Tangent Line Problems **Name:** Answers
AP Calculus

1) Find all points on the graph of $y = x^3 - 3x$ where the tangent line is parallel to the line whose equation is given by $y = 9x + 4$. Write the equation of one of those tangent lines.

↑ slope = 9

$y' = 3x^2 - 3$

$9 = 3x^2 - 3$

$12 = 3x^2$

$4 = x^2$

$\boxed{x = \pm 2}$

Points: $(2, 2)$ $(-2, -2)$

$f(2) = (2)^3 - 3(2) = 2$

$f(-2) = (-2)^3 - 3(-2) = -2$

$y - 2 = 9(x - 2)$

or

$y + 2 = 9(x + 2)$

2) Find a and b so that the graph of $y = ax^2 + bx$ is tangent to the line $y = -2$ at $x = 1$.

↑ slope = 0 at $x = 1$ and $(1, -2)$ is on function

$y' = 2ax + b$

at $x = 1$ $y' = 2a(1) + b$

$2a + b = 0$

$-2 = a + b$

$-2 - a = b$

$-2 = a(1)^2 + b(1)$

$-2 = a + b$

Solve system to find $a + b$

$2a + -2 - a = 0$

$\boxed{a = 2}$ $b = -2 - 2 = -4$

$\boxed{b = -4}$

3) Given the equation $x^2 + 2xy - y^2 + x = 2$
write the equation of the tangent line and the equation of the normal line at the point where x = 1 and the curve is above the x-axis

at $x=1$: $(1)^2 + 2(1)y - y^2 + (1) = 2$

$1 + 2y - y^2 + 1 = 2$

$2y - y^2 = 0$

$y(2-y) = 0$

$y = 0, \; y = 2$ Use (1, 2) since above x-axis

tangent & normal lines → find slope

$$2x + 2y + 2x\frac{dy}{dx} - 2y\frac{dy}{dx} + 1 = 0$$

$$\frac{dy}{dx}(2x - 2y) = -2x - 2y - 1$$

$$\frac{dy}{dx} = \frac{-2x - 2y - 1}{2x - 2y}$$

at (1, 2) $\frac{dy}{dx} = \frac{-2(1) - 2(2) - 1}{2(1) - 2(2)} = \frac{-7}{-2} = \frac{7}{2}$

tangent line: $y - 2 = \frac{7}{2}(x - 1)$

normal line: $y - 2 = -\frac{2}{7}(x - 1)$

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