Scarsdale High School
Mathematics Department

Math 453 Curriculum
June 201l

Textbook: Calculus I with Precalculus – A One-Year Course, 2nd Edition, by Larson, Hostetler and Edwards; Houghton Mifflin, 2006. (ISBN: 0-618-56806-9)

Supplementary Materials: 1. Calculus in Motion (CIM) by Audrey Weeks
2. TI89-Titanium Packet by Roger Cappucci

*** Technology enhancement submitted by Lynn Potter, Joe Nista, and Roger Cappucci.**

Notes: 1. The major concepts in this course will be applied to polynomial functions. There are times, however, when rational functions or square root functions will be utilized. Trigonometric functions, exponential functions and logarithmic functions have intentionally been excluded to allow for depth over breadth.

2. The length of the course is 104 lessons. Days for testing have been included in the number of lessons per unit.

I. Functions (6 Lessons) [Sections: 1.1, 1.2, 1.4, 1.5]

1. Definition of function ***(www.mathwarehouse.com/algebra/relation/math-function.php)***
2. Representing a function graphically and through an equation (function notation)
3. Define even and odd functions
4. Define one-to-one and many-to-one
5. Linear Functions (See p. 88: 67 – 74)
6. Define and graph piecewise functions
7. Algebra of functions: sum, difference, product, quotient and composition
   (http://www.youtube.com/watch?v=828y5EtC9LQ) [Melanie Kennelly]
   (Note: it is not necessary to consider the domain of the "new" function.)
8. Inverse of a function

II. Introducing the TI89; Review of Polynomial Functions; Solving Polynomial Equations (7 Lessons) [Roger's Packet; Sections 2.2; 2.3]

1. Introduction to the basic operations found on the home screen of the TI89.
2. Graphing on the TI89 (Discuss the importance of reasonable window dimensions.)
3. Apply commands from the graph screen: *Value, Zero, Max/Min, Intersection*
4. Use *Table* setup and *Table* to find values of a function
5. Definition of Polynomial Function
6. Finding the zeros (real and imaginary) of a polynomial function
   Review: Rational Root Theorem, Synthetic Division, etc.
7. End Behavior model of a polynomial function (Use the TI89 to illustrate this concept.)

III. Rational Functions (7 Lessons) [Section 2.6]

1. Define Rational Function

(http://zonalandeducation.com/mmts/functionInstitute/rationalFunctions/definition/definition.html)

2. Analyze the equation of a rational function and find: *intercepts, symmetry, deleted point(s), vertical asymptotes, end behavior model (horizontal asymptote, polynomial asymptote), check for intersection with the EBM, Number Line Analysis to determine if the graph is above/below the x-axis.*
3. Sketch the graph of the rational function by hand

Notes: 1. Use the TI89 to demonstrate/reinforce *numerically* (table) the behavior of the graph to the left and right of the vertical asymptote and the end behavior.
2. The concept of *limit* will be introduced in this unit without explicitly using the term "limit" but rather "*x* approaches a number *a* from the left or right" or "as *x* increases or decreases without bound the value of the function …"

IV. Limits & Their Properties (19 Lessons) [Sections 3.1 – 3.5]

1. Evaluate a limit numerically (Table of Values / Graphically-use *ZOOM* commands to illustrate. ***(www.analyzemath.com/calculus/limits/introduction.html)***
2. Identify when a limit *fails to exist*
3. Properties of Limits (Constant, Constant times a function, sum, difference, product, quotient, power)
4. Evaluate limits by substitution
5. Evaluate a limit algebraically: *Reducing a Rational Expression; Simplifying a Complex Fraction; Rationalizing a Numerator or Denominator with a Square Root*)
6. Definition of Continuity at a Point
7. Types of Discontinuity – *Point, Jump, Infinite*
8. Removable vs. Non-Removable Discontinuity
9. Redefine a function to remove the discontinuity at $x=a$
10. Evaluating limits involving the greatest integer function and piecewise defined functions
    (Note: Students may not have learned about the greatest integer function in previous math courses.)
11. Solve continuity problems with piecewise functions and a variable of a. (p. 235: 49, 50)
12. State and apply: The Intermediate Value Theorem and The Location Principle
13. Limit of a function as $x \to \pm\infty$.
14. **Review graphing a rational function and incorporate limits into the "analysis" of the function**

V. The Derivative and Techniques of Differentiation (30 Lessons) [Sections 4.1 – 4.5]

(http://www.youtube.com/watch?v=0v6kB21KmMc) [2 questions of calculus-Burger]

1. Define and calculate the average rate of change in the value of a function (p. 49); the definition of a secant line
2. Definition of a line *tangent* to a curve at a point; slope of a curve
3. Limit definition of the derivative function

(http://www.sosmath.com/tables/derivative/derivative.html) [Definition Derivative/formulas]

4. Alternate definition of the derivative (p. 257)
5. Derivative notation: $f'(x)$ and $\frac{dy}{dx}$
6. Establish without proof: "Differentiability implies continuity."
   "Continuity does not imply differentiability."(e.g. $y = |x|$ at $x = 0$.)
   "If a function is not continuous at a point then it is not differentiable at that point."
7. Define vertical tangent informally (e.g. $y = \sqrt[3]{x}$ at $x = 0$)

Note: Establish the following techniques of differentiating without proof. In certain cases it is advisable to use the TI89, CIM or Calculus Applets to motivate the rules.

8. Techniques of Differentiation: The Constant Rule, The Constant Multiple Rule, Sum and Difference Rules, Product Rule, Quotient Rule
9. Find the equation of a line tangent to a curve at a given point.
10. Higher Order Derivatives
11. Definition of Instantaneous Velocity and Acceleration; Apply the definitions to solve problems
12. The Chain Rule; ***(http://www.youtube.com/watch?v=PqzFTmJbrmg) [Chain Rule]***
13. Implicit Differentiation

VI. Applications of the Derivative (21 Lessons) [Sections 4.6; 5.1 – 5.6]

1. Related Rate Problems ***(Calculus In Motion – Weeks)***
2. Definition of Extrema
3. State and apply the Extreme Value Theorem
4. Definition of Relative Extrema
5. Define Critical Number and find the critical number(s) of a function
6. Establish: "If *f* has a relative min or max at *x=c* then *c* is a critical number of *f*." Also establish that the converse of this theorem is not true.
7. Find extrema on a closed interval
8. State: Rolle's Theorem and the Mean Value Theorem
9. Apply the MVT to find the value(s) of "*c*"
10. Increasing and Decreasing intervals for a function (First Derivative Test)
11. Points of Inflection and Concavity of a function (2^{nd} Derivative test)
12. Curve Sketching **[Include above concepts to polynomial and rational functions.]**
13. Optimization Problems **[Applications in several areas of study should be included.]**

VII. Anti Differentiation: Applications - Area (10 Lessons)
[Sections 6.1, 6.2, 6.4, *6.5, *6.6]

1. The Basic Rules of Anti-Differentiating
2. The Power Rule for Anti-Differentiating
3. RRAM, LRAM and MRAM for approximating area under a curve

*4. Numerical approximation of an integral by the Trapezoidal Method

5. The Fundamental Theorem of Calculus (FTC)
   (http://www.youtube.com/watch?v=ODvljvv4vrw) [FTC Part 2 – Burger]
   (a) Evaluating a definite integral
   (b) Finding the area bounded by a curve defined on [a, b] and the x-axis

*6. Area bounded by two curves
   (http://www.youtube.com/watch?v=sD0N16GSP3Y) [Area b/w 2 curves]

*7. Integration by substitution

*If time permits.

VIII. Review for Final Exam (4 Lessons)