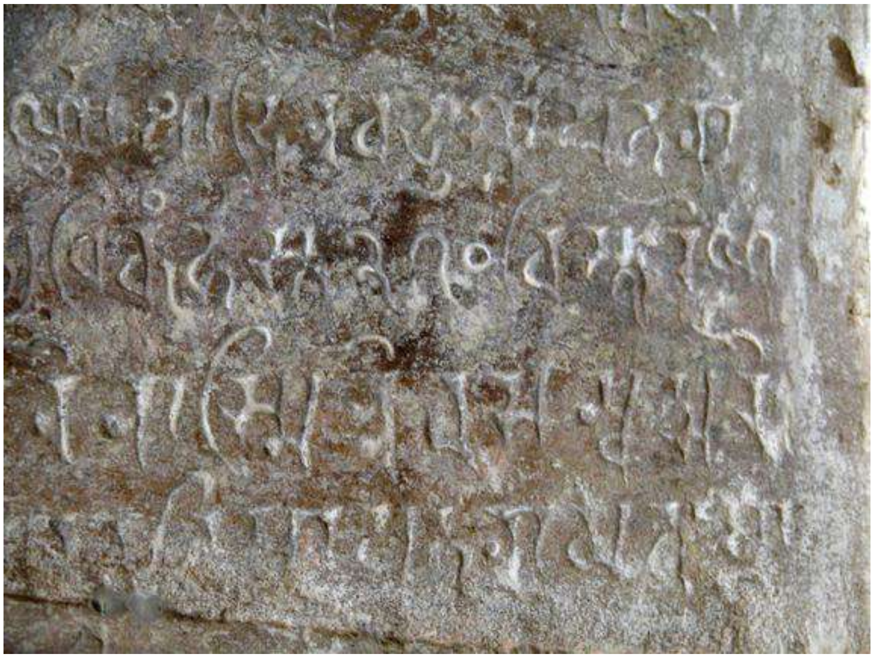
Understanding ancient Indian mathematics

S.G. Dani



A portion of a dedication tablet in a rock-cut Vishnu temple in Gwalior built in 876 AD. The number 270 seen in the inscription features the oldest extant zero in India. Courtesy: Bill Casselman

It is high time we studied our mathematical heritage with diligence and objectivity

Quite often I find that conversations, with people from various walks of life, on ancient Indian mathematics slide to "Vedic mathematics" of the "16 sutras" fame, which is supposed to endow one with magical powers of calculation. Actually, the "16 sutras" were introduced by Bharati Krishna Tirthaji, who was the Sankaracharya of Puri from 1925 until he passed away in 1960, associating with them procedures for certain arithmetical or algebraic computations. Thus, this so-called "Vedic mathematics (VM)" is essentially a 20th century phenomenon.

Neither the "sutras" nor the procedures that they are supposed to yield, or correspond to, have anything to do with either the Vedas, or even with any post-Vedic mathematical tradition of yore in India. The image that it may conjure up of ancient rishis engaged in such arithmetical exercises as are taught to the children in the name of VM, and representing the solutions through word-strings of a few words in modern styled Sanskrit, with hardly any sentence structure or grammar, is just too far from the realm of the plausible. It would have amounted to a joke, but for the aura it has acquired on account of various factors, including the general ignorance about the knowledge in ancient times. It is a pity that a long tradition of over 3,000 years of learning and pursuit of mathematical ideas has come to be perceived by a large section of the populace through the prism of something so mundane and so lacking in substance from a mathematical point of view, apart from not being genuine.

Tall claims

The colossal neglect involved is not for want of pride about the achievements of our ancients; on the contrary, there is a lot of writing on the topic, popular as well as technical, that is full of unsubstantiated claims conveying an almost supreme knowledge our forefathers are supposed to have possessed. But there is very little understanding or appreciation, on an intellectual plane, of the specics of their knowledge or achievements in real terms.

In the colonial era this variety of discourse emerged as an antithesis to the bias that was manifest in the works of some Western scholars. Due to the urgency to respond to the adverse propaganda on the one hand and the lack of resources in addressing the issues at a more profound level on the other, recourse was often taken to short-cuts, which involved more assertiveness than substance. There were indeed some Indian scholars, like Sudhakar Dvivedi, who adhered to a more intellectual approach, but they were a minority. Unfortunately, the old discourse has continued long after the colonial context is well past, and long after the world community has begun to view the Indian achievements with considerable objective curiosity and interest. It is high time that we switch to a mode betting a sovereign and intellectually self-reliant society, focussing on an objective study and critical assessment, without the reference frame of "what they say" and how "we must assert ourselves."

Ancient India has indeed contributed a great deal to the world's mathematical heritage. The country also witnessed steady mathematical developments over most part of the last 3,000 years, throwing up many interesting mathematical ideas well ahead of their appearance elsewhere in the world, though at times they lagged behind, especially in the recent centuries. Here are some episodes from the fascinating story that forms a rich fabric of the sustained intellectual endeavour.

Vedic knowledge

The mathematical tradition in India goes back at least to the Vedas. For compositions with a broad scope covering all aspects of life, spiritual as well as secular, the Vedas show a great fascination for large numbers. As the transmission of the knowledge was oral, the numbers were not written, but expressed as combinations of powers of 10. It would be reasonable to believe that when the decimal place value system for written numbers came into being it owed a great deal to the way numbers were discussed in the older compositions.

The decimal place value system of writing numbers, together with the use of '0,' is known to have blossomed in India in the early centuries AD, and spread to the West through the intermediacy of the Persians and the Arabs. There were actually precursors to the system, and various components of it are found in other ancient cultures such as the Babylonian, Chinese, and Mayan. From the decimal representation of the natural numbers, the system was to evolve further into the form that is now commonplace and crucial in various walks of life, with decimal fractions becoming part of the number system in 16th century Europe, though this again has some intermediate history involving the Arabs. The evolution of the number system represents a major phase in the development of mathematical ideas, and arguably contributed greatly to the overall advance of science and technology. The cumulative history of the number system holds a lesson that progress of ideas is an inclusive phenomenon, and while contributing to the process should be a matter of joy and pride to those with allegiance to the respective contributors, the role of others also ought to be appreciated.

It is well-known that Geometry was pursued in India in the context of construction of vedis for the yajnas of the Vedic period. The *Sulvasutras* contain elaborate descriptions of construction of vedis and enunciate various geometric principles. These were composed in the rst millennium BC, the earliest Baudhayana Sulvasutra dating back to about 800 BC. Sulvasutra geometry did not go very far in comparison to the Euclidean geometry developed by the Greeks, who appeared on the scene a little later, in the seventh century BC. It was, however, an important stage of development in India too. The Sulvasutra geometers were aware, among other things, of what is now called the Pythagoras theorem, over 200 years before Pythagoras (all the four major Sulvasutras contain an explicit statement of the theorem), addressed (within the framework of their geometry) issues such as nding a circle with the same area as a square and vice versa, and worked out a very good approximation to the square root of two, in the course of their studies.

Though it is generally not recognised, the *Sulvasutra* geometry was itself evolving. This is seen, in particular, from the differences in the contents of the four major extant Sulvasutras. Certain revisions are especially striking. For instance, in the early Sulvasutra period the ratio of the circumference to the diameter was, as in other ancient cultures, thought to be three, as seen in a sutra of Baudhayana, but in the Manava Sulvasutra, a new value was proposed, as three-and-one-fth. Interestingly, the sutra describing it ends with an exultation "not a hair-breadth remains," and though we see that it is still substantially off the mark, it is a gratifying instance of an advance made. In the Manava Sulvasutra one also nds an improvement over the method described by Baudhayana for nding the circle with the same area as that of a given square.

The Jain tradition has also been very important in the development of mathematics in the country. Unlike for the Vedic people, for Jain scholars the motivation for mathematics came not from ritual practices, which indeed were anathema to them, but from the contemplation of the cosmos. Jains had an elaborate cosmography in which mathematics played an integral role, and even largely philosophical Jain works are seen to incorporate mathematical discussions. Notable among the topics in the early Jain works, from about the fifth century BC to the second century AD, one may mention geometry of the circle, arithmetic of numbers with large powers of 10, permutations and combinations, and categorisations of innities (whose plurality had been recognised).

As in the Sulvasutra tradition, the Jains also recognised, around the middle of the rst millennium BC, that the ratio of the circumference of the circle to its diameter is not three. In "Suryaprajnapti," a Jain text believed to be from the fourth century BC, after recalling the "traditional" value three for it, the author discards that in favour the square root of 10. This value for the ratio, which is reasonably close to the actual value, was prevalent in India over a long period and is often referred as the Jain value. It continued to be used long after Aryabhata introduced the well-known value 3.1416 for the ratio. The Jain texts also contain rather unique formulae for lengths of circular arcs in terms of the length of the corresponding chord and the bow (height) over the chord, and also for the area of regions subtended by circular arcs together with their chords. The means for the accurate determination of these quantities became available only after the advent of Calculus. How the ancient Jain scholars arrived at these formulae, which are close approximations, remains to be understood.

Jain tradition

After a lull of a few centuries in the early part of the rst millennium, pronounced mathematical activity is seen again in the Jain tradition from the 8th century until the middle of the 14th century. *Ganitasarasangraha* of Mahavira, written in 850, is one of the well-known and inuential works. Virasena (8th century), Sridhara (between 850 and 950), Nemicandra (around 980 CE), Thakkura Pheru (14th century) are some more names that may be mentioned. By the 13th and 14th centuries, Islamic architecture had taken root in India and in *Ganitasarakaumudi* of Thakkura Pheru, who served as treasurer in the court of the Khilji Sultans in Delhi, one sees a combination of the native Jain tradition with Indo-Persian literature, including work on the calculation of areas and volumes involved in the construction of domes, arches, and tents used for residential purposes.

Mathematical astronomy or the Siddhanta tradition has been the dominant and enduring mathematical tradition in India. It ourished almost continuously for over seven centuries, starting with Aryabhata (476-550) who is regarded as the founder of scientic astronomy in India, and extending to Bhaskara II (1114-1185) and beyond. The essential continuity of the tradition can be seen from the long list of prominent names that follow Aryabhata, spread over centuries: Varahamihira in the sixth century, Bhaskara I and Brahmagupta in the seventh century, Govindaswami and Sankaranarayana in the ninth century, Aryabhata II and Vijayanandi in the 10th century, Sripati in the 11th century, Brahmadeva and Bhaskara II in the 12th century, and Narayana Pandit and Ganesa from the 14th and 16th centuries respectively.

Aryabhatiya, written in 499, is basic to the tradition, and even to the later works of the Kerala school of Madhava (more on that later). It consists of 121 verses divided into four chapters — Gitikapada, Ganitapada, Kalakriyapada and Golapada. The rst, which sets out the cosmology, contains also a verse describing a table of 24 sine differences at intervals of 225 minutes of arc. The second chapter, as the name suggests, is devoted to mathematics *per se*, and includes in particular procedures to nd square roots and cube roots, an approximate expression for 'pi' (amounting to 3.1416 and specied to be approximate), formulae for areas and volumes of various geometric gures, and shadows, formulae for sums of consecutive integers, sums of squares, sums of cubes and computation of interest. The other two chapters are concerned with astronomy, dealing with distances and relative motions of planets, eclipses and so on.

Influential work

Brahmagupta's *Brahmasphutasiddhanta* is a voluminous work, especially for its time, on Siddhanta astronomy, in which there are two chapters, Chapter 12 and Chapter 18, devoted to general mathematics. Incidentally, Chapter 11 is a critique on earlier works including *Aryabhatiya*; as in other healthy scientific communities this tradition also had many, and often bitter, controversies. Chapter 12 is well-known for its systematic treatment of arithmetic operations, including with negative numbers; the notion of negative numbers had eluded Europe until the middle of the second millennium. The chapter also contains geometry, including in particular his famous formula for the area of a quadrilateral (stated without the condition of cyclicity of the quadrilateral that is needed for its validity — a point criticised by later mathematicians in the tradition). Chapter 18 is devoted to the kuttaka and other methods, including for solving second-degree indeterminate equations. An identity described in the work features also in some current studies where it is referred as the Brahmagupta identity. Apart from this, Chapter 21 has verses dealing with trigonometry. *Brahmasphutasiddhanta* considerably influenced mathematics in the Arab world, and in turn the later developments in Europe. Bhaskara II is the author of the famous mathematical texts *Lilavati* and *Bijaganita*. Apart from being an accomplished mathematician he was a great teacher and populariser of mathematics. *Lilavati*, which literally means 'one who is playful,' presents mathematics in a playful way, with several verses directly addressing a pretty young woman, and examples presented through reference to various animals, trees, ornaments, and so on. (Legend has it that the book is named after his daughter after her wedding failed to materialise on account of an accident with the clock, but there is no historical evidence to that effect.) The book presents, apart from various introductory aspects of arithmetic, geometry of triangles and quadrilaterals, examples of applications of the Pythagoras theorem, trirasika, kuttaka methods, problems on permutations and combinations, etc. The *Bijaganita* is an advanced-level treatise on Algebra, the first independent work of its kind in Indian tradition. Operations with unknowns, kuttaka and chakravala methods for solutions of indeterminate equations are some of the topics discussed, together with examples. Bhaskara's work on astronomy, *Siddhantasiromani* and *Karana kutuhala*, contain several important results in trigonometry, and also some ideas of Calculus.

The works in the Siddhanta tradition have been edited on a substantial scale and there are various commentaries available, including many from the earlier centuries, and works by European authors such as Colebrook, and many Indian authors including Sudhakara Dvivedi, Kuppanna Sastri and K.V. Sarma. The two-volume book of Datta and Singh and the book of Saraswati Amma serve as convenient references for many results known in this tradition. Various details have been described, with a comprehensive discussion, in the recent book by Kim Plofker. The *Bakhshali* manuscript, which consists of 70 folios of bhurjapatra (birch bark), is another work of signicance in the study of ancient Indian mathematics, with many open issues around it. The manuscript was found buried in a eld near Peshawar, by a farmer, in 1881. It was acquired by the Indologist A.F.R. Hoernle, who studied it and published a short account on it. He later presented the manuscript to the Bodleian Library at Oxford, where it has been since then. Facsimile copies of all the folios were brought out by Kaye in 1927, which have since then been the source material for the subsequent studies. The date of the manuscript has been a subject of much controversy since the early years, with the estimated dates ranging from the early centuries of CE to the 12th century.

Takao Hayashi, who produced what is perhaps the most authoritative account so far, concludes that the manuscript may be assigned sometime between the eighth century and the 12th century, while the mathematical work in it may most probably be from the seventh century. Carbon dating of the manuscript could settle the issue, but efforts towards this have not materialised so far.

A formula for extraction of square-roots of non-square numbers found in the manuscript has attracted much attention. Another interesting feature of the *Bakhshali* manuscript is that it involves calculations with large numbers (in decimal representation).

Kerala school

Let me nally come to what is called the Kerala School. In the 1830s, Charles Whish, an English civil servant in the Madras establishment of the East India Company, brought to light a collection of manuscripts from a mathematical school that ourished in the north-central part of Kerala, between what are now Kozhikode and Kochi. The school, with a long teacher-student lineage, lasted for over 200 years from the late 14th century well into the 17th century. It is seen to have originated with Madhava, who has been attributed by his successors many results presented in their texts. Apart from Madhava, Nilakantha Somayaji was another leading personality from the school. There are no extant works of Madhava on mathematics (though some works on astronomy are known). Nilakantha authored a book called *Tantrasangraha* (in Sanskrit) in 1500 AD. There have also been expositions and commentaries by many other exponents from the school, notable among them being *Yuktidipika* and *Kriyakramakari* by Sankara, and *Ganitayuktibhasha* by Jyeshthadeva which is in Malayalam. Since the middle of the 20th century, various Indian scholars have researched on these manuscripts and the contents of most of the manuscripts have been looked into. An edited translation of the latter was produced by K.V. Sarma and it has recently been published with explanatory notes by K. Ramasubramanian, M.D. Srinivas and M.S. Sriram. An edited translation of *Tantrasangraha* has been brought out more recently by K. Ramasubramanian and M.S. Sriram.

The Kerala works contain mathematics at a considerably advanced level than earlier works from anywhere in the world. They include a series expansion for 'pi' and the arc-tangent series, and the series for sine and cosine functions that were obtained in Europe by Gregory, Leibnitz and Newton, respectively, over 200 years later. Some numerical values for 'pi' that are accurate to 11 decimals are a highlight of the work. In many ways, the work of the Kerala mathematicians anticipated calculus as it developed in Europe later, and in particular involves manipulations with indenitely small quantities (in the determination of circumference of the circle and so on) analogous to the innitesimals in calculus; it has also been argued by some authors that the work is indeed calculus already.

Honouring the tradition

A lot needs to be done to honour this rich mathematical heritage. The extant manuscripts need to be cared for to prevent deterioration, catalogued properly with due updates and, most important, they need to be studied diligently and the ndings placed in proper context on the broad canvass of the world of mathematics, from an objective standpoint. Let the occasion of the 125th birth anniversary of the genius of Srinivasa Ramanujan, a global mathematician to the core, inspire us as a nation, to apply ourselves to this task.

(The author is Distinguished Professor, School of Mathematics, Tata Institute of Fundamental Research, Mumbai.)

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