www.MathWorksheetsGo.com
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I. Model Problems
II. Practice
III. Challenge Problems
IV. Answer Key

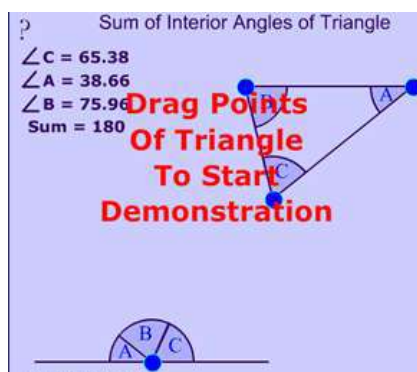**Web Resources**
Sum of Interior Angles
www.mathwarehouse.com/geometry/triangles/

Interactive Demonstration of Sum of Interior Angles
(drag vertices of triangle and see formula in action)

www.MathWorksheetsGo.com

Triangle Sum Theorem

Preliminary Information: The measures of the three interior angles of any triangle in a plane always sums to 180°. For example, in the triangle below at left, $55° + 40° + 85° = 180°$. This relationship may be expressed more generally using algebra as $x + y + z = 180°$, as in the triangle below right.
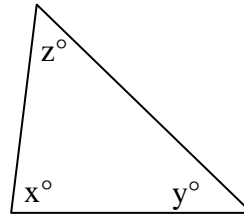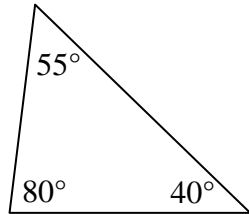
Part 1: Model Problems

Example 1: Determine the unknown angle in the triangle pictured below:
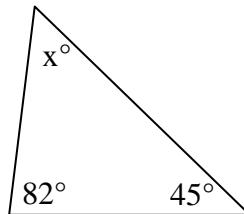
Step 1: Set up an equation to represent the situation:

Since the three angles must sum to 180°, we write

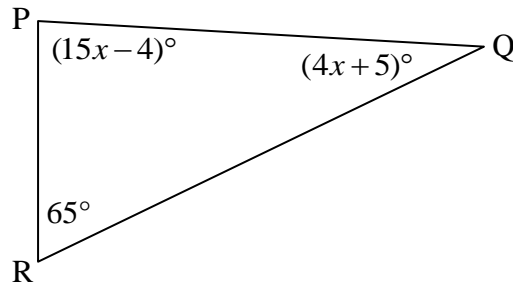$$x + 82 + 45 = 180$$

Step 2: Solve for the unknown variable:

$$x + 127 = 180$$
$$-127 = -127$$
$$x = 53°$$

Step 3: Check for reasonableness: Since $53 + 82 + 45 = 180$, this solution is reasonable.

Example 2: Determine the measure of $\angle P$ in the triangle pictured below.

Step 1: Set up an equation to represent the situation: Since the three interior angles must sum to 180°, we write

$$(15x-4)+(4x+5)+65=180$$

By combining like terms, we obtain

$$15x+4x-4+5+65=180$$
$$19x+66=180$$
$$-66=-66$$
$$19x=114$$
$$\frac{19x}{19}=\frac{114}{19}$$
$$x=6$$

Step 2: Solve for the unknown variable:

$$m\angle P=15x-4$$
$$m\angle P=15(6)-4$$
$$m\angle P=90-4$$
$$m\angle P=86°$$

So the measure of $\angle P$ is 86°.

Step 3: Check for reasonableness: We must obtain the measure of $\angle Q$ to determine if the three angles do, in fact, sum to 180°:

$$m\angle Q=4x+5$$
$$m\angle Q=4(6)+5$$
$$m\angle Q=24+5$$
$$m\angle Q=29°$$
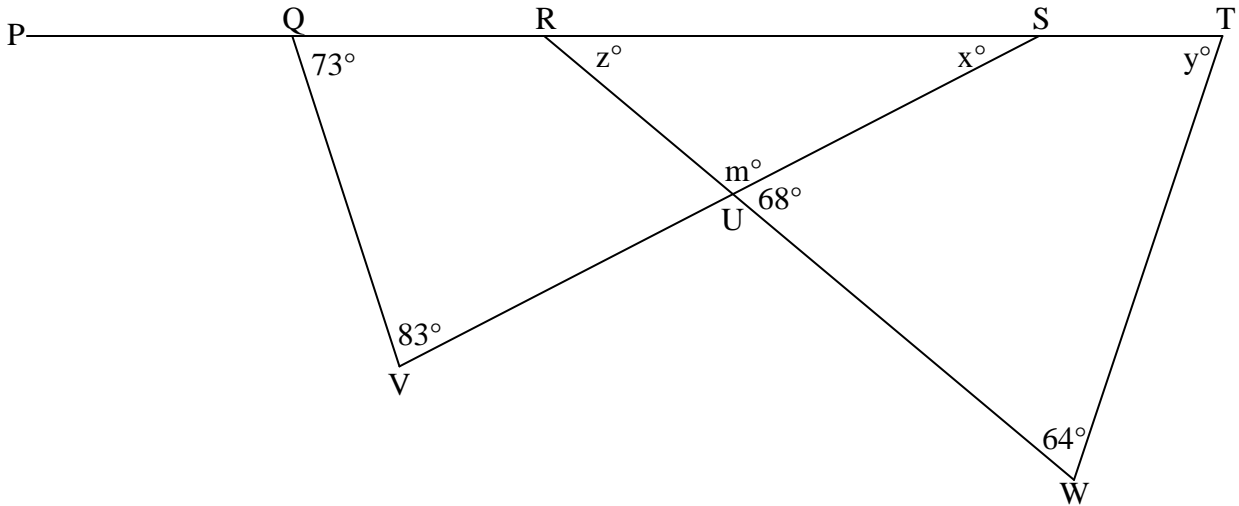
So we check the sum of all three angles:

$$m\angle P+m\angle Q+m\angle R=180$$
$$86+29+65=180 \ \text{☺}$$
$$180=180$$

Example 3: Determine the measures of all unknown angles in the figure below:

Step 1: Set up an equation to represent the sum of the three angles of a triangle. In the figure above, there are three triangles: $\Delta SQV, \Delta TRW, and \ \Delta RSU$. In ΔSQV , we know two of the three angles, so we write an equation to represent it:

$$m\angle QSV + m\angle SQV + m\angle V = 180$$
$$x + 73 + 83 = 180$$
$$x + 156 = 180$$
$$-156 = -156$$
$$x = 24°$$

Step 2: Determine the value of m: Because the angles marked m° and 68° form a

linear pair, they are supplementary. So

$$m + 68 = 180 \ or \ m = 180 - 68 = 112°$$

Step 3: Continue to set up equations to represent the sum of the three angles of a triangle. In ΔRSU , we know two of the three angles, so we write an equation to represent it:

$$m\angle RSU + m\angle SUR + m\angle URS = 180$$
$$x + m + z = 180$$
$$24 + 112 + z = 180$$
$$136 + z = 180$$
$$-136 = -136$$
$$z = 44°$$

The figure is drawn again here for clarity. We now focus on the third triangle:
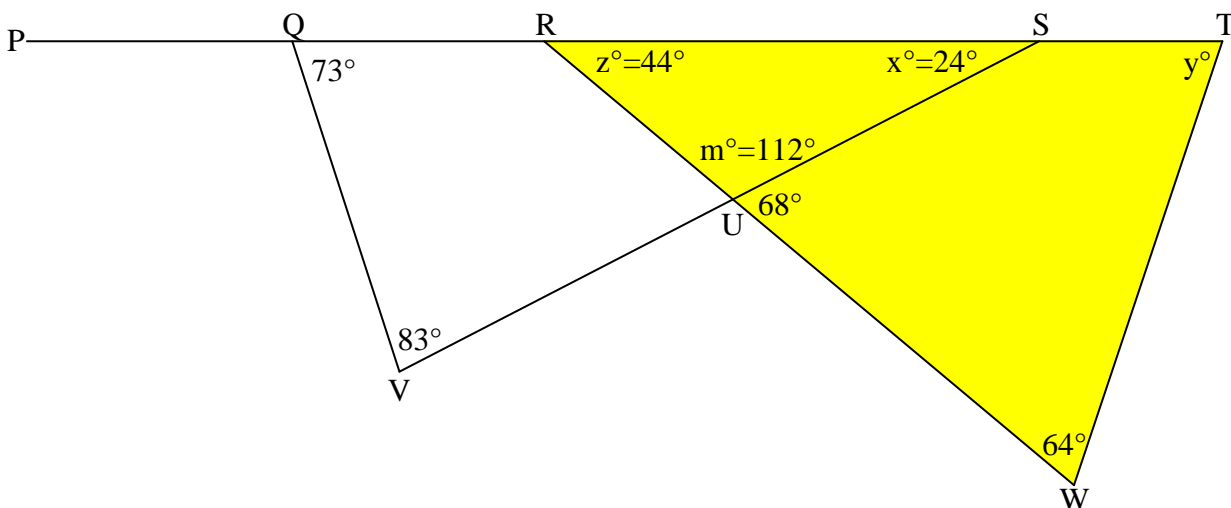
Step 4: Continue to set up equations to represent the sum of the three angles of a triangle. In ΔTRW , we know two of the three angles, so we write an equation to represent it:

$$m\angle TRW + m\angle RWT + m\angle WTR = 180$$
$$z + 64 + y = 180$$
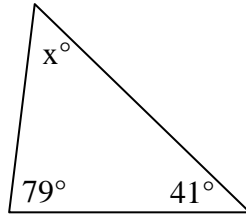$$44 + 64 + y = 180$$
$$y + 108 = 180$$
$$-108 = -108$$
$$y = 72°$$

Step 6: Check for reasonableness. Since we used three different triangles, we should check that all three have interior angles that sum to 180°:
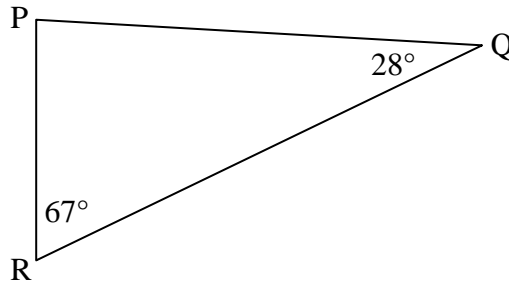
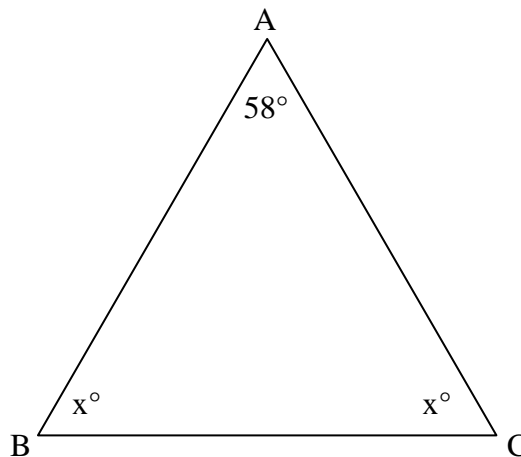| ΔSQV	ΔTRW	ΔRSU
$73 + 83 + 24 = 180$ ☺ $180 = 180$	$44 + 64 + 72 = 180$ ☺ $180 = 180$	$44 + 24 + 112 = 180$ ☺ $180 = 180$

Part 2: Practice

1. Determine the unknown angle in the triangle pictured below:

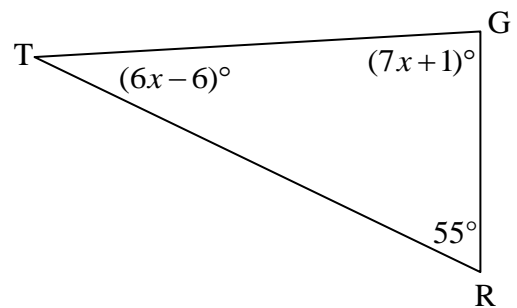2. Determine the measure of ∠*P* in the triangle pictured below.

3. Determine the measure of ∠*B* in the triangle pictured below.
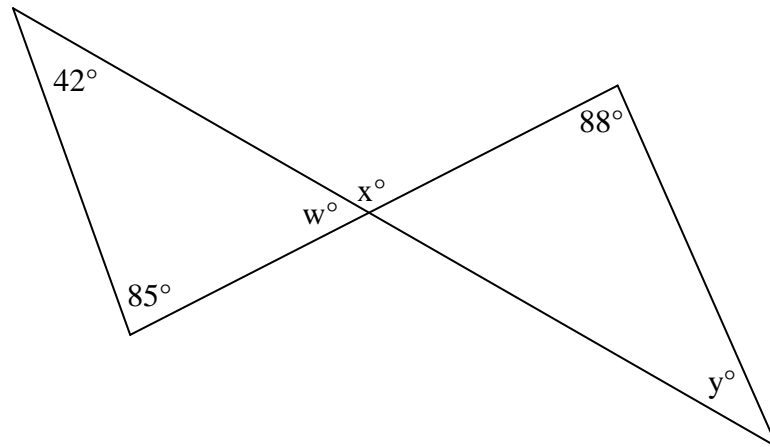

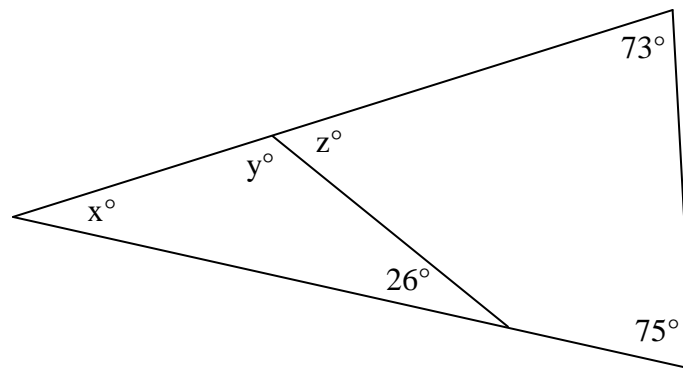4. Multiple Choice: What is the correct measure of ∠*T* in the triangle pictured below?
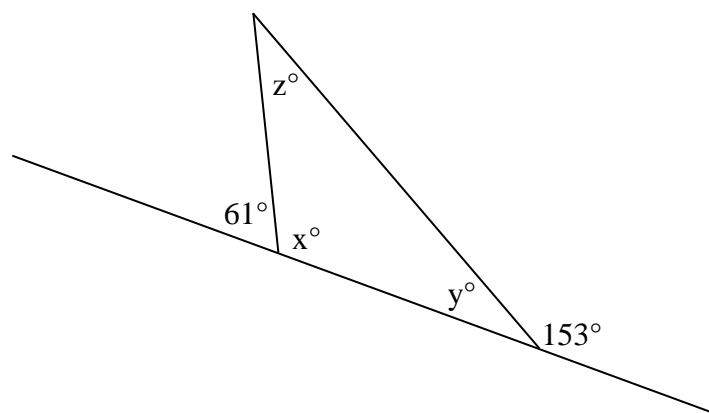
A) 10°
B) 12°
C) 54°
D) 71°

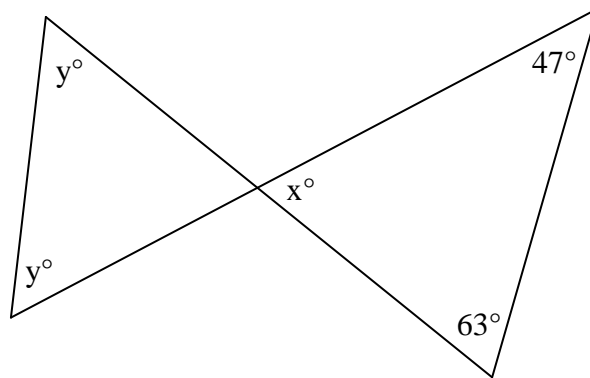5. Determine the measures of the unknown angles in the figure.

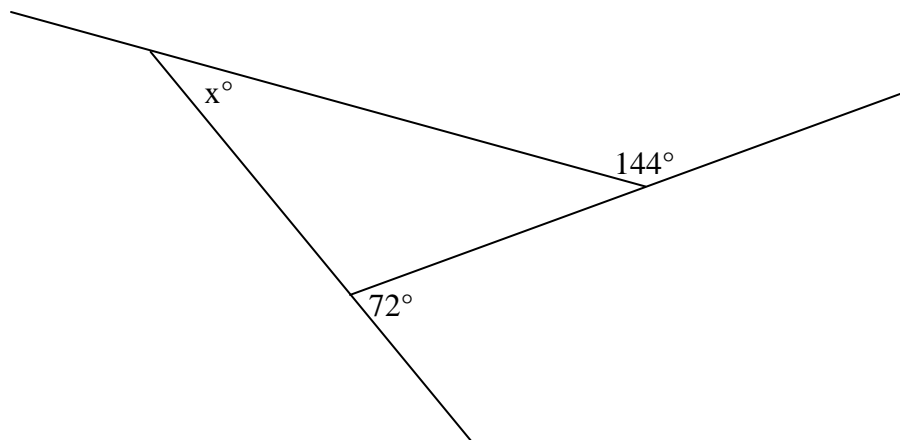6. Determine the measures of the unknown angles in the figure.

7. Determine the measures of the unknown angles in the figure.

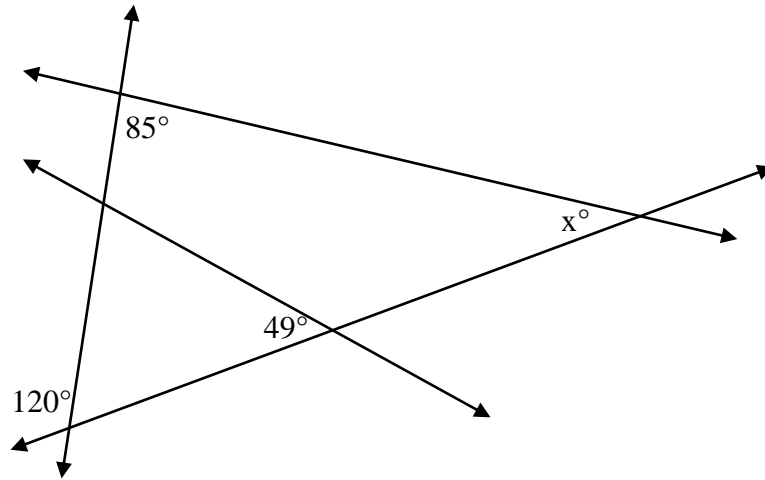8. Determine the measures of the unknown angles in the figure.

9. Error Analysis: Erik and David both got the same answer when they worked the following problem. However, their teacher graded their solutions very differently. Which student, if any, was correct?
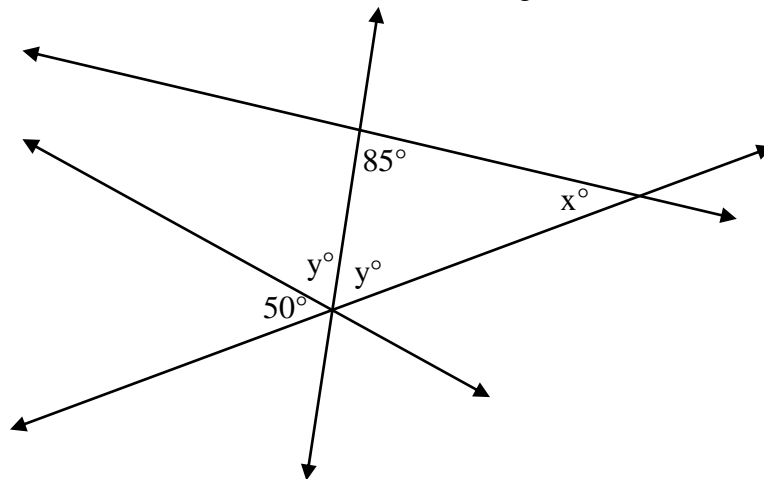

Erik's solution	David's solution
$x + 72 + 144 = 180$	$180 - 144 = 36; \quad 180 - 72 = 108$
$x + 216 = 180$	$x + 36 + 108 = 180$
$216 - 180 = 36$	$x + 144 = 180$
$x = 36°$	$-144 = -144$
	$x = 36°$

Part 3: Challenge Problems

10. Determine the measure of the unknown angle.

85°

x°

49°

120°

11. Determine the measures of the unknown angles.

85°

x°

y° y°

50°
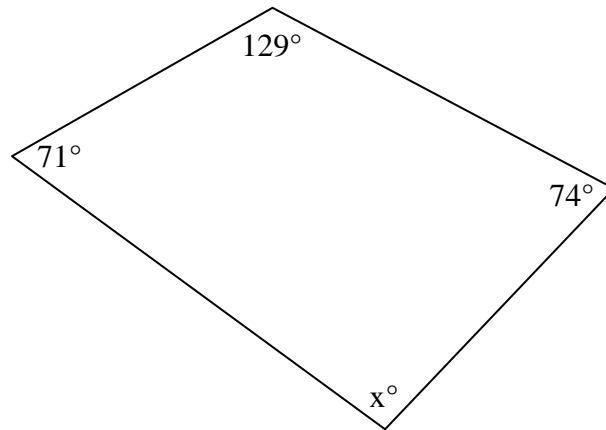
12. Consider the quadrilateral (4-sided polygon) at right.

    a) On the figure, draw an auxiliary line from A to C.

    b) How many triangles are formed?

    c) How many degrees, total, are in the interior angles of each triangle?

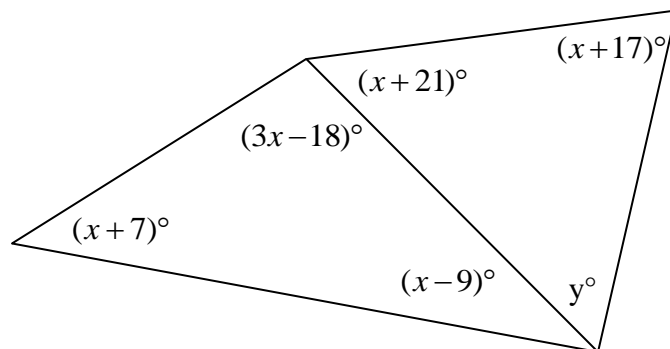   d) Make a conjecture: what is the sum of the interior angles in every quadrilateral?

A

D

B

C
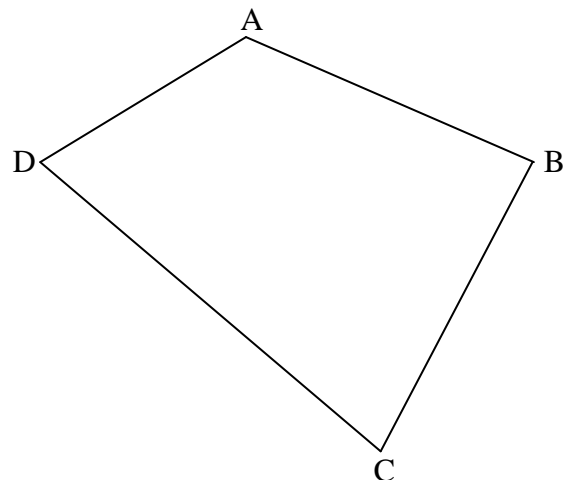
13. Determine the value of the unknown.

129°

71°

74°

x°

14. Determine the value of the unknowns.

74°

72°

115°

x°

w°

z°

y°

45°

55°

v°
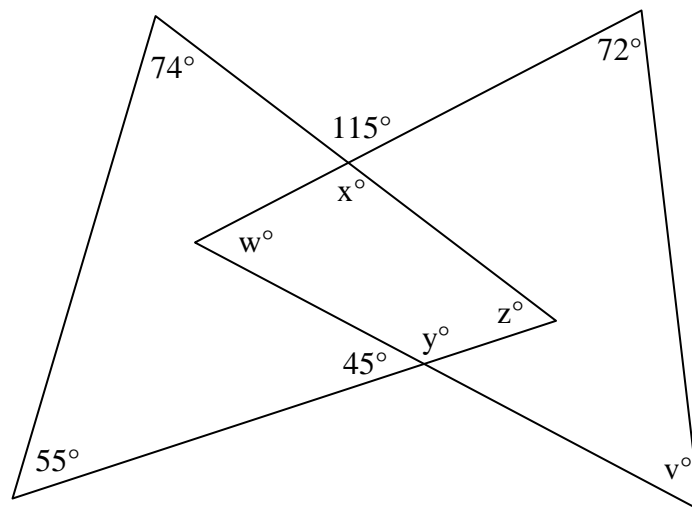
15. Determine the values of the unknown variables.

$(x+17)°$

$(x+21)°$

$(3x-18)°$

$(x+7)°$

$(x-9)°$

y°

Part 4: Answer Key

1. x = 60°

2. 85°

3. 61°

4. C

5. w = 53°; x = 127°; y = 39°

6. x = 32°; y = 122°; z = 58°

7. x = 119°; y = 27°; z = 34°

8. x = 70°; y = 55°

9. Erik's solution is incorrect; he just adds up all three quantities to 180. If he continued his algebra steps correctly, he would arrive at a solution of x = −36°, which isn't possible. David's solution is correct.

10. 35°

11. x = 30°; y = 65°

12. a) See figure at right.

    b) Two triangles are formed.

    c) Each triangle has 180°

    d) The sum of the interior angles of a quadrilateral is 360°.
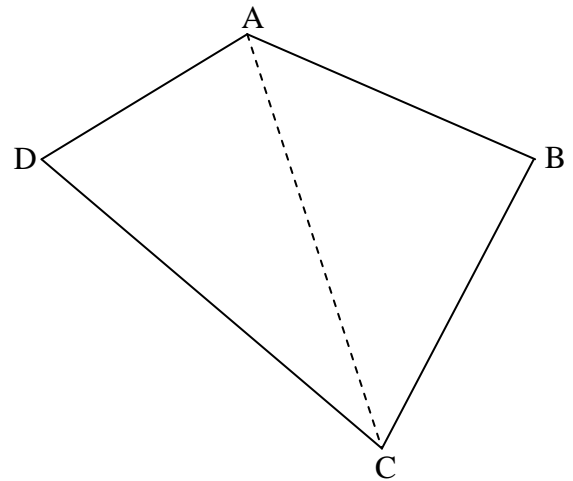
13. x = 86°

14. v = 49°; w = 59°; x = 115°; y = 135°; z = 51°

15. x = 40°; y = 62°