

TRIGONOMETRY PACKET

6 - 10

Hamby

Trigonometry Snow Packet #6 AND #7

(Packet #6 and #7 has 9 pages)

Hi everyone! I hope this finds you doing well. Today in your packet we are going to bring together several different activities, including some reading, a graphic organizer, an article of local interest, and an online video. As we look at these pieces of our lesson, my hope is that you begin to get an idea of one way Trig is used in the “real world.” And for those of you planning a career in science, math, healthcare or veterinary work, I hope you have your interest peaked a little bit! **Hint:** please don’t take pics of your work and send them to your friends so they can copy, because 1) that’s not cool, 2) your teacher is smarter than that and your teacher actually cares, 3) I haven’t given you very many assignments for Trig application, so why not give it a try? 4) what else do you really have to do...? 5) by copying you’re taking away your opportunity to grow, and 5) What else does Mrs. Hamby have to do besides actually read your work for authenticity?

So, here we go....

1. Locate the “Trigonometry in Real Life” page. This is a modified graphic organizer; if we were in class, it would have been a foldable. If you have the ability, print it; or, If you don’t have a paper copy, you should use a piece of paper and write (and label) the information down that it asks you to do.

2. Locate the “Physical Principles of Ultrasound”/“Mechanics of Ultrasound Waves” Article.

Work through the graphic organizer and read the article at the same time. Notice the graphic organizer says “summarize paragraph 1 here”, so you read paragraph 1 and summarize on the organizer. Continue through the article and complete the organizer activities.

3. Locate the “Video Notes” section of your graphic organizer. If you have access to the internet, watch the video and take notes for this section. It may help you to watch the video all the way through, and then play it again to take notes on it. If you don’t have internet, perhaps you can find someone or somewhere who does. Here is the link:

<https://www.animalultrasoundassociation.org/sine-waves-ultrasound/>

4. Locate the article “Historic Breakthrough: WVU Rockefeller Neuroscience Team...”
Read the article, and answer the 12 questions that go with it. Also, complete the 5 “take-aways” for this article on your graphic organizer. Again, you can write on your own paper or print these documents and write on them.

TRIGONOMETRY IN REAL LIFE

"Physical Principles of Ultrasound" Article

1: Summarize paragraph 1 here:

2: From paragraph 2: What is required in order for a medical ultrasound to work?

3: Define the terms from paragraph 3-

Wavelength-

Cycle-

Amplitude-

Period-

Velocity-

4: Complete each statement from paragraphs 4, 5, & 6:

1. Wavelength and frequency have an _____ relationship.
2. Velocity of sound in tissues is _____.
3. Average velocity of sound in human tissue is _____.
4. Transducers convert _____ to _____.
5. The transducer _____ and _____.

“Physical Principles of Ultrasound” Article

5: Answer these questions from paragraphs 9 and 10.

- What is PRF?
- How is an ultrasound image produced?
- What role does the amplitude of the returning waves play?

Video Notes—

<https://www.animalultrasoundassociation.org/sine-waves-ultrasound/>

Look for: How does a transducer work? How is ultrasound related to trig?

From "Historic Breakthrough @ WVU" Article

- Choose 5 "take-aways" from the WVU article. Include items you find intriguing, surprising, exciting, personally significant or scientifically significant. Write about them here at the bullet points below; write in a CRAVE format, including one bullet for each "take-away."
- Use **C**-complete sentences, **R**-restate the question, **A**-be accurate in your statements in reflecting info from the article, **V**-use math/science vocabulary, **E**-explain why you feel they are find intriguing, surprising, exciting, personally significant or scientifically significant.

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Physical Principles of Ultrasound

Pat F. Fulgham

The Mechanics of Ultrasound Waves

The image produced by ultrasound is the result of the interaction of mechanical ultrasound waves with biologic tissues and materials. Because ultrasound waves are transmitted at frequent intervals and the reflected waves received by the transducer, the images can be reconstructed and refreshed rapidly, providing a real-time image of the organs being evaluated. Ultrasound waves are **mechanical waves** which require a physical medium (such as tissue or fluid) to be transmitted. Medical ultrasound imaging utilizes frequencies in the one million cycles per second (or MHz) range. Most transducers used in urology vary from 2.5 to 18 MHz, depending on the application.

Ultrasound waves are created by applying alternating current to piezoelectric crystals within the transducer. Alternating expansion and contraction of the piezoelectric crystals creates a mechanical wave which is transmitted through a coupling medium (usually gel) to the skin and then into the body. The waves that are produced are **longitudinal waves**. This means that the particle motion is in the same direction as the propagation of the wave (Fig. 2.1). This longitudinal wave produces areas of rarefaction and compression of tissue in the direction of travel of the ultrasound wave.

The compression and rarefaction of molecules is represented graphically as a sine wave alternating between a positive and negative

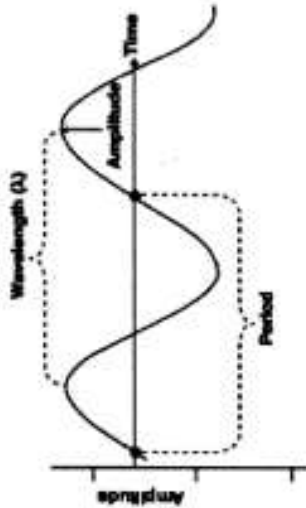
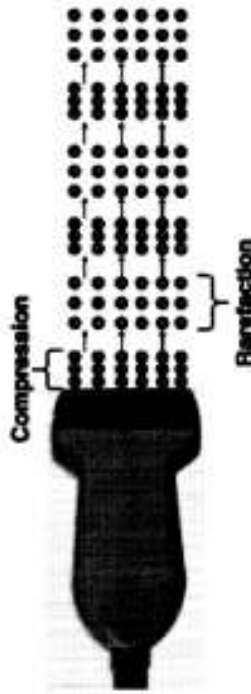


Fig. 2.2 Characteristics of a sound wave: the amplitude of the wave is a function of the acoustical power used to generate the mechanical compression wave and the medium through which it is transmitted

deflection from the baseline. A **wavelength** is described as the distance between one peak of the wave and the next peak. One complete path traveled by the wave is called a **cycle**. One cycle per second is known as 1 Hertz (Hertz). The **amplitude** of a wave is the maximal excursion in the positive or negative direction from the baseline, and the **period** is the time it takes for one complete cycle of the wave (Fig. 2.2).

The velocity with which a sound wave travels through tissue is a product of its frequency and its wavelength. The velocity of sound in tissues is constant. Therefore, as the frequency of the sound wave changes, the wavelength must also change. The average velocity of sound in human tissues is 1540 m/s. Wavelength and frequency vary in an inverse relationship. Velocity equals frequency times wavelength (Fig. 2.3). As the frequency diminishes from 10 to 1 MHz, the wavelength increases from 0.15 to 1.5 mm. This has important consequences for the choice of transducer depending on the indication for imaging.

Fig. 2.1 Longitudinal waves. The expansion and contraction of piezoelectric crystals caused by the application of alternating current to the crystals causes compression and rarefaction of molecules in the body



$$v = f\lambda$$

velocity = frequency x wavelength

Fig. 2.3 Since the velocity of sound in tissue is a constant, the frequency and wavelength of sound must vary inversely

Ultrasound Image Generation

The image produced by an ultrasound machine begins with the transducer. **Transducer** comes from the Latin **transducere**, which means to convert. In this case, electrical impulses are converted to mechanical sound waves via the **piezoelectric effect**.

In ultrasound imaging the transducer has a dual function as a sender and a receiver. Sound waves are transmitted into the body where they are at least partially reflected. The piezoelectric effect occurs when alternating current is applied to a crystal containing dipoles (1). Areas of charge within a piezoelectric element are distributed in patterns which yield a "net" positive and negative

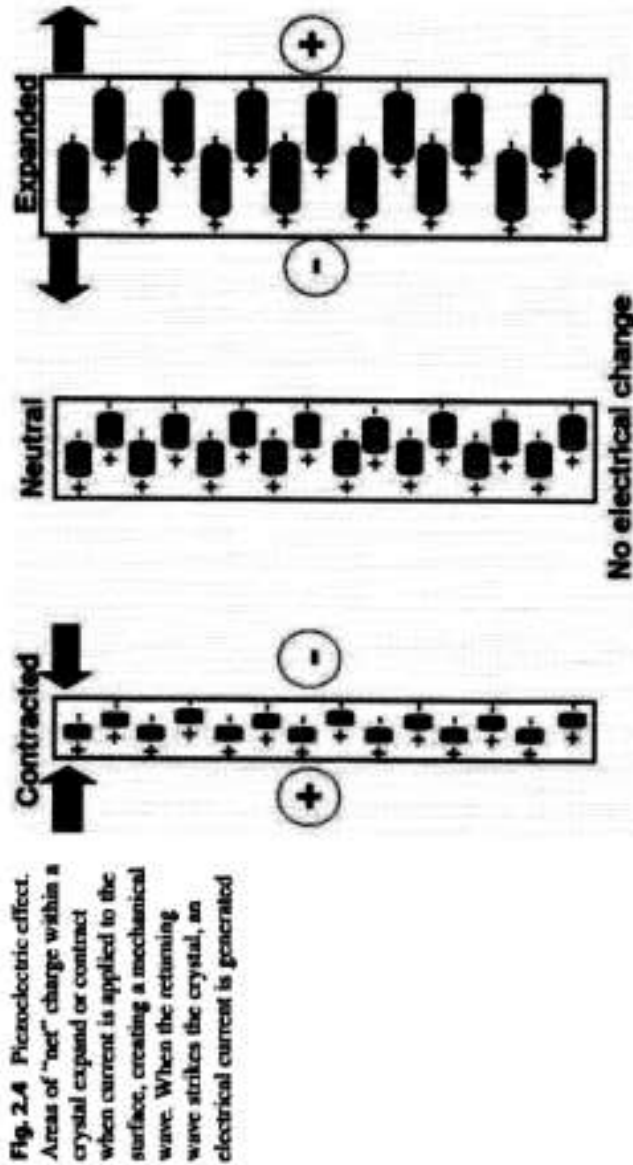


Fig. 2.4 Piezoelectric effect. Areas of "net" charge within a crystal expand or contract when current is applied to the surface, creating a mechanical wave. When the returning wave strikes the crystal, an electrical current is generated.

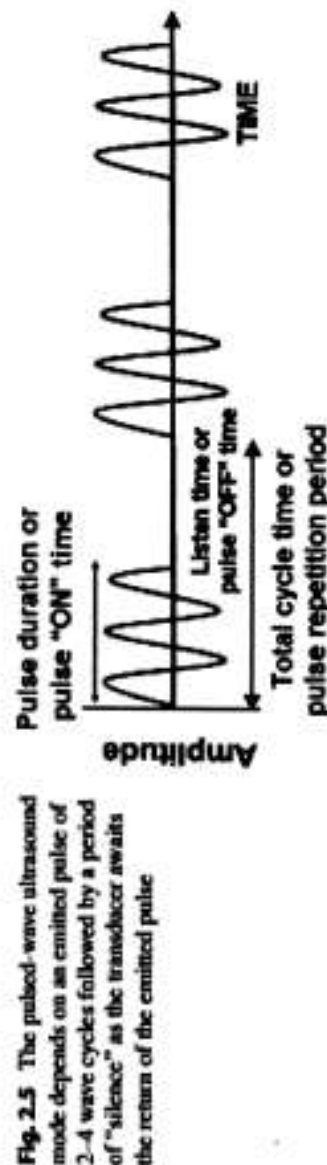


Fig. 2.5 The pulsed-wave ultrasound mode depends on an emitted pulse of 2-4 wave cycles followed by a period of "silence" as the transducer awaits the return of the emitted pulse.

and then interpreting the returning echoes for duration of transit and amplitude. This "image" is rapidly refreshed on a monitor to give the impression of continuous motion. Frame refresh rates are typically 12-30 per second. The sequence of events depicted in Fig. 2.7 is the basis for all "scanned" modes of ultrasound including the familiar gray-scale ultrasound.

orientation. When alternating charge is applied to the two element faces, a relative contraction or elongation of the charged areas occurs resulting in a mechanical expansion and then a contraction of the element. This results in a mechanical wave which is transmitted to the patient (Fig. 2.4).

Reflected mechanical sound waves are received by the transducer and converted back into electrical energy via the piezoelectric effect. The electrical energy is interpreted within the ultrasound instrument to generate an image which is displayed upon the screen.

For most modes of ultrasound, the transducer emits a limited number of wave cycles (usually two to four) called a pulse. The frequency of the two to four waves within each cycle is usually in the 2.5-14 MHz range. The transducer is then "silent" as it awaits the return of the reflected waves from within the body (Fig. 2.5). The transducer serves as a receiver more than 99 % of the time.

Pulses are sent out at regular intervals usually between 1 and 10 kHz which is known as the pulse repetition frequency (PRF). By timing the pulse from transmission to reception, it is possible to calculate the distance from the transducer to the object reflecting the wave. This is known as ultrasound ranging (Fig. 2.6). This sequence is known as pulsed-wave ultrasound.

The amplitude of the returning waves determines the brightness of the pixel assigned to the reflector in an ultrasound image. The greater the amplitude of the returning wave, the brighter the pixel assigned. Thus, an ultrasound unit produces an "image" by first causing a transducer to emit a series of ultrasound waves at specific frequencies and intervals

Packet #6 & #7 continued

Historic breakthrough: WVU Rockefeller Neuroscience team first to use ultrasound to treat Alzheimer's



by John Dahlia NCWV Media Business Editor

Oct 28, 2018

The research team celebrates at the WVU Rockefeller Neuroscience Institute moments after successfully performing the first procedure in the world of a phase II trial using focused ultrasound to treat a patient with early stage Alzheimer's. At the center is Judi Polak, the courageous first patient. Holding her hand is Dr. Marc Haut, who is flanked by Dr. Jeff Carpenter and Dr. Ali R. Rezai.

- [Photo courtesy of WVU Medicine](#)

(paragraphs have been collapsed for better use of space here)

MORGANTOWN — World-leading brain experts at West Virginia University's Rockefeller Neuroscience Institute are celebrating the historic breakthrough Alzheimer patients around the globe have been awaiting. "For Alzheimer's, there's not that many treatments available, despite hundreds of clinical trials over the past two decades and billions of dollars spent," said Dr. Ali R. Rezai, a neurosurgeon at WVU who led the team of investigators that successfully performed a phase II trial using focused ultrasound to treat a patient with early stage Alzheimer's. The WVU team tested the innovative treatment in collaboration with INSIGHTEC, an Israeli medical technology company. Earlier this year, INSIGHTEC was approved by the U.S. Food and Drug Administration to begin a phase II clinical trial of the procedure, and selected the WVU Rockefeller Neuroscience Institute as the first site in the United States for that trial. Last summer, researchers at Sunnybrook Health Sciences Centre in Toronto reported the results of a phase I safety trial showing they could reversibly open the blood-brain barrier in Alzheimer's patients. The procedure in West Virginia involved the use of ultrasound waves focused through a specialized helmet with more than 1,000 probes targeting a precise spot in the brain, Rezai explained, coupled with microscopic bubbles. "And when we put a different frequency of ultrasound on the bubbles, they start oscillating," he said. The reaction opens up the brain-blood barrier — a nearly impenetrable shield between the brain's blood vessels and cells that make up brain tissue. "It's protected on one end for us to function but also prevents larger molecules or chemotherapy or medications or anti-bodies or immune system cells or amino therapy or stem cells to get in," he said. In this case, the West Virginia team targeted the hippocampus and the memory and cognitive centers of the brain that are impacted by plaques found in patients with Alzheimer's. "Plaques are these clusters of proteins that accumulate and they block-up the brain's connectivity," he said. "In animal studies it showed that these plaques are cleared with ultrasound technology.

The first patient, a person Rezai called a pioneer and hero, is West Virginia health care worker and former WVU Children's Hospital Neonatal Intensive Care Unit nurse Judi Polak. "I think that with Alzheimer's there's so much in the unknown and I've been with Health Science for a long time and I understand that we need to be able to step forward and look into the future," Polak said. But getting to this point was a long journey beginning five years ago when she was first diagnosed with early-onset Alzheimer's. "That took me a while to deal with," Polak admitted while sitting with her husband of 36 years, Mark Polak. "It was hard to say that I have Alzheimer's. I didn't want to be the person who felt sorry for myself and so we looked at clinical trials as a way to help not only me but other people too." Early-onset Alzheimer's is an uncommon form of dementia that strikes people younger than age 65. Of all the people who have Alzheimer's disease, according to research conducted by the Mayo Clinic, about 5 percent develop symptoms before age 65. Judi Polak's willingness to be the center of a study or research experiment in hopes of finding a cure for Alzheimer's took an emotional toll, Mark Polak said, referring to a controlled drug-placebo trial at the University of Pittsburgh several years ago. "Guess what, the drug didn't work," he said with contempt. "Just like every drug that has been tried doesn't work." However, Judi Polak's patience and persistence appears to have paid off. The procedure, which lasted three hours, safely and successfully opened her blood-brain barrier for a record 36 hours. "It was opened longer than they expected," Mark Polak said. "They were actually, I think both excited and scared. The team was ecstatic." One member of the team Mark Polak mentioned is Dr. Jeff Carpenter, a professor of neurology, neurosurgery and an interventional neuroradiologist at WVU. "This is really step one," Carpenter said of the successful trial. "This is to make sure it's safe and hopefully we can decrease some of the big plaques in that part of the brain." Carpenter is what he jokingly called the "technical guy" on Rezai's team with 18 years of experience working MRI technology and interventional radiology. "It's a combination of knowing MRI very well and also being used to actually treating patients," Carpenter said. "This treatment marries MRI guidance with ultrasound targeting. "It really uses all the things I've been working with." Carpenter, a native of Fairmont, credited Rezai's work and ultimately the leadership at WVU Medicine for supporting the research needed. "It is really nice to be able to do this level of work this close to home," he added. The potential benefits of the first and subsequent treatments will take several years to fully evaluate, Rezai said. Two more similar procedures are scheduled for Judi Polak; one on Tuesday and a final test in November. "I am hopeful that focused ultrasound opening of the blood-brain barrier will prove to be a valuable treatment option for Judi Polak and other patients with early Alzheimer's who are confronting the enormous challenges associated with the disease on a daily basis," Rezai said. Although Rezai stopped short of giving any immediate results from the first treatment, Polak said she noticed a change the next day. "I think I could speak clearer and did not wait as long in answering questions," she said. "Sometimes in the past things would leave my mind and I couldn't remember things." "This is man on the moon stuff," Mark Polak said of his wife's success in the first trial. "Maybe we're on to something."

Questions on “Historic Breakthrough”: WVU/Alzheimer’s

1. Where did this procedure happen?
2. What phase of Alzheimer’s was the test subject in?
3. What is INSIGHTEC? How were they helpful?
4. Ultrasound waves went through what, in the procedure?
5. How many probes were used?
6. What makes the microscopic bubbles oscillate? What does that cause?
7. What area of the brain was targeted?
8. What is a plaque?
9. How long did the procedure last?
10. How long was the blood-brain barrier open?
11. How long before Mrs. Polak noticed results? What were the results?
12. What were the “next steps” in her treatment?

Trigonometry Snow Packet #8

Trigonometric Identities

Reciprocal Identities		Quotient Identities
$\sin \theta = \frac{1}{\csc \theta}$	$\csc \theta = \frac{1}{\sin \theta}$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
$\cos \theta = \frac{1}{\sec \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\cot \theta = \frac{\cos \theta}{\sin \theta}$
$\tan \theta = \frac{1}{\cot \theta}$	$\cot \theta = \frac{1}{\tan \theta}$	
Pythagorean Identities $\sin^2 \theta + \cos^2 \theta = 1$ $\tan^2 \theta + 1 = \sec^2 \theta$ $1 + \cot^2 \theta = \csc^2 \theta$		

Sometimes, it is helpful to use a Pythagorean Identity, and re-arrange it for another function. Like,

$$\sin^2 \theta + \cos^2 \theta = 1$$

could be arranged as

$$\cos^2 \theta = 1 - \sin^2 \theta$$

by subtracting $\sin^2 \theta$ from both sides.

Using the Reciprocal and Quotient Identities simplify each as much as possible.

To do this, we use identities from above to substitute into the expression and simplify. Like this:

EXAMPLE 1: $\tan(\theta) \cdot \cos(\theta)$

$$\frac{\sin \theta}{\cancel{\cos \theta}} \cdot \cancel{\cos \theta} = \sin \theta$$

EXAMPLE 2: $\csc(\theta) \cdot \tan(\theta) - \sec(\theta)$

$$\frac{1}{\cancel{\sin \theta}} \cdot \frac{\sin \theta}{\cos \theta} - \frac{1}{\cos \theta} = \frac{1}{\cos \theta} - \frac{1}{\cos \theta} = 0$$

EXAMPLE 3: $\sin^2 \theta (\csc^2 \theta - 1)$

$$\sin^2 \theta (\cot^2 \theta) \text{ by rearranging } 1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin^2 \theta \left(\frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta} \right) = \cos^2 \theta$$

****Sometimes, if the identity you choose doesn't work, you need to try again using another identity or another strategy.**

Packet #8 continued

Using the Reciprocal and Quotient Identities simplify each as much as possible.

If you don't have a printed copy/printer, write these on your own paper.

1. $\sin \theta \cdot \sec \theta$

2. $\cot(\theta) \cdot \cos(\theta) \cdot \sin(\theta)$

3. $\sin(\theta) \cdot \cot(\theta) + \cos(\theta)$

4. $\frac{\sec^2 \theta - 1}{\sin^2 \theta}$

Hint: re-arrange a
Pythagorean identity,
then substitute

5. $\frac{\cot(\theta)}{\csc(\theta)}$

Hint: substitute, then
keep change flip

6. $\cot \theta \cdot \sec \theta$

7. $\cos^2 \theta (\sec^2 \theta - 1)$

Hint: re-arrange a
Pythagorean identity,
then substitute

8. $\frac{\cot(\theta) \cdot \sin(\theta)}{\cos(\theta)} + \cos(\theta) \cdot \sec(\theta)$

Trigonometry Snow Packet #9

Trigonometric Identities

Reciprocal Identities		Quotient Identities
$\sin \theta = \frac{1}{\csc \theta}$	$\csc \theta = \frac{1}{\sin \theta}$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
$\cos \theta = \frac{1}{\sec \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\cot \theta = \frac{\cos \theta}{\sin \theta}$
$\tan \theta = \frac{1}{\cot \theta}$	$\cot \theta = \frac{1}{\tan \theta}$	

Pythagorean Identities
$\sin^2 \theta + \cos^2 \theta = 1$
$\tan^2 \theta + 1 = \sec^2 \theta$
$1 + \cot^2 \theta = \csc^2 \theta$
<p>Sometimes, it is helpful to use a Pythagorean Identity, and rearrange is for another function. Like,</p> $\sin^2 \theta + \cos^2 \theta = 1$ <p>could be arranged as</p> $\cos^2 \theta = 1 - \sin^2 \theta$ <p>by subtracting $\sin^2 \theta$ from both sides.</p>

Using the Reciprocal and Quotient Identities

verify the following trigonometric identities.

EXAMPLE 1: Verify:

$\sec \theta \cdot \cot \theta \cdot \sin \theta = 1$ We are trying to verify (prove) that $\sec \theta \cdot \cot \theta \cdot \sin \theta$ is the same as 1.

$$\frac{1}{\cancel{\cos \theta}} \cdot \frac{\cancel{\cos \theta}}{\cancel{\sin \theta}} \cdot \cancel{\sin \theta} = 1$$

EXAMPLE 2: Verify:

$\frac{\sec^2 \theta - 1}{\csc^2 \theta - 1} = \tan^4 \theta$ We are trying to verify (prove) that $\frac{\sec^2 \theta - 1}{\csc^2 \theta - 1}$ is the same as $\tan^4 \theta$

We could rearrange 2 of the Pythagorean Identities, and then substitute:

$$\frac{\tan^2 \theta}{\cot^2 \theta}, \text{ which is } \frac{\tan^2 \theta}{\frac{1}{\tan^2 \theta}}, \text{ and after dividing (keep change flip), we get } \tan^4 \theta$$

EXAMPLE 3: Verify:

$(\sec \theta + \csc \theta)(\cos \theta - \sin \theta) = \cot \theta - \tan \theta$ We are trying to verify (prove) that $(\sec \theta + \csc \theta)(\cos \theta - \sin \theta)$ is the same as $\cot \theta - \tan \theta$

We could multiply by using FOIL: $\sec \theta \cos \theta - \sec \theta \sin \theta + \csc \theta \cos \theta - \csc \theta \sin \theta$.

Then, substitute using identities: $\frac{1}{\cos \theta} \cos \theta - \frac{1}{\cos \theta} \sin \theta + \frac{1}{\sin \theta} \cos \theta - \frac{1}{\sin \theta} \sin \theta$.

Which then equals: $1 - \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} - 1$ which is

$$- \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \text{ which is } -\tan \theta + \cot \theta \text{ or } \cot \theta - \tan \theta$$

Packet #9 continued

Using the Reciprocal and Quotient Identities verify the following trigonometric identities.

1. $\cot(\theta) \cdot \sec(\theta) \cdot \sin(\theta) = 1$

2. $\frac{\cot(\theta)}{\csc(\theta)} = \cos(\theta)$

3. $\frac{\sin(\theta)\sec(\theta)}{\tan(\theta)} + \cos(\theta) \cdot \sec(\theta) = 2$

Simplify the fraction first, then simplify the $\cos\theta\sec\theta$ then add.

4. $\tan(\theta) + \sec(\theta) = \frac{1+\sin(\theta)}{\cos(\theta)}$

First, substitute for tan and sec, then add those resulting fractions.

5. $\frac{1}{\csc(\theta)} + \tan(\theta) \cdot \cos(\theta) = 2\sin(\theta)$

6. $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

First, distribute and cancel. Then, get a common denominator (cos). Lastly, substitute an identity.

7. $\csc^2\theta - \csc^2\theta\cos^2\theta = 1$

First, factor out a GCF $\csc^2\theta$.
Then, substitute an identity.

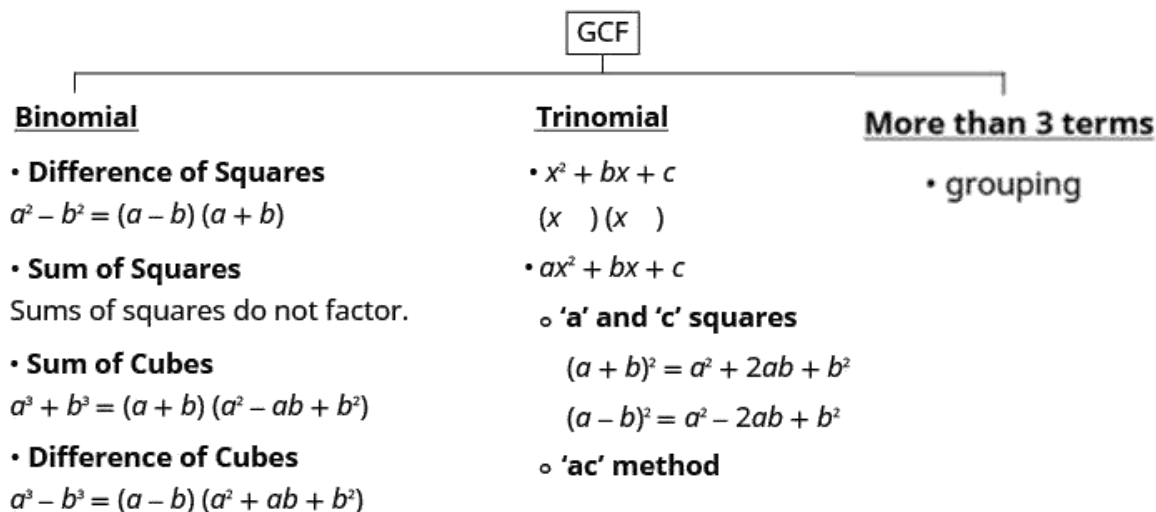
8. $\frac{(\sec\theta+1)(\sec\theta-1)}{\sin^2\theta} = \sec^2\theta$

First, FOIL multiply. Then, substitute identities and simplify.

Trigonometry Snow Packet #10

Factoring Review. Sometimes we need to use factoring to help us verify using identities.

Here's some reminders of factoring forms:



Factor Completely; always check for a GCF to take out first!

- | | | |
|----------------------------|---------------------------|-----------------------|
| 1. $9x^2 - 4$ | 2. $x^3 + 64$ | 3. $200x^2 - 50$ |
| 4. $7x^3 + 14x^2 + 7x$ | 5. $2x^3 - 4x^2 - 3x - 6$ | 6. $3x^2 + 81$ |
| 7. $2x^2 - x - 3$ | 8. $x^2 + 3x - 10$ | 9. $x^2 + 8x + 16$ |
| 10. $4x^2 - 20x + 25$ | 11. $2x^2 - 5x + 2$ | 12. $3x^2 - 11x - 20$ |
| 13. $x^3 - 3x^2 - 5x + 15$ | 14. $3x^4 - 11x^2 - 20$ | 15. $4x^2 - 49$ |

Packet #10 continued:

NOW...Try to see the structures with Trig mixed in!

EXAMPLE 1: Factor.

$\sin^2\theta - \cos^2\theta$ This is a difference of 2 squares...

using the format $x^2 - y^2 = (x + y)(x - y)$

$$=(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)$$

EXAMPLE 2: Factor.

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$ This is a perfect square trinomial

using the format $x^2 \mp 2xy + y^2 = (x + y)(x + y)$

$$=(\sin\theta + \cos\theta)(\sin\theta + \cos\theta)$$

EXAMPLE 3: Factor.

$\cot^2\theta + 3\cot\theta + 2$ This is a trinomial where we find factors of the back # that add to the middle

using the format $x^2 + 3x + 2 = (x + 2)(x + 1)$

$$=(\cot\theta + 2)(\cot\theta + 1)$$

EXAMPLE 4: Factor.

$4\tan^2\theta + \tan\theta - 3$ This is a trinomial where we guess/check or use the ac method:

using the format $4x^2 + x - 3 = (4x - 3)(x + 1)$

$$=(4\tan\theta - 3)(\tan\theta + 1)$$

You Try!

1. $\tan^2\theta - 1$

2. $\sec^2\theta - \tan^2\theta$

3. $\cot^2\theta + 5\cot\theta + 6$

4. $2\sin^2\theta - 7\sin\theta - 4$

5. $\sin^2\theta + \sin^2\theta\cot^2\theta$

6. $1 + 2\sin\theta + \sin^2\theta$