Teaching static equilibrium and torque through the ISLE approach

Eugenia Etkina and Gorazd planinsic

For today's workshop

Rename yourself

First name Where you teach - high school or college Country

Eugenia University USA

Links to the document for today's workshop

April 2025 Static equilibrium and torque

ALG Chapter 8 Third edition.docx

ALG Chapter 8 SolutionsThird edition.docx

OALG Chapter 8 Final.docx

Need to know

Sit on a chair with your back straight and your feet on the floor in front of the chair. Without using your hands and without tilting forward, try to stand up.

Team 1

ALG 8.1.1 Observe and find a pattern

Use a pencil eraser to push at several points on the edge of a thin, flat, irregularly-shaped piece of plywood or cardboard that you put on the smooth surface. **Work with your group members to identify a pattern in the direction of the forces that do not cause the object to rotate.**

Hint: Draw lines on the object in the direction of the forces.

https://mediaplayer.pearsoncmg.com/assets/_frames.true/secs-experiment-video-17

Team 2

ALG 8.1.1 Observe and find a pattern

Use a pencil eraser to push at several points on the edge of a thin, flat, irregularly-shaped piece of plywood or cardboard that you put on the smooth surface. **Work with your group members to identify a pattern in the direction of the forces that do not cause the object to rotate.**

Hint: Draw lines on the object in the direction of the forces.

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Team 3

ALG 8.1.1 Observe and find a pattern

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Team 4

ALG 8.1.1 Observe and find a pattern

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Hint: Draw lines on the object in the direction of the forces.

https://mediaplayer.pearsoncmg.com/assets/_frames.true/secs-experiment-video-17

Time for telling

New model: a rigid body - an object for which the distance between any two points does not change during any process.

Center of mass (a qualitative definition)

The center of mass of an object is a point where a force exerted on the object pointing directly toward or away from that point will not cause the object to turn. The location of this point depends on the mass distribution of the object.

Team 1

OALG 8.1.2 Observe and explain

Use the same piece of hard cardboard with the known location of the center of mass of your object. Then take a pin and use it to support the object at the center of mass. Why doesn't the object tilt?

Need a video here or have the object with me

Time for telling

When multiple forces are exerted on a rigid body, the center of mass of the rigid body accelerates translationally according to Newton’s second law.

Team 1

ALG 8.2.1 Observe and find a pattern

Work with your group to conduct the following observational experiments: Place the whiteboard marker horizontally on the table and secure it in place with some play-dough so it cannot roll. Place the meter stick on top of the marker so that it balances at the 50 cm mark and doesn't touch the surface of the table. Note: If your meter stick doesn't balance at 50 cm exactly, add either a paper clip or some play-dough to a suitable place on it so that it does. Now place different numbers of washers on the left and on the right of the balance point. Figure out where you need to place the washers in order for the system to balance and complete the following table:

a. For each situation, draw a picture of the meter stick showing all of the forces exerted on it. In other words, sketch the apparatus and draw an extended-body force diagram for the meter stick, showing forces and points where those forces are exerted on the meter stick.

b. Find a pattern that relates the distances of the washers from the balance point and the magnitudes of the forces the washers exert on the meter stick. What is the relationship between the positions (as measured from the middle) of the left and right groups of washers, and the force that each group exerts on the ruler?

Number of washers on the left	**Distance of left washer group from the middle**	**Number of washers on the right**	**Distance of right washer group from the middle**
1	20 cm	1	*20 cm*
1	20 cm	2	*10 cm*
2	30 cm	3	*20 cm*
4	30 cm	3	*40 cm*

Team 1 sketch and pattern

Team 2

ALG 8.2.1 Observe and find a pattern

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Team 2 sketch and pattern

Team 3

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Team 3 sketch and pattern

Team 4

ALG 8.2.1 Observe and find a pattern

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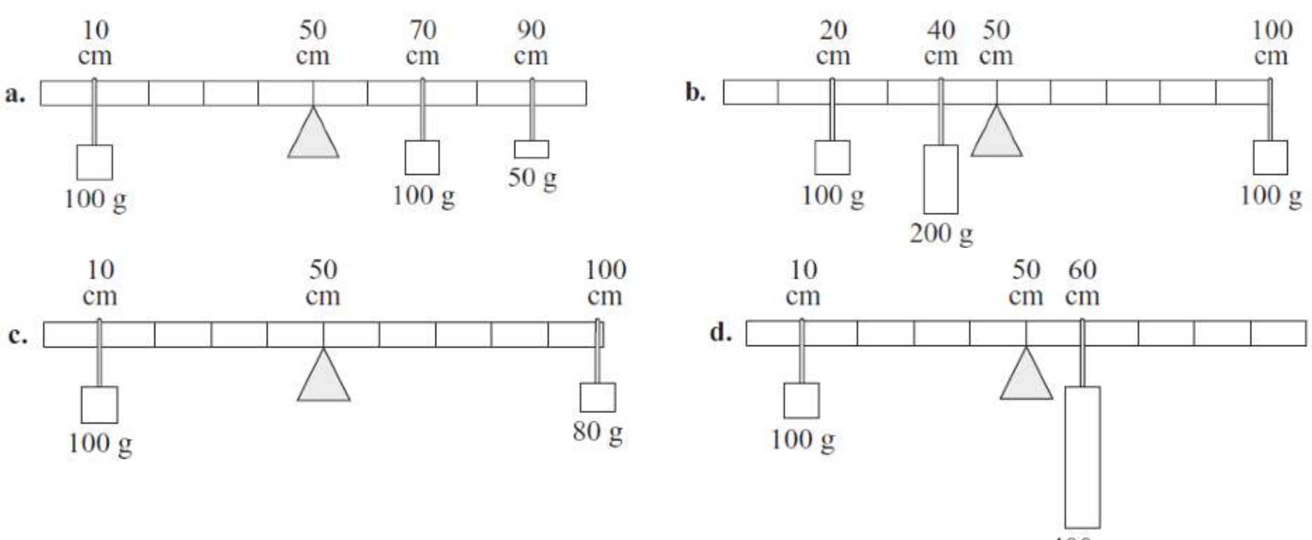
Team 4 sketch and pattern

Team 1

ALG 8.2.3 Observe and find a pattern A meter stick is balanced at its center. When you hang different mass blocks from different positions on the stick, as shown in the illustrations that follow, the stick remains balanced. Work together to devise a rule to explain this behavior (or extend the rules developed in the previous activities). Be sure that the rule is compatible with all of the experiments.

a. Draw all forces exerted on the stick by other objects. Remember the force exerted by Earth on the stick.

b. Describe the rule you devised, both in words and mathematically.



Team 2

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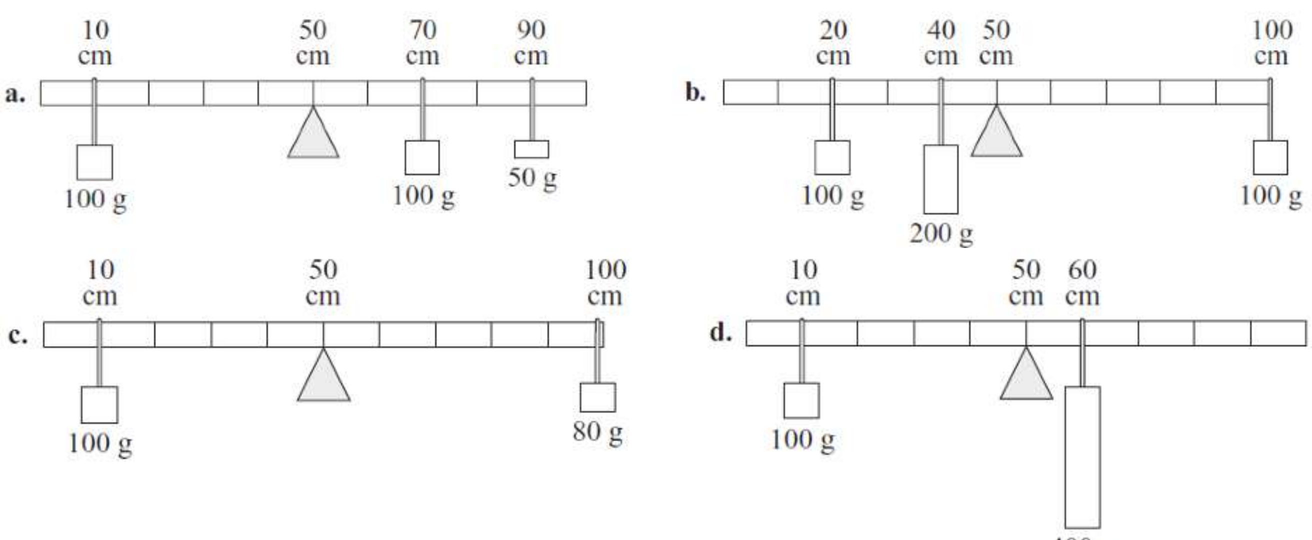

Team 3

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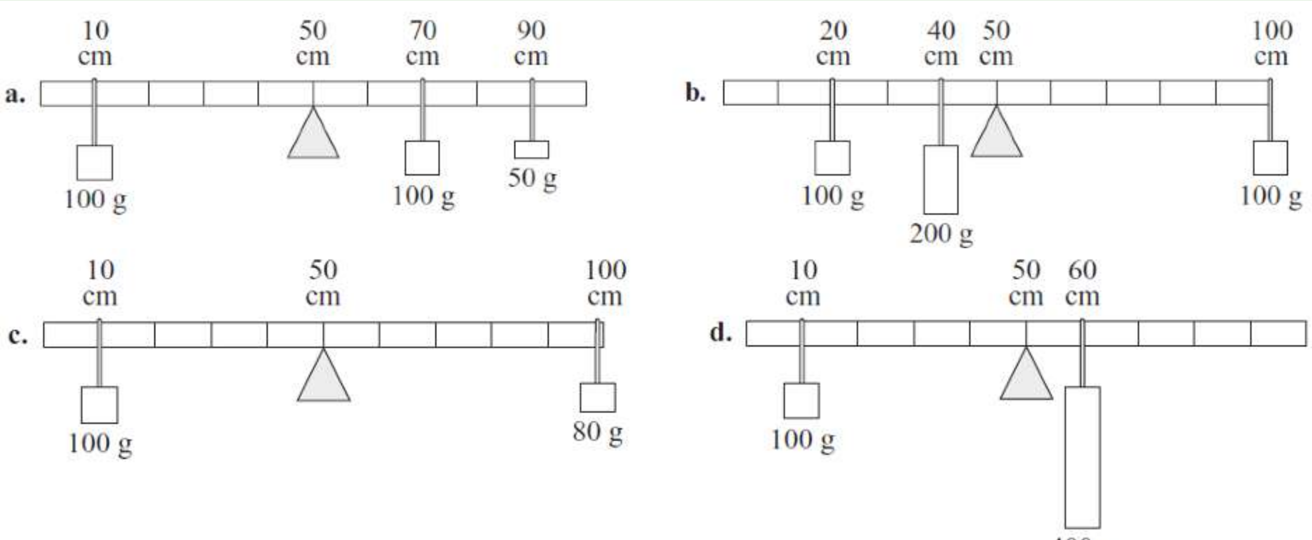

Team 4

ALG 8.2.3 Observe and find a pattern A meter stick is balanced at its center. When you hang different mass blocks from different positions on the stick, as shown in the illustrations that follow, the stick remains balanced. Work together to devise a rule to explain this behavior (or extend the rules developed in the previous activities). Be sure that the rule is compatible with all of the experiments.

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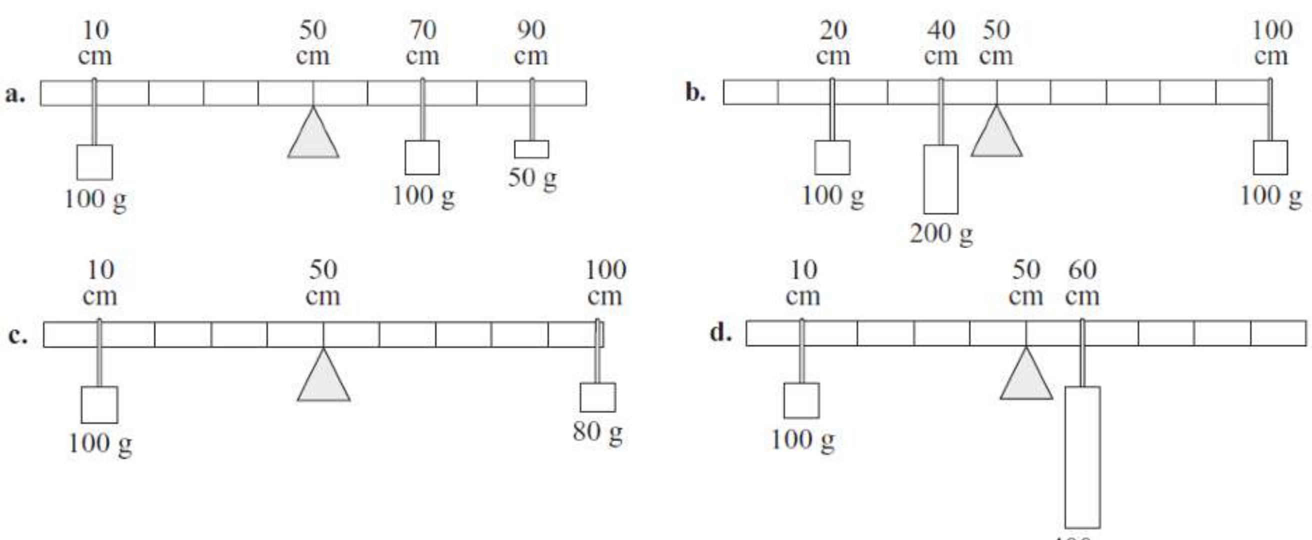


All together

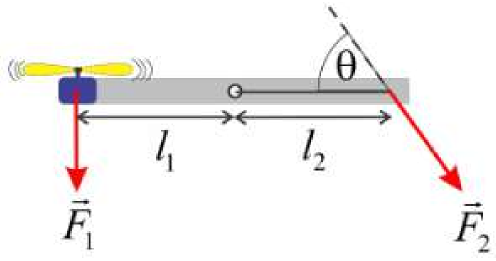
ALG 8.2.4. Link to the document with solutions

[ALG Chapter 8 SolutionsThird edition.docx]

So far, we have come up with a mathematical description of the turning ability of the force as ± Fl, where “plus” and “minus” refer to the direction of rotation (counterclockwise or clockwise), F denotes the magnitude of the force, and l is the distance from the point of application of this force to the axis of rotation. However, in all the cases we have seen so far, the direction of the force was has been perpendicular to the line connecting the axis of rotation and the point where the force is exerted.

What happens if this is not true?

We can investigate this question by placing a metal ruler fixed at its center of mass on a support so that it rotates freely in the horizontal plane around the center of mass (this which is where the axis of rotation is). At the left end of the ruler, we fix a fan. When the fan is on, it pushes the air away from the observer and therefore, the air pushes the fan in the opposite direction, exerting the force F1, making the ruler rotate counterclockwise. At the right end of the ruler, we fix a force meter connected to a computer that displays the force reading. The force meter pulls the ruler in the clockwise direction, exerting a force F2. If our mathematical expression for the rotational ability of the force is correct, then the reading of the force meter should remain the same in the experiment, as long as the fan setting does not change.

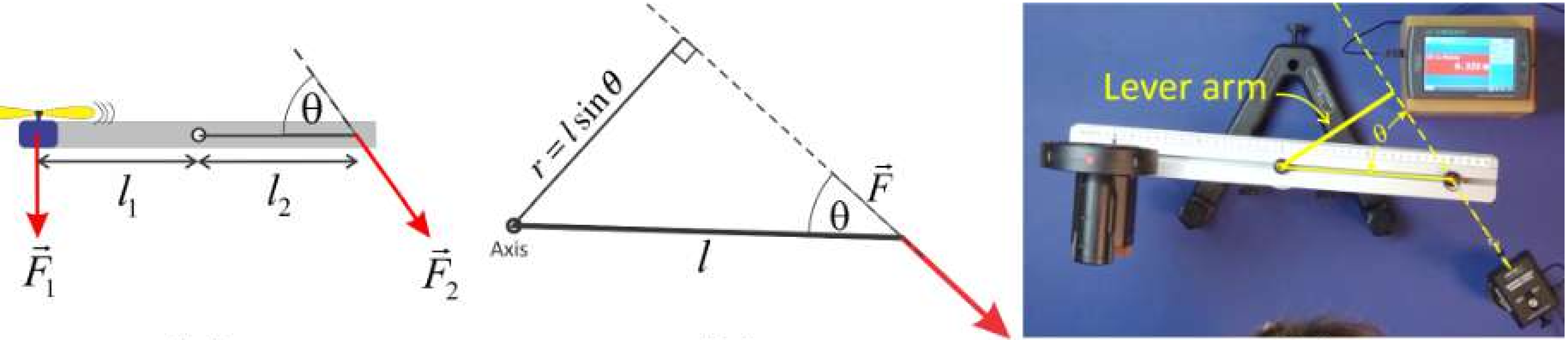


(a)

https://www.youtube.com/watch?v=5YMkiyKwIGU

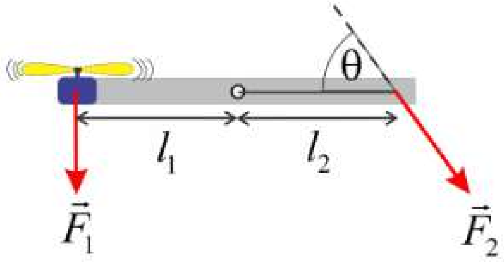
However, as you saw in the video, which shows the results of this experiment, this is not the case at all! Without moving the point where the force is exerted on the ruler, we changed the magnitude of the force without setting the ruler into rotational motion. This means that the angle q between the line along which the force is exerted and the line connecting the point of application of the force to the axis of rotation matters!

Let’s find a pattern in the relationship between the direction of the force and the turning ability of the force (*Fl)*. To account for the role of the direction of the force, we record the angle q that the force makes with the ruler and the sin of the same angle. (We use the sin of the angle because the force parallel to the ruler does not cause rotation.) The table shows the data that we collected from the video.

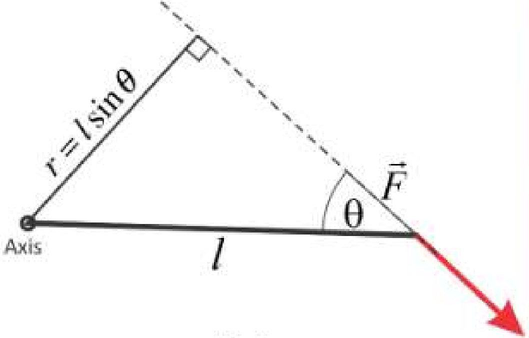


(a) (b) (c)

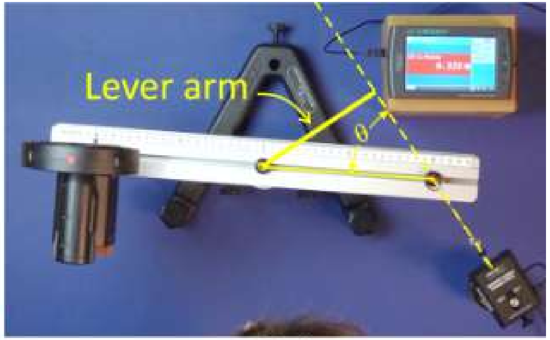
F_1 (N)	l_1 (m)	$F_1 l_1$	θ_1 (°)	$F_1 l_1 \sin\theta_1$	F_2 (N)	l_2 (m)	$F_2 l_2$	θ_2 (°)	$F_2 l_2 \sin\theta_2$
0.25	0.20	0.050	90° 1.00	0.050	0.25	0.20	0.050	90° 1.00	0.050
0.25	0.20	0.050	90° 1.00	0.050	0.26	0.20	0.052	71° 0.95	0.049
0.25	0.20	0.050	90° 1.00	0.050	0.28	0.20	0.056	64° 0.90	0.050
0.25	0.20	0.050	90° 1.00	0.050	0.33	0.20	0.066	50° 0.77	0.051



(a)



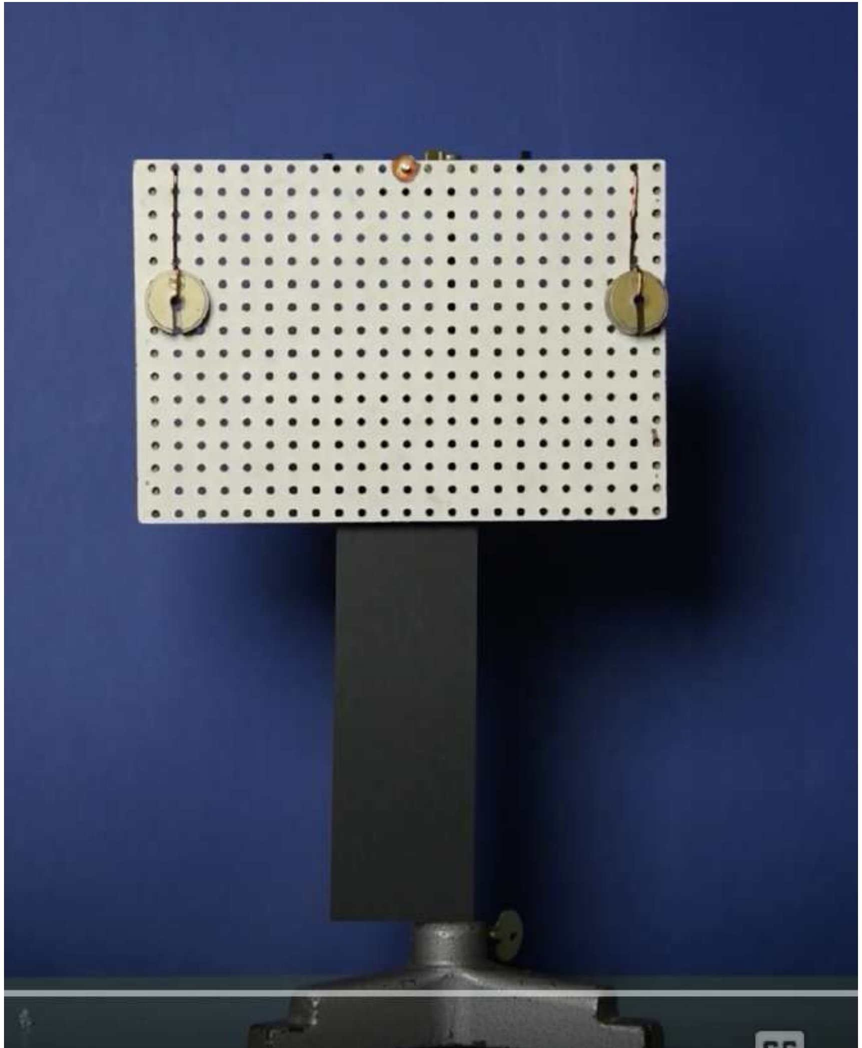
(b)



(c)

Test your idea

Predict what will happen to the equilibrium of the board when we move the hanging object on the right down the board and thus increase the distance between the object and the axis of rotation?



Compare your prediction to the outcome of the experiment

https://youtu.be/5LqbtTey2xs

Time for telling

What is torque?

Use the patterns and rules that you devised in the previous activities to summarize what you know about the sum of the forces exerted on an object that is in equilibrium and the sum of the torques caused by these forces.

Condition 1. Translational (force) condition of static equilibrium An object modeled as a rigid body is in translational static equilibrium with respect to a particular observer if it is at rest with respect to that observer and the components of the sum of the forces exerted on it in the perpendicular x- and y-directions are zero:

$$\Sigma F_{\text{on O}\,x} = F_{1\text{ on O}\,x} + F_{2\text{ on O}\,x} + \cdots + F_{n\text{ on O}\,x} = 0 \tag{8.2x}$$

$$\Sigma F_{\text{on O}\,y} = F_{1\text{ on O}\,y} + F_{2\text{ on O}\,y} + \cdots + F_{n\text{ on O}\,y} = 0 \tag{8.2y}$$

The subscript n indicates the number of forces exerted by external objects on the rigid body.

Condition 2. Rotational (torque) condition of static equilibrium A rigid body is in turning or rotational static equilibrium if it is at rest with respect to the observer and the sum of the torques $\Sigma\tau$ (positive counterclockwise torques and negative clockwise torques) about any axis of rotation produced by the forces exerted on the object is zero:

$$\Sigma\tau = \tau_1 + \tau_2 + \cdots + \tau_n = 0 \tag{8.3}$$

Return to the “need to know”

Explain why you cannot get off the chair. In your teams use the whiteboard to draw a force diagram, consider torques, etc. Then take a screenshot of your whiteboard and paste it to the slide for your team.

Team 1 Why can’t you get off the chair?

Team 2 Why can’t you get off the chair?

Team 3 Why can’t you get off the chair?

Team 4 Why can’t you get off the chair?

Center of mass - quantitatively

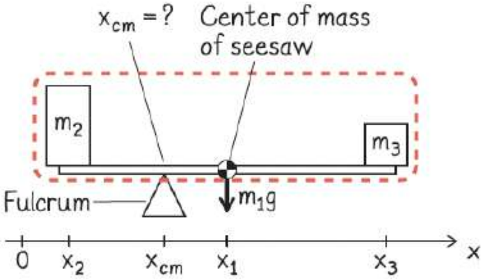
EXAMPLE 8.4 **Supporting a seesaw with two people**

Find an expression for the position of the center of mass of a system that consists of a uniform seesaw of mass m_1 and two people of masses m_2 and m_3 sitting at the ends of the seesaw beam ($m_2 > m_3$).

Sketch and translate The figure at right shows a labeled sketch of the situation. The two people are represented as blocks. We choose the seesaw and two blocks as the system and construct a mathematical equation that lets us calculate the center of mass of that system. We place the x-axis along the seesaw with its origin at some arbitrary position on the left side of the seesaw. The center of mass of the seesaw beam is at x_1 and the two blocks rest at x_2 and x_3. At what *position x* should we pl

not rotate—

sum of all torques exerted on the system is zero. This position is the center of mass of the three-object system.



Center of mass (quantitative definition) If we consider an object as consisting of parts 1, 2, 3, . . . n whose centers of masses are located at the coordinates (x_1, y_1); (x_2, y_2); (x_3, y_3); . . . (x_n, y_n), then the center of mass of this whole object is at the following coordinates:

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots + m_ny_n}{m_1 + m_2 + m_3 + \cdots + m_n} \quad (8.4)$$

Team 1 read the text of the activity but use the video of the experiments to do the calculations

ALG 8.4.1 Design an application experiment

You have a ruler of unknown mass and a small 50-g object. Design an experiment to determine the mass of the ruler using your knowledge of static equilibrium.

a. Draw a picture of the experimental set-up.

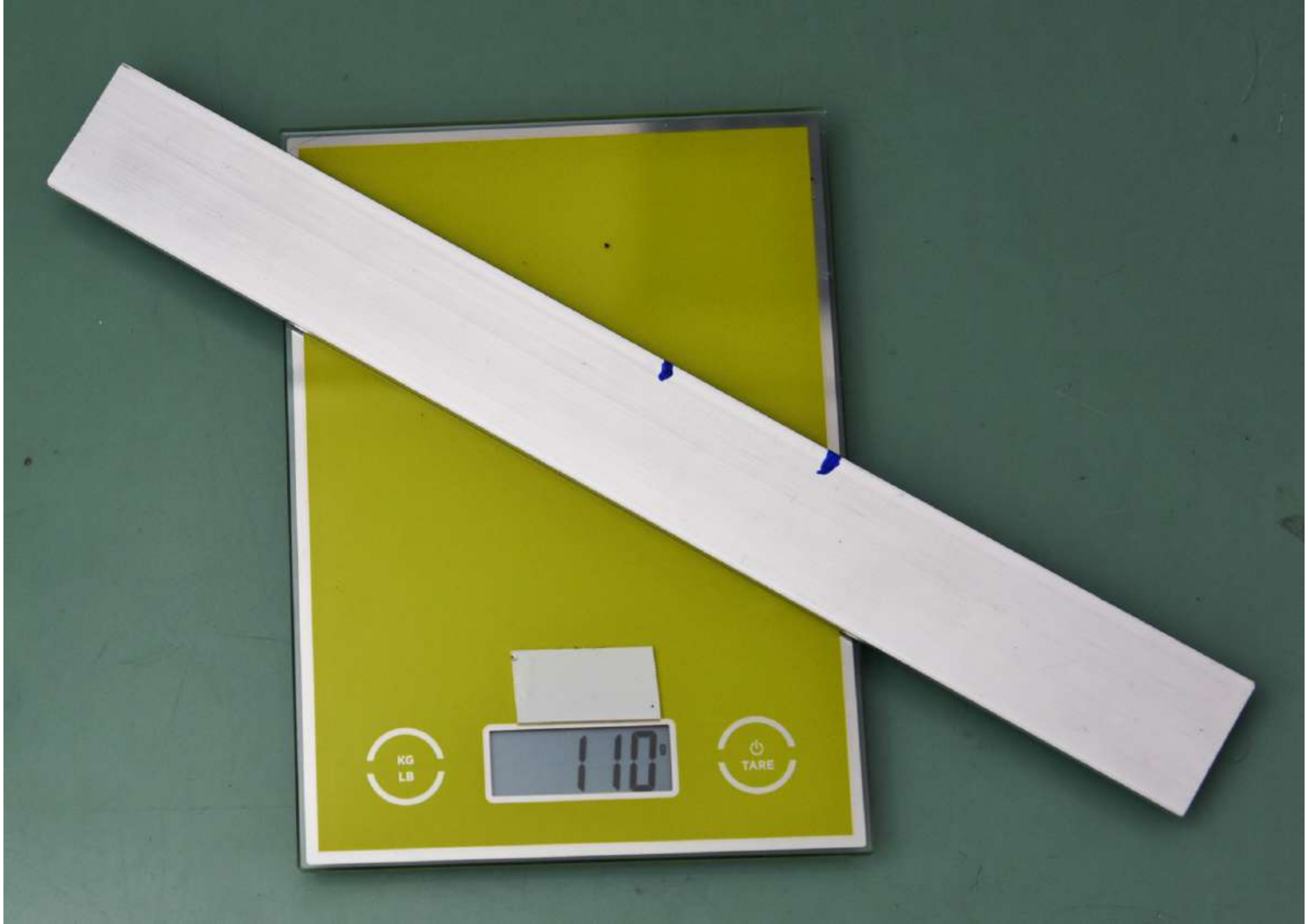
b. Describe the procedure in words.

c. Apply the concepts of equilibrium to develop equations that can be used to predict the mass of the meter stick. Then predict the mass.

d. Use a scale to measure the mass and compare the result to the predicted value.

e. How can you explain the difference between the predicted and the measured value?

https://www.youtube.com/watch?v=k38Og8b0Jf0


KG
LB
110
TARE

Team 1

Test an idea

Your friend says that the mass of any object is distributed evenly around its center of mass. Design an experiment to test your friend’s idea. You have a ruler, a set of small objects of different masses, masking tape, and a mass measuring scale.

Use the data from the video in the previous experiment.

Stability of equilibrium

You have probably observed that it is easier to balance and avoid falling while standing in a moving bus or subway train if you spread your feet apart in the direction of motion. By doing this you are increasing the area of support, the area of contact between an object and the surface it is supported by.

a. Draw force diagrams and consider the torques exerted on two people by the gravitational force and normal forces at their feet when they are on a slowing-down train. Person A is standing with their feet close together and person B is standing with their feet wide apart.

b. Who is more likely to fall?

c. When is equilibrium stable? (What does it mean “stable”?)

Stable equilibrium: when the object returns to the equilibrium position after

Testing experiment

You have a box of crackers and the same mass wooden block. The cracker box is bigger than the wooden block. Compare the angle that you can tilt the cracker box before it tips to the angle that you can tip the wooden block. Explain your prediction.

A new situation

https://www.youtube.com/watch?v=UEnkN939ZLw

Watch the first 1.5 minutes without sound. What did you notice?

Take a pencil. Can you balance it vertically using the tip?

https://www.youtube.com/watch?v=3FKI_qMTk48

Time for telling: when is such equilibrium stable?

Time for telling

When is such equilibrium stable?

What did you learn today?