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5 Rational Expressions

5–1 Quotients of Monomials

Objective: To simplify quotients using the law of exponents.

Vocabulary

Multiplication rule for fractions Let p, q, r, and s be real numbers with $q \neq 0$ and $s \neq 0$. Then

$$\frac{p}{q} \cdot \frac{r}{s} = \frac{pr}{qs}.$$

Rule for simplifying fractions Let p, q, and r be real numbers with $q \neq 0$ and $r \neq 0$. Then

$$\frac{pr}{qr} = \frac{p}{q}.$$

Law of exponents Let m and n be positive integers and a and b be real numbers, with $a \neq 0$ and $b \neq 0$ when they are divisors. Then:

1. $a^m \cdot a^n = a^{m+n}$
2. $(ab)^m = a^m b^m$
3. $(a^m)^n = a^{mn}$

4a. If $m > n$, $\frac{a^m}{a^n} = a^{m-n}$

4b. If $n > m$, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$

5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Simplifying a quotient of monomials A quotient of monomials having integral coefficients is simplified when:

1. the integral coefficients have no common factors except 1 and -1;
2. each base appears only once; and
3. there are no powers of powers [such as $(a^2)^3$].

Example 1 Simplify: **a.** $\frac{45}{60}$ **b.** $\frac{12xy^3}{21x^2y}$

Solution **a.** The GCF of 45 and 60 is 15.

$$\frac{45}{60} = \frac{3 \cdot \cancel{15}}{4 \cdot \cancel{15}} = \frac{3}{4}$$

b. The GCF of $12xy^3$ and $21x^2y$ is $3xy$.

$$\frac{12xy^3}{21x^2y} = \frac{4y^2 \cdot \cancel{3xy}}{7x \cdot \cancel{3xy}} = \frac{4y^2}{7x}$$

Use the method shown in Example 1 to simplify. Assume that no denominator equals 0.

1. $\frac{18}{48}$ **2.** $\frac{36}{63}$ **3.** $\frac{100}{24}$ **4.** $\frac{54}{90}$ **5.** $\frac{8t}{2t^2}$

6. $\frac{16x^4}{24x}$ **7.** $\frac{18y}{4y^3}$ **8.** $\frac{3a^2b}{9ab^2}$ **9.** $\frac{50st^2}{25st^3}$ **10.** $\frac{15p^3q^2}{18p^2q}$

Example 2 Simplify: **a.** $\frac{3^6}{3^4}$ **b.** $\frac{7x^2}{x^5}$ **c.** $\left(\frac{y^4}{5}\right)^2$

Solution

a. $\frac{3^6}{3^4} = 3^{6-4} = 3^2 = 9$

b. $\frac{7x^2}{x^5} = \frac{7}{x^{5-2}} = \frac{7}{x^3}$

c. $\left(\frac{y^4}{5}\right)^2 = \frac{(y^4)^2}{5^2} = \frac{y^8}{25}$

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5–1 *Quotients of Monomials* *(continued)*

Use the method shown in Example 2 to simplify. Assume that no denominator equals 0.

11. $\frac{8^7}{8^5}$ **12.** $\frac{4^2}{4^5}$ **13.** $\frac{x^3}{x^4}$ **14.** $\frac{t^9}{t^4}$

15. $\frac{5a^3}{a^5}$ **16.** $\frac{y^6}{8y^3}$ **17.** $\left(\frac{x^2}{9}\right)^2$ **18.** $\left(\frac{4}{z^3}\right)^2$

Example 3 Simplify: **a.** $\frac{24x^5y}{56x^3y^7}$ **b.** $\frac{4r}{t} \cdot \frac{r^3}{t^2}$ **c.** $\frac{3a}{4c}\left(\frac{2c}{a^3}\right)^2$ **d.** $\frac{(uw^2)^3}{(u^2w)^2}$

Solution **a.** $\frac{24x^5y}{56x^3y^7} = \frac{24}{56} \cdot \frac{x^5}{x^3} \cdot \frac{y^1}{y^7}$

$= \frac{3}{7} \cdot \frac{x^2}{1} \cdot \frac{1}{y^6}$

$= \frac{3x^2}{7y^6}$

b. $\frac{4r}{t} \cdot \frac{r^3}{t^2} = \frac{4r \cdot r^3}{t \cdot t^2}$

$= \frac{4r^4}{t^3}$

c. $\frac{3a}{4c}\left(\frac{2c}{a^3}\right)^2 = \frac{3a}{4c} \cdot \frac{4c^2}{a^6}$

$= \frac{3 \cdot 4}{4} \cdot \frac{a^1}{a^6} \cdot \frac{c^2}{c^1}$

$= \frac{3c}{a^5}$

d. $\frac{(uw^2)^3}{(u^2w)^2} = \frac{u^3w^6}{u^4w^2}$

$= \frac{u^3}{u^4} \cdot \frac{w^6}{w^2}$

$= \frac{1}{u} \cdot \frac{w^4}{1}$

$= \frac{w^4}{u}$

Simplify. Assume that no denominator equals 0.

19. $\frac{27x^3y^5}{18x^4y^2}$ **20.** $\frac{10c^4d^2}{-12cd}$ **21.** $\frac{-8s^4t}{16st^4}$ **22.** $\frac{5u^2v^6}{15u^3v^8}$

23. $\frac{6x^2}{y^2} \cdot \frac{x}{y^2}$ **24.** $\frac{m^3}{2n} \cdot \frac{m^4}{n^3}$ **25.** $\frac{2a}{b} \cdot \frac{3b^3}{4a^2}$ **26.** $\frac{5xy}{2} \cdot \frac{8x^2}{15y^3}$

27. $\frac{p^3}{q}\left(\frac{3q}{p}\right)^2$ **28.** $\frac{2r^3}{t^3}\left(\frac{-t}{2r^2}\right)^2$ **29.** $\frac{(x^2y^3)^2}{(x^3y)^2}$ **30.** $\frac{(ab^2c)^2}{(a^3bc^2)^3}$

Mixed Review Exercises

Find the solution set of each inequality.

1. $x^2 + 7x + 6 > 0$ **2.** $3x + 5 \le 2$ **3.** $r^2 > 4$

4. $3 - 2x < -9$ **5.** $|3x - 2| \le 4$ **6.** $-2 < 3a + 1 < 4$

7. $x^2 - x \le 2$ **8.** $|x + 3| > 2$ **9.** $-3a < -9$ or $a - 3 \le -6$

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5–2 Zero and Negative Exponents

Objective: To simplify expressions involving the exponent zero and negative integral exponents.

Vocabulary

Zero exponent Let a be any *nonzero* real number. Then: $a^0 = 1$

Examples: $5^0 = 1$ $\quad \left(\frac{1}{3}\right)^0 = 1$ $\quad (-200)^0 = 1$

Note that 0^0 is undefined.

Negative exponent Let a be any *nonzero* real number and n be a positive integer. Then:

$$a^{-n} = \frac{1}{a^n}$$

Examples: $2^{-1} = \frac{1}{2^1}$ $\quad 4^{-2} = \frac{1}{4^2}$ $\quad (-5)^{-3} = \frac{1}{(-5)^3}$

Example 1 Write in simplest form without negative or zero exponents.

a. 10^{-3} **b.** $-2x^{-2}$ **c.** $(-2x)^{-2}$ **d.** $3^{-1}x^0y^{-4}$

Solution We will assume that the variables are restricted so that no denominator will be zero.

a. $10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$ **b.** $-2x^{-2} = -2 \cdot \frac{1}{x^2} = -\frac{2}{x^2}$

c. $(-2x)^{-2} = \frac{1}{(-2x)^2} = \frac{1}{4x^2}$ **d.** $3^{-1}x^0y^{-4} = \frac{1}{3^1} \cdot 1 \cdot \frac{1}{y^4} = \frac{1}{3y^4}$

Write in simplest form without negative or zero exponents.

1. 10^{-2} **2.** $(-4)^{-1}$ **3.** 17^0 **4.** $5t^{-3}$

5. $-8a^{-2}$ **6.** $(-8a)^{-2}$ **7.** $2^{-1}h^{-5}k^0$ **8.** $3^2u^{-1}v^{-2}$

Example 2 Write in simplest form without negative or zero exponents.

a. $5^{-3} \cdot 5^{-1}$ **b.** $(-4 \cdot 3^{-2})^{-2}$ **c.** $\frac{2a^{-2}b^3}{5^0ab^{-2}}$

Solution The laws of exponents hold for negative and zero exponents as well as for positive exponents.

a. $5^{-3} \cdot 5^{-1} = 5^{-3+(-1)} = 5^{-4} = \frac{1}{5^4} = \frac{1}{625}$

b. $(-4 \cdot 3^{-2})^{-2} = (-4)^{-2}(3^{-2})^{-2}$

$= \frac{1}{(-4)^2} \cdot 3^4$

$= \frac{81}{16}$

c. $\frac{2a^{-2}b^3}{5^0ab^{-2}} = \frac{2 \cdot a^{-2-1} \cdot b^{3-(-2)}}{1}$

$= 2a^{-3}b^5$

$= \frac{2b^5}{a^3}$

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5-2 *Zero and Negative Exponents* (continued)

Write in simplest form without negative or zero exponents.

9. $3^{-2} \cdot 3^{-1}$ **10.** $(-2)^2 \cdot (-2)^{-5}$ **11.** $(5^{-2})^2$ **12.** $(-2^{-1})^{-2}$

13. $(3 \cdot 8^{-2})^{-1}$ **14.** $(4^{-1} \cdot 6^{-1} \cdot 7^0)^{-1}$ **15.** $\frac{.4^2}{4^{-1}}$ **16.** $\frac{x^{-1}}{x^{-2}}$

17. $\frac{z^{-6}}{z^2}$ **18.** $\frac{s^{-3}t^{-4}}{s^{-2}t^0}$ **19.** $\frac{9ab^{-2}}{-3a^{-3}b^{-1}}$ **20.** $\frac{18u^4v^{-5}}{u^{-1}v^7}$

Example 3 Write in simplest form without negative or zero exponents.

a. $\left(\frac{4}{5}\right)^{-3}$ **b.** $\left(\frac{a^2}{b}\right)^{-3}\left(\frac{b^{-1}}{a}\right)^2$

Solution **a.** $\left(\frac{4}{5}\right)^{-3} = \frac{4^{-3}}{5^{-3}}$

$$= 4^{-3} \cdot \frac{1}{5^{-3}}$$
$$= \frac{1}{4^3} \cdot 5^3$$
$$= \frac{5^3}{4^3}$$
$$= \frac{125}{64}$$

b. $\left(\frac{a^2}{b}\right)^{-3}\left(\frac{b^{-1}}{a}\right)^2 = \frac{(a^2)^{-3}}{b^{-3}} \cdot \frac{(b^{-1})^2}{a^2}$

$$= \frac{a^{-6}}{b^{-3}} \cdot \frac{b^{-2}}{a^2}$$
$$= \frac{a^{-6}b^{-2}}{a^2b^{-3}}$$
$$= a^{-8}b^1$$
$$= \frac{b}{a^8}$$

Note: $\frac{1}{a^{-n}} = a^n$ since a^{-n} and a^n are reciprocals.

Write in simplest form without negative or zero exponents.

21. $\left(\frac{5}{2}\right)^{-2}$ **22.** $\left(\frac{2}{3}\right)^{-3}$ **23.** $\left(\frac{2}{x^2y^{-3}}\right)^{-2}$ **24.** $\left(\frac{3pq^{-1}}{p^{-2}q}\right)^{-1}$

25. $4z(2y^2z)^{-2}$ **26.** $\frac{(2a^{-1})^{-3}}{2(a^{-1})^{-2}}$ **27.** $\left(\frac{x^3}{y^{-1}}\right)^{-2}\left(\frac{y^3}{x^{-2}}\right)^2$ **28.** $\left(\frac{c^2}{d^{-6}}\right)^0\left(\frac{c^{-5}}{d^{-1}}\right)^2$

Example 4 Write without using fractions: **a.** $\frac{7}{1000}$ **b.** $\frac{3a}{b^2c^3}$

Solution **a.** $\frac{7}{1000} = \frac{7}{10^3} = 7 \times 10^{-3}$ **b.** $\frac{3a}{b^2c^3} = 3ab^{-2}c^{-3}$

Write without using fractions.

29. $\frac{9}{10{,}000}$ **30.** $\frac{3}{100}$ **31.** $\frac{11}{100{,}000}$

32. $\frac{a^3}{b^2}$ **33.** $\frac{6x^3}{yz^2}$ **34.** $\frac{2hk^3}{j^4}$

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5–3 Scientific Notation and Significant Digits

Objective: To use scientific notation and significant digits.

Vocabulary

Scientific notation A number represented in the form $m \times 10^n$, where $1 \leq m < 10$ and n is an integer.

Examples: $8{,}320{,}000 = 8.32 \times 10^6$ $\quad 0.00079 = 7.9 \times 10^{-4}$

Significant digit A significant digit of a number written in decimal form is any nonzero digit or any zero that has a purpose other than placing the decimal point.

Examples: $0.\underbrace{602}$ $\quad \underbrace{3054}$ $\quad \underbrace{81.0}$ $\quad 0.000\underbrace{90}$ (brackets indicate significant digits)

Symbol $\approx$ (is approximately equal to)

Example 1 Write each number in scientific notation. If the number is an *integer* and ends in zeros, assume that the zeros are not significant.

a. 34.070 **b.** 0.000242 **c.** 5,070,000 **d.** 0.068×10^3

Solution

a. $34.070 = 3.4070 \times 10^1$ decimal point moved 1 place to the *left*

b. $0.000242 = 2.42 \times 10^{-4}$ decimal point moved 4 places to the *right*

c. $5{,}070{,}000 = 5.07 \times 10^6$ decimal point moved 6 places to the *left*

d. $0.068 \times 10^3 = (6.8 \times 10^{-2}) \times 10^3$ decimal point moved 2 places to the *right*

$= 6.8 \times (10^{-2} \times 10^3)$ associative property used

$= 6.8 \times 10^1$ $\quad a^m \cdot a^n = a^{m+n}$

Write each number in scientific notation. If the number is an integer and ends in zeros, assume that the zeros are not significant.

1. 750	**2.** 347,000	**3.** 89.2	**4.** 0.037
5. 2100	**6.** 34	**7.** 0.00086	**8.** 51.080
9. 9,006,000	**10.** 0.07	**11.** 0.00401	**12.** 958.05
13. 0.8490	**14.** 0.0000265	**15.** 70,030	**16.** 2570.20
17. 302×10^2	**18.** 0.51×10^{-2}	**19.** 0.840×10^3	**20.** 6376×10^{-1}

Example 2 Write each number in decimal form: **a.** 5×10^{-2} **b.** 8.46×10^3

Solution **a.** $5 \times 10^{-2} = 0.05$ move decimal point 2 places to the *left*

b. $8.46 \times 10^3 = 8460$ move decimal point 3 places to the *right*

Write each number in decimal form.

21. 3×10^4	**22.** 10^{-5}	**23.** 6.80×10^{-3}	**24.** 2.7×10^6
25. 5.02×10^{-4}	**26.** 7×10^{-4}	**27.** 9.000×10^2	**28.** 1.40×10^5

5–3 Scientific Notation and Significant Digits *(continued)*

Example 3

a. $8.97 \times 10^5 < 1.36 \times 10^8$	The number with the larger exponent is greater.
b. $5.72 \times 10^{-1} > 4.88 \times 10^{-1}$	The exponents are equal, so compare decimals: $5.72 > 4.88$.
c. $6.13 \times 10^6 < (3 \times 10^3)^2$	Since $(3 \times 10^3)^2 = 9 \times 10^6$, and $6.13 < 9$.

Replace the $\underline{?}$ with $>$ or $<$ to make a true statement.

29. $5.3 \times 10^6 \underline{?} 2.56 \times 10^6$

30. $6.17 \times 10^4 \underline{?} 3.27 \times 10^5$

31. $4.3 \times 10^{-2} \underline{?} 1.2 \times 10^{-1}$

32. $(2 \times 10^3)^2 \underline{?} 9.24 \times 10^3$

Example 4 Find a one-significant-digit estimate of x, where $x = \dfrac{9300 \times 78.4}{0.0018 \times 226}$.

Solution

$$x = \frac{9.3 \times 10^3 \times 7.84 \times 10^1}{1.8 \times 10^{-3} \times 2.26 \times 10^2}$$ Write each number in scientific notation.

$$= \frac{9 \times 10^3 \times 8 \times 10^1}{2 \times 10^{-3} \times 2 \times 10^2}$$ Round each decimal to a whole number.

$$= \frac{9 \times 8}{2 \times 2} \times 10^{3+1-(-3)-2}$$ Compute, and give the result to one significant digit.

$$= 18 \times 10^5$$

$$\approx 20 \times 10^5 = 2 \times 10^6$$ Note that $20 = 2 \times 10^1$.

So, $x \approx 2 \times 10^6$, or 2,000,000.

Find a one-significant-digit estimate of the following.

33. $\dfrac{26.1 \times 0.73}{0.00012 \times 3800}$ **34.** $\dfrac{0.642 \times 3890}{12.6 \times 0.00024}$ **35.** $\dfrac{0.0373 \times 0.561}{0.0017 \times 41.5}$ **36.** $\dfrac{88.3 \times 0.057}{46{,}000 \times 0.0019}$

Mixed Review Exercises

Simplify.

1. $\dfrac{24a^4b}{30ab^2}$ **2.** $\left(\dfrac{5r}{3s}\right)^2$ **3.** $\dfrac{4x}{y} \cdot \dfrac{x^2}{3y} \cdot \dfrac{9x}{y}$

4. $\dfrac{(a^2b)^2}{(ab^2)^3}$ **5.** $\dfrac{(-p)^4}{-p^4}$ **6.** $\dfrac{28h^4}{21h^4}$

Express in simplest form without negative or zero exponents.

7. $(x^{-2}y^{-3})^{-1}$ **8.** $\left(\dfrac{u^{-1}}{v}\right)^{-2}$ **9.** $\dfrac{a^{-3}b^{-2}}{a^{-4}b^5}$

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5-4 Rational Algebraic Expressions

Objective: To simplify algebraic expressions.

Vocabulary

Rational algebraic expression (or **rational expression)** An algebraic expression that can be expressed as a quotient of polynomials.

Examples: $\frac{1}{x}$ $\quad \frac{5y + 15}{5}$ $\quad \frac{z^2 - 4}{2z(z + 2)}$

Simplified rational expression A quotient of polynomials whose greatest common factor is 1.

Examples: $\frac{2a}{3b}$ $\quad \frac{y + 3}{1}$ $\quad \frac{z - 2}{2z}$

Zero of a function A number r is a zero of a function f if $f(r) = 0$.

Example: 2 is a *zero* of $f(x) = x^2 + x - 6$ because $f(2) = 2^2 + 2 - 6 = 0$.

Example 1 Simplify: **a.** $\frac{8x^2 + 16x}{4x}$ **b.** $\frac{x^2 - 16}{x^2 + 7x + 12}$

Solution **a.** $\frac{8x^2 + 16x}{4x} = \frac{{}^{2}\cancel{8x}(x + 2)}{\cancel{4x}^{1}}$ Factor and then simplify.

$$= \frac{2(x + 2)}{1}$$

$$= 2(x + 2)$$

b. $\frac{x^2 - 16}{x^2 + 7x + 12} = \frac{\cancel{(x + 4)}(x - 4)}{(x + 3)\cancel{(x + 4)}}$ Factor and then simplify.

$$= \frac{x - 4}{x + 3}$$

Example 2 Simplify $(2 - x - 3x^2)(9x^2 - 4)^{-1}$.

Solution $(2 - x - 3x^2)(9x^2 - 4)^{-1} = \frac{2 - x - 3x^2}{9x^2 - 4}$ Definition: $a^{-n} = \frac{1}{a^n}$

$$= \frac{-(3x^2 + x - 2)}{(3x + 2)(3x - 2)} \quad \text{Factor.}$$

$$= \frac{-\cancel{(3x - 2)}(x + 1)}{(3x + 2)\cancel{(3x - 2)}} \quad 2 - 3x = -(3x - 2)$$

$$= \frac{-(x + 1)}{3x + 2}$$

$$= -\frac{x + 1}{3x + 2}$$

5–4 *Rational Algebraic Expressions* *(continued)*

Simplify.

1. $\dfrac{5x + 15}{10}$

2. $\dfrac{3y - 9}{6}$

3. $\dfrac{9s^3 + 27s^2}{6s}$

4. $\dfrac{r^2 + 9r + 18}{(r + 6)^2}$

5. $\dfrac{(p - 4)^2}{2p^2 - 9p + 4}$

6. $\dfrac{k^2 - 5k + 4}{k^2 + 2k - 3}$

7. $\dfrac{m^2 + 5m - 14}{m^2 - 4}$

8. $\dfrac{a^2 + 4a - 5}{a^3 - a}$

9. $\dfrac{t^2 - 9}{t^3 - 6t^2 + 9t}$

10. $(x - y)(y - x)^{-1}$

11. $(a^2 + ab)(a^2 - b^2)^{-1}$

12. $\dfrac{2x^2 + 5x - 3}{1 - 4x^2}$

13. $\dfrac{16 - 9b^2}{3b^2 + 11b - 20}$

14. $\dfrac{c^2 - d^2}{c^2 + 2cd + d^2}$

15. $\dfrac{y^4 - 16}{(y + 2)^2(y^2 + 4)}$

Example 3 Let $f(x) = \dfrac{2x^2 + x - 1}{x^3 - 9x}$.

a. Find the domain of f. **b.** Find the zeros of f, if there are any.

Solution $f(x) = \dfrac{2x^2 + x - 1}{x^3 - 9x} = \dfrac{(2x - 1)(x + 1)}{x(x - 3)(x + 3)}$

a. The function f will be undefined at any value for which the denominator equals 0. Find those values.

$x(x - 3)(x + 3) = 0$

$x = 0$ or $x - 3 = 0$ or $x + 3 = 0$ Use the zero-product property.

$x = 0$ $x = 3$ $x = -3$

$\therefore$ the domain of f consists of all real numbers except 0, 3, and -3.

b. $f(x) = 0$ if and only if $(2x - 1)(x + 1) = 0$. That is, if the numerator of a fraction equals 0, then the fraction equals 0.

$(2x - 1)(x + 1) = 0$

$2x - 1 = 0$ or $x + 1 = 0$ Use the zero-product property.

$x = \dfrac{1}{2}$ $x = -1$

$\therefore$ the zeros of f are $\dfrac{1}{2}$ and -1.

Find (a) the domain and (b) the zeros, if any, of each function.

16. $f(x) = \dfrac{x^2 - 4}{x^2 - 4x}$

17. $g(x) = \dfrac{x^2 + 5x - 24}{x^2 - 36}$

18. $F(y) = \dfrac{y^2 - y - 12}{2y^2 - 5y + 2}$

19. $h(t) = \dfrac{4t^2 - 11t + 6}{(3t + 1)^2}$

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5–5 *Products and Quotients of Rational Expressions*

Objective: To multiply and divide rational expressions.

Vocabulary

Division rule for fractions Let p, q, r, and s be real numbers with $q \neq 0$, $r \neq 0$, and $s \neq 0$. Then $\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \cdot \frac{s}{r} = \frac{ps}{qr}$. (To divide by a fraction, you multiply by its *reciprocal.*)

Example: $\frac{3}{8} \div \frac{2}{5} = \frac{3}{8} \cdot \frac{5}{2} = \frac{3 \cdot 5}{8 \cdot 2} = \frac{15}{16}$

Example 1 Simplify: **a.** $\frac{14}{9} \cdot \frac{15}{28}$ **b.** $\frac{x^2 + x - 2}{x^2 - 4} \cdot \frac{x^2 - 5x - 6}{x^2 - 2x + 1}$

Solution Factor where possible, then multiply. Divide out common factors.

a. $\frac{14}{9} \cdot \frac{15}{28} = \frac{14 \cdot 15}{9 \cdot 28} = \frac{5}{6}$

b. $\frac{x^2 + x - 2}{x^2 - 4} \cdot \frac{x^2 - 5x - 6}{x^2 - 2x + 1} = \frac{(x + 2)(x - 1)}{(x + 2)(x - 2)} \cdot \frac{(x - 6)(x + 1)}{(x - 1)^2}$

$$= \frac{\cancel{(x + 2)}\cancel{(x - 1)}(x - 6)(x + 1)}{\cancel{(x + 2)}(x - 2)\cancel{(x - 1)}(x - 1)}$$

$$= \frac{(x - 6)(x + 1)}{(x - 2)(x - 1)}$$

Simplify.

1. $\frac{18}{5} \cdot \frac{10}{27}$
2. $\frac{4}{3} \cdot 12$
3. $\frac{x}{y^2} \cdot \frac{y}{x}$
4. $\frac{2p}{q} \cdot \frac{p}{8q^2}$
5. $\frac{5a}{2b^3} \cdot \frac{4b^2}{a}$
6. $\frac{3}{z + 2} \cdot \frac{z^2 - 4}{6}$
7. $\frac{2y^2 - 50}{2y - 10} \cdot \frac{4y - 2}{6y + 30}$
8. $\frac{t^2 + 3t}{t^2 + 2t - 3} \cdot \frac{t + 1}{t}$
9. $\frac{k^2 - 2k - 8}{2k^2 + 5k + 3} \cdot \frac{2k + 3}{k - 4}$

Example 2 Simplify: **a.** $\frac{-18}{25} \div \frac{6}{5}$ **b.** $\frac{6a^2b}{5c^3} \div \frac{ab^2}{10c}$

Solution **a.** Multiply by the reciprocal.

$$\frac{-18}{25} \div \frac{6}{5} = \frac{-18}{25} \cdot \frac{5}{6}$$

$$= \frac{-18 \cdot 5}{25 \cdot 6} = -\frac{3}{5}$$

b. You can divide out factors common to the numerator and denominator *before* you write the product as a single fraction.

$$\frac{6a^2b}{5c^3} \div \frac{ab^2}{10c} = \frac{\overset{a}{\cancel{6a^2b}}}{\underset{c^2}{\cancel{5c^3}}} \cdot \frac{\overset{2}{\cancel{10c}}}{\underset{b}{\cancel{ab^2}}} = \frac{12a}{bc^2}$$

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5–5 Products and Quotients of Rational Expressions *(continued)*

Example 3 Simplify $\frac{2x - 14}{x^2 - 2x - 35} \div \frac{6x^3}{x^2 - 25}$.

Solution

$$\frac{2x - 14}{x^2 - 2x - 35} \div \frac{6x^3}{x^2 - 25} = \frac{2x - 14}{x^2 - 2x - 35} \cdot \frac{x^2 - 25}{6x^3}$$
$$= \frac{2(x - 7)}{(x - 7)(x + 5)} \cdot \frac{(x + 5)(x - 5)}{6x^3}$$
$$= \frac{x - 5}{3x^3}$$

Simplify.

10. $\frac{20}{9} \div \frac{5}{12}$
11. $\frac{-3}{8} \div \frac{4}{9}$
12. $\frac{45a^2}{8} \div \frac{35a}{4}$
13. $\frac{21t^3}{5} \div \frac{7t^2}{-5}$
14. $\frac{6bc^2}{15a^2} \div \frac{2bc}{5a}$
15. $\frac{u + 1}{16} \div \frac{u + 1}{4}$
16. $\frac{r - 3}{2r^2} \div \frac{r^2 - 4r + 3}{2r}$
17. $\frac{x^2 - 9}{x^2 + 6x + 9} \div \frac{x - 3}{x + 3}$
18. $\frac{y^2 - 4y}{y^2 + 2y} \div \frac{y^2 - 9y + 20}{2y^2 - 9y - 5}$

Example 4 Simplify:

$$\frac{3m^2n}{2} \div \frac{n}{4m} \div \frac{15m^4}{2}$$

Solution

$$\frac{3m^2n}{2} \div \frac{n}{4m} \div \frac{15m^4}{2} = \frac{3m^2n}{2} \cdot \frac{4m}{n} \cdot \frac{2}{15m^4}$$
$$= \frac{\cancel{3m^2n} \cdot 4\cancel{m} \cdot \cancel{2}}{\cancel{2} \cdot \cancel{n} \cdot \underset{5m}{\cancel{15m^4}}} = \frac{4}{5m}$$

Simplify.

19. $5xy \div \frac{10x^2}{y^2} \div \frac{y^3}{x}$
20. $\frac{12a^2}{b} \div \frac{2}{3ab} \div \frac{54a^3}{b}$
21. $\frac{24c^3}{d} \div \frac{40c}{de^2} \cdot \frac{5}{9e}$
22. $\frac{3p + 6}{9p} \cdot \frac{12p}{p^2 - 4} \div \frac{18p^3}{2p - 4}$

Mixed Review Exercises

Express in scientific notation. Assume that the zeros at the end of any integer are not significant.

1. 0.00276
2. 0.5
3. 7634
4. 72,000,000
5. 38.20

Simplify.

6. $\frac{20r^5s^2}{32r^3s^5}$
7. $\frac{3k + 9}{2k + 6}$
8. $\frac{(6x^2)^2}{(4x^3)^2}$
9. $\frac{4z^2 - 4}{8z^2 + 16z + 8}$

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5–6 Sums and Differences of Rational Expressions

Objective: To add and subtract rational expressions.

Vocabulary

Least common multiple (LCM) of two or more polynomials The common multiple having least degree and least positive factors.

Examples: The LCM of $2x$, $6x^2$, and $3x$ is $6x^2$.
The LCM of $x(x + 1)$ and $(x + 1)(x - 1)$ is $x(x + 1)(x - 1)$.
The LCM of $x + 2$ and $x + 4$ is $(x + 2)(x + 4)$.

Example 1 Simplify: **a.** $\frac{5}{6} + \frac{13}{6} - \frac{7}{6}$ **b.** $\frac{2x - 5}{x - 3} - \frac{x - 1}{x - 3}$

Solution With fractions having the same denominator, add or subtract the numerators and write the result over the common denominator.

a. $\frac{5}{6} + \frac{13}{6} - \frac{7}{6} = \frac{5 + 13 - 7}{6}$

$= \frac{11}{6}$

b. $\frac{2x - 5}{x - 3} - \frac{x - 1}{x - 3} = \frac{2x - 5 - (x - 1)}{x - 3}$

$= \frac{2x - 5 - x + 1}{x - 3}$

$= \frac{x - 4}{x - 3}$

Example 2 Simplify: **a.** $\frac{5}{6} + \frac{1}{8} - \frac{1}{3}$ **b.** $\frac{1}{3a} - \frac{1}{4a} + \frac{2}{a^2}$

Solution With fractions having different denominators, rewrite the fractions using their *least common denominator* (LCD), which is the LCM of the denominators.

a. $6 = 2 \cdot 3$ and $8 = 2^3$. So the LCD $= 2^3 \cdot 3 = 24$.

$$\frac{5}{6} + \frac{1}{8} - \frac{1}{3} = \frac{5 \cdot 4}{6 \cdot 4} + \frac{1 \cdot 3}{8 \cdot 3} - \frac{1 \cdot 8}{3 \cdot 8}$$

$$= \frac{20}{24} + \frac{3}{24} - \frac{8}{24}$$

$$= \frac{20 + 3 - 8}{24} = \frac{15}{24} = \frac{5}{8}$$

b. The LCD for $3a$, $4a$, and a^2 is $12a^2$.

$$\frac{1}{3a} - \frac{1}{4a} + \frac{2}{a^2} = \frac{1 \cdot 4a}{3a \cdot 4a} - \frac{1 \cdot 3a}{4a \cdot 3a} + \frac{2 \cdot 12}{a^2 \cdot 12}$$

$$= \frac{4a}{12a^2} - \frac{3a}{12a^2} + \frac{24}{12a^2}$$

$$= \frac{4a - 3a + 24}{12a^2}$$

$$= \frac{a + 24}{12a^2}$$

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5–6 *Sums and Differences of Rational Expressions* *(continued)*

Simplify.

1. $\frac{7}{8} - \frac{3}{8} + \frac{1}{8}$

2. $\frac{1}{2} + \frac{1}{3} + \frac{3}{5}$

3. $\frac{5}{6} + \frac{2}{5} - \frac{8}{15}$

4. $\frac{3}{4} + \frac{5}{18} - \frac{7}{9}$

5. $\frac{5}{2x} - \frac{3}{2x}$

6. $\frac{3}{5x^3y} - \frac{2}{xy^2}$

7. $\frac{x}{x+1} + \frac{1}{x+1}$

8. $\frac{8t+4}{t-2} - \frac{6t-1}{t-2}$

9. $\frac{2}{3z} + \frac{7}{12z}$

10. $\frac{3}{rs} - \frac{4}{rs^2}$

11. $\frac{3m-2}{6} - \frac{m-3}{9}$

12. $\frac{2n+1}{3n} + \frac{2-3n}{4n}$

Example 3 Simplify $\frac{3}{x^2+x-2} - \frac{5}{x^2-x-6}$.

Solution $x^2 + x - 2 = (x+2)(x-1)$
$x^2 - x - 6 = (x-3)(x+2)$ $\Big\{$ Factor the denominators to find the LCD.

So the LCD is $(x+2)(x-1)(x-3)$.

$$\begin{aligned}
\frac{3}{x^2+x-2} - \frac{5}{x^2-x-6} &= \frac{3}{(x+2)(x-1)} - \frac{5}{(x-3)(x+2)} \\
&= \frac{3(x-3)}{(x+2)(x-1)(x-3)} - \frac{5(x-1)}{(x-3)(x+2)(x-1)} \\
&= \frac{3(x-3)-5(x-1)}{(x+2)(x-1)(x-3)} \\
&= \frac{3x-9-5x+5}{(x+2)(x-1)(x-3)} \\
&= \frac{-2x-4}{(x+2)(x-1)(x-3)} \\
&= \frac{-2\cancel{(x+2)}}{\cancel{(x+2)}(x-1)(x-3)} \\
&= \frac{-2}{(x-1)(x-3)}, \text{ or } -\frac{2}{(x-1)(x-3)}
\end{aligned}$$

Simplify.

13. $\frac{2}{k-3} + \frac{4}{k+3}$

14. $\frac{c+1}{c} - \frac{c}{c+1}$

15. $\frac{y}{y-1} + \frac{4}{y+1}$

16. $\frac{5m+1}{2m^2-2m} - \frac{3}{2m-2}$

17. $\frac{1}{x^2-3x} - \frac{1}{x^2-9}$

18. $\frac{1}{z^2-4} + \frac{1}{(z-2)^2}$

19. $\frac{3}{p^2-3p+2} - \frac{2}{p^2-1}$

20. $\frac{1}{x^2+x-2} + \frac{1}{x^2-5x+4}$

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5–7 Complex Fractions

Objective: To simplify complex fractions.

Vocabulary

Complex fraction A fraction that has a fraction or powers with negative exponents in its numerator or denominator (or both).

Symbol $\frac{a}{b}$ means $a \div b$

Example 1 Simplify $\dfrac{\frac{7}{15} + \frac{1}{5}}{2 + \frac{2}{9}}$.

Solution *Method 1:* Simplify the numerator and denominator separately.

$$\frac{\frac{7}{15} + \frac{1}{5}}{2 + \frac{2}{9}} = \frac{\frac{7}{15} + \frac{3}{15}}{\frac{18}{9} + \frac{2}{9}} = \frac{\frac{10}{15}}{\frac{20}{9}} = \frac{10}{15} \div \frac{20}{9} = \frac{10}{15} \cdot \frac{9}{20} = \frac{3}{10}$$

Method 2: Multiply the numerator and the denominator by the LCD.

The LCD for $\frac{7}{15}$, $\frac{1}{5}$, and $\frac{2}{9}$ is 45.

$$\frac{\frac{7}{15} + \frac{1}{5}}{2 + \frac{2}{9}} = \frac{\left(\frac{7}{15} + \frac{1}{5}\right)45}{\left(2 + \frac{2}{9}\right)45} = \frac{\left(\frac{7}{15}\right)45 + \left(\frac{1}{5}\right)45}{(2)45 + \left(\frac{2}{9}\right)45} = \frac{21 + 9}{90 + 10} = \frac{30}{100} = \frac{3}{10}$$

Simplify.

1. $\dfrac{\frac{3}{2} - 1}{\frac{5}{6} - \frac{2}{3}}$

2. $\dfrac{1 + \frac{2}{5}}{\frac{5}{2} - \frac{2}{5}}$

3. $\dfrac{\frac{1}{2} + \frac{1}{5}}{\frac{1}{6} + \frac{1}{8}}$

4. $\dfrac{\frac{4}{9} + \frac{1}{4}}{2 - \frac{1}{3}}$

Example 2 Simplify $\dfrac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y}}$.

Solution

Method 1: $$\frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y}} = \frac{\frac{y - x}{xy}}{\frac{y + x}{xy}} = \frac{y - x}{xy} \div \frac{y + x}{xy} = \frac{y - x}{xy} \cdot \frac{xy}{y + x} = \frac{y - x}{y + x}$$

Method 2: $$\frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y}} = \frac{\left(\frac{1}{x} - \frac{1}{y}\right)xy}{\left(\frac{1}{x} + \frac{1}{y}\right)xy} = \frac{\left(\frac{1}{x}\right)xy - \left(\frac{1}{y}\right)xy}{\left(\frac{1}{x}\right)xy + \left(\frac{1}{y}\right)xy} = \frac{y - x}{y + x}$$

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5-7 Complex Fractions *(continued)*

Example 3 Simplify $\frac{1 - m^{-1}}{1 - m^{-2}}$.

Solution Use the definition $a^{-n} = \frac{1}{a^n}$.

$$\frac{1 - m^{-1}}{1 - m^{-2}} = \frac{1 - \frac{1}{m}}{1 - \frac{1}{m^2}} = \frac{\frac{m - 1}{m}}{\frac{m^2 - 1}{m^2}}$$

$$= \frac{m - 1}{m} \div \frac{m^2 - 1}{m^2}$$

$$= \frac{(m - 1)}{m} \cdot \frac{m^2}{(m + 1)(m - 1)}$$

$$= \frac{m}{m + 1}$$

You can also use Method 2 to simplify the complex fraction above.

Simplify.

5. $\frac{\frac{1}{x}}{\frac{1}{y}}$

6. $\frac{\frac{m}{2} + \frac{2}{m}}{\frac{m + 2}{2m}}$

7. $\frac{a - 1}{1 - \frac{1}{a}}$

8. $\frac{c - \frac{1}{c}}{1 + \frac{1}{c}}$

9. $\frac{s + \frac{s}{t}}{1 + \frac{1}{t}}$

10. $\frac{1 + \frac{1}{z + 1}}{1 + \frac{3}{z - 1}}$

11. $\frac{r^{-2} + 1}{r - 1}$

12. $\frac{y^{-1} + x^{-1}}{y^{-2} - x^{-2}}$

13. $\frac{9 - k^{-2}}{3k^{-1} - k^{-2}}$

Mixed Review Exercises

Simplify.

1. $\frac{2}{3t} - \frac{t}{6t^2}$

2. $\frac{6}{a^2 - 4a} \cdot \frac{a^2 - 16}{10a}$

3. $\frac{49c^2}{25d} \div \frac{42c^4}{15d^2}$

4. $\frac{r}{r - 3} - \frac{1}{r + 3}$

5. $-\frac{10xy^4z}{15x^2yz}$

6. $\frac{5p - 10}{p^2 - 4p + 4}$

Find the unique solution of each system. Check your answer by using substitution.

7. $x + 2y = 6$
   $x - 2y = 4$

8. $3x + 7y = -4$
   $2x + 5y = -3$

9. $4x + 5y = -8$
   $3x - 4y = -6$

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5-8 Fractional Coefficients

Objective: To solve equations and inequalities having fractional coefficients.

Example 1 Solve $\frac{x^2}{5} = \frac{3x}{10} + \frac{1}{2}$.

Solution

$\frac{x^2}{5} = \frac{3x}{10} + \frac{1}{2}$ The LCD is 10.

$10\left(\frac{x^2}{5}\right) = 10\left(\frac{3x}{10} + \frac{1}{2}\right)$ Multiply *both* sides of the equation by the LCD to clear the denominators.

$2x^2 = 3x + 5$

$2x^2 - 3x - 5 = 0$ Make one side 0.

$(2x - 5)(x + 1) = 0$ Factor the polynomial.

$2x - 5 = 0$ or $x + 1 = 0$ Use the zero-product property.

$x = \frac{5}{2}$ or $x = -1$

$\therefore$ the solution set is $\left\{\frac{5}{2}, -1\right\}$.

Example 2 Solve $\frac{x}{6} - \frac{x-3}{4} \le \frac{x+1}{8}$.

Solution

$\frac{x}{6} - \frac{x-3}{4} \le \frac{x+1}{8}$ The LCD is 24.

$24\left(\frac{x}{6} - \frac{x-3}{4}\right) \le 24\left(\frac{x+1}{8}\right)$ Multiply *both* sides of the inequality by the LCD.

$4x - 6(x - 3) \le 3(x + 1)$ Use the distributive property and combine similar terms.

$4x - 6x + 18 \le 3x + 3$

$-2x + 18 \le 3x + 3$

$-5x \le -15$ Divide both sides by -5 and reverse the inequality sign.

$x \ge 3$

$\therefore$ the solution set is $\{x: x \ge 3\}$.

Solve each open sentence.

1. $\frac{2x}{3} - \frac{x}{6} = -1$
2. $\frac{4t}{3} + \frac{3t}{10} = \frac{7}{5}$
3. $\frac{h}{4} \le \frac{3}{2} - \frac{h}{5}$
4. $\frac{d}{6} - \frac{2}{3} \ge \frac{d}{4}$
5. $\frac{3u}{14} - \frac{5-u}{21} = \frac{2}{7}$
6. $\frac{3r-4}{5} - \frac{2r+1}{4} = -\frac{1}{2}$
7. $\frac{y+3}{2} + \frac{3}{5} \ge \frac{y+1}{10}$
8. $\frac{c+8}{12} - \frac{3c-5}{15} < \frac{c}{20}$
9. $\frac{p^2}{6} - \frac{3p}{4} = \frac{15}{4}$
10. $\frac{2a}{5} - \frac{a^2}{3} = \frac{1}{15}$
11. $\frac{x^2}{4} + \frac{x-1}{3} = 0$
12. $\frac{u(u+6)}{5} = \frac{u-1}{2}$

5-8 *Fractional Coefficients* *(continued)*

Example 3 How much pure alcohol must be added to 15 oz of a 60% solution of rubbing alcohol to change it to a 70% solution?

Solution Percents can be thought of as fractions since percent means *divided by 100.*

Step 1 The problem asks for the number of ounces of alcohol added to the 60% solution.

Step 2 Let x = number of ounces of alcohol to be added. Then $15 + x$ = number of oz in the 70% solution. } Show the known facts in a table.

	oz of solution ×	% alcohol =	oz of alcohol
60% solution	15	60%	0.60(15)
Alcohol added	x	100%	$1x$
70% solution	$15 + x$	70%	$0.70(15 + x)$

Step 3 alcohol in 60% solution + alcohol added = alcohol in 70% solution

$$0.60(15) + 1x = 0.70(15 + x)$$

or $$60(15) + 100x = 70(15 + x)$$ { To clear decimals, multiply both sides by 100.

Step 4
$$900 + 100x = 1050 + 70x$$
$$30x = 150$$
$$x = 5$$

Step 5 *Check:*
$$0.60(15) + 1(5) \stackrel{?}{=} 0.70(15 + 5)$$
$$9 + 5 \stackrel{?}{=} 0.70(20)$$
$$14 = 14 \ \checkmark$$

∴ 5 oz of alcohol must be added.

Solve.

13. How many liters of pure acid must be added to 5 L of a solution that is 20% acid to make a solution that is 60% acid?
14. A nurse has 6 L of a 3% boric acid solution. How much of a 10% boric acid solution must he add to produce a 4% solution?
15. How many gallons of cream that is 23% butterfat and milk that is 3% butterfat must a dairy farmer mix to make 30 gallons of milk that is 4% butterfat?

Complete the chart and solve.

16. John Gordon invested \$1000, part at 5% and the rest at 6.5%. The income from the 5% investment exceeded the income from the 6.5% investment by \$1.70. How much did he invest at each rate?

	Amount invested ×	Rate =	Interest earned
Investment at 5%	x	?	?
Investment at 6.5%	?	?	?

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5–9 Fractional Equations

Objective: To solve and use fractional equations.

Vocabulary

Fractional equation An equation in which a variable occurs in a denominator.

Extraneous root A root of a transformed equation that is not a root of the original equation.

CAUTION Since multiplying an equation by a polynomial may produce extraneous roots, you must *always check* each root of the new equation in the *original* equation.

Example Solve $\frac{12}{t^2 - 4} - \frac{3}{t - 2} = -1.$

Solution

$$\frac{12}{(t + 2)(t - 2)} - \frac{3}{t - 2} = -1$$ The LCD is $(t + 2)(t - 2)$.

$$(t + 2)(t - 2)\left[\frac{12}{(t - 2)(t + 2)} - \frac{3}{t - 2}\right] = (t + 2)(t - 2)(-1)$$ Multiply both sides by the LCD.

$$\begin{aligned} 12 - 3(t + 2) &= -1(t^2 - 4) \\ 12 - 3t - 6 &= -t^2 + 4 \\ t^2 - 3t + 2 &= 0 \\ (t - 2)(t - 1) &= 0 \longrightarrow t - 2 = 0 \text{ or } t - 1 = 0 \\ & \qquad\qquad\quad\; t = 2 \text{ or } \quad t = 1 \end{aligned}$$

Check the possible solutions in the *original* equation.

When $t = 2$: $\frac{12}{2^2 - 4} - \frac{3}{2 - 2} \stackrel{?}{=} -1$

$\frac{12}{0} - \frac{3}{0} \stackrel{?}{=} -1$

not defined

2 is an extraneous root.

When $t = 1$: $\frac{12}{1^2 - 4} - \frac{3}{1 - 2} \stackrel{?}{=} -1$

$\frac{12}{-3} - \frac{3}{-1} \stackrel{?}{=} -1$

$-4 + 3 = -1$ √

1 is a root of the original equation.

∴ the solution set is $\{1\}$.

Solve and check. If an equation has no solution, say so.

1. $\frac{3}{y} - \frac{1}{2y} = \frac{5}{4}$
2. $\frac{4}{3z} + \frac{2}{z} = \frac{5}{6}$
3. $\frac{6}{x + 1} = \frac{3}{x - 2}$
4. $\frac{12}{a} = \frac{4}{a - 4}$
5. $\frac{3}{r - 3} + 9 = \frac{r}{r - 3}$
6. $\frac{12}{n} = \frac{12}{n + 1} + 1$
7. $\frac{6p}{2p - 1} - 3 = \frac{3}{p}$
8. $\frac{7}{k - 3} - \frac{3}{k - 4} = \frac{1}{2}$
9. $\frac{9}{m + 5} - \frac{1}{m - 5} = \frac{3m}{m^2 - 25}$
10. $\frac{60}{d^2 - 36} + 1 = \frac{5}{d - 6}$
11. $\frac{2}{b^2 - 2b} - \frac{1}{b} = \frac{1}{3}$
12. $\frac{5}{x - 2} + \frac{x^2 - 4}{x^2 + 3x - 10} = \frac{x}{x + 5}$

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5–9 *Fractional Equations* *(continued)*

Vocabulary

Work rate The fractional part of a job done in a given unit of time.
Example: Lenny can paint a room in 3 h. His work rate is $\frac{1}{3}$ job per hour.

Special rate formulas work rate × time = work done rate × time = distance

Complete each table and solve.

13. Stan can load his truck in 24 min. If Chris helps him, it takes 15 min to load the truck. How long does it take Chris alone?

Let x = the time it takes Chris alone.

	Work rate × Time = Work done		
Stan	?	15	?
Chris	$\frac{1}{x}$	15	?

Stan's part of job + Chris' part of job = Whole job
? + ? = 1

14. Bonnie can complete her paper route in 45 min. When her sister Jean helps her it takes them 18 min to complete the route. How long would it take Jean alone?

Let x = the time it takes Jean alone.

	Work rate × Time = Work done		
Bonnie	?	18	?
Jean	$\frac{1}{x}$	18	?

15. An express train travels 150 km in the same time that a freight train travels 100 km. The average speed of the freight train is 20 km/h less than that of the express train. Find the speed of each train.

Use the fact that time = $\frac{\text{distance}}{\text{rate}}$.

	Distance	Rate	Time
Express	?	r	?
Freight	?	$r - 20$	?

time for express train = time for freight train

16. Helen can ride 15 km on her bicycle in the same time it takes her to walk 6 km. If her rate riding is 6 km/h faster than her rate walking, how fast does she walk?

	Distance	Rate	Time
Riding	?	?	?
Walking	?	r	?

Mixed Review Exercises

Simplify.

1. $\frac{x^2 - 4}{2 - x}$ **2.** $\frac{72m^2n^3}{27mn^4}$ **3.** $\frac{1 + a^{-1}}{a^{-2} - 1}$ **4.** $\frac{k^2 - k - 6}{k^2 - 2k - 8}$