Taylor Remainder
AP Calculus BC

Name: Answers

1) Estimating $\sin(\frac{13}{12}\pi)$ with a Taylor polynomial centered at $x = \pi$, what is the least degree polynomial that assures an error smaller than 0.001?

$f(x) = \sin x$

$\max f^{(n+1)}(x) = 1$

$$|R_n(x)| \le \left| \frac{\max f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1} \right|$$

$$R_n\left(\tfrac{13\pi}{12}\right) = \left| \frac{1}{(n+1)!}\left(\frac{13\pi}{12} - \pi\right)^{n+1} \right| < .001$$

$$\left| \frac{1}{(n+1)!}\left(\frac{1}{12}\pi\right)^{n+1} \right| < .001$$

$$\boxed{n=3}$$

Use guess + check

n	$R_n(x)$
1	$\frac{1}{2}(\frac{1}{12}\pi)^2 \approx .0342$
2	$\frac{1}{6}(\frac{1}{12}\pi)^3 \approx .00299$
3	$\frac{1}{24}(\frac{1}{12}\pi)^4 \approx .00019$

2) Estimating $e^{1.4}$ with a Taylor polynomial centered at $x = 2$, what is the least degree polynomial that assures an error smaller than 0.001?

$$|R_n(1.4)| \le \left| \frac{e^2}{(n+1)!}(1.4-2)^{n+1} \right| < .001 \qquad \boxed{n=5}$$

Guess + check

n	$R_n(1.4)$
3	$\left\lvert\frac{e^2}{4!}(-.6)^4\right\rvert \approx .0399$
4	.004788
5	.0004788 ✓

3) If the 6th degree Maclaurin polynomial is used to compute $\cos\frac{\pi}{4}$ what is the Lagrange remainder?

$$|R_6| \le \left| \frac{1}{(6+1)!}\left(\frac{\pi}{4}\right)^{6+1} \right| \approx .0000365762$$

4) If the 5th degree Maclaurin polynomial is used to compute compute $sin\frac{\pi}{6}$ what is the Lagrange remainder?

$$\left|R_5\left(\frac{\pi}{6}\right)\right| \le \left|\frac{1}{6!}\left(\frac{\pi}{6}\right)^6\right| \approx .000028619$$

6) Let f be a function given by $f(x)=\cos\left(3x-\frac{\pi}{10}\right)$ and let $P_4(x)$ be the 4th degree Taylor polynomial about f at $x=0$. Use the Lagrange error bound to show that $\left|f\left(\frac{1}{4}\right)-P_4\left(\frac{1}{4}\right)\right|<\frac{1}{500}$.

$f'(x) = -3\sin(3x-\frac{\pi}{10})$

$f''(x) = -9\cos(3x-\frac{\pi}{10})$

$f'''(x) = 27\sin(3x-\frac{\pi}{10})$

$f^{(4)}(x) = 81\cos(3x-\frac{\pi}{10})$

$f^{(5)}(x) = 243\sin(3x-\frac{\pi}{10})$

So $\max|f^{(5)}(x)| = 243$

$$\left|R_4\left(\frac{1}{4}\right)\right| \le \left|\frac{243}{5!}\left(\frac{1}{4}\right)^5\right|$$

$$R_4\left(\frac{1}{4}\right) \le .001977539$$

✓ $.001977539 < .002$