Section I—Part A

Number of Questions	Time	Use of Calculator
28	55 Minutes	No

Directions:

Use the answer sheet provided in the previous page. All questions are given equal weight. There is no penalty for unanswered questions. However, 1/4 of the number of the incorrect answers will be subtracted from the number of correct answers. Unless otherwise indicated, the domain of a function f is the set of all real numbers. The use of a calculator is *not* permitted in this part of the exam.

1. $\int_0^8 x^{2/3}dx$

(A) $\frac{1}{3}$ (B) $\frac{96}{5}$ (C) $\frac{4}{3}$
(D) $-\frac{1}{3}$ (E) $-\frac{96}{5}$

2. The $\lim_{x\to-\infty} \frac{x^2+4x-5}{x^3-1}$ is

(A) 0 (B) $\frac{1}{3}$ (C) 5
(D) $-\infty$ (E) ∞

3. What is the $\lim_{x\to-2} f(x)$, if

$$f(x) = \begin{cases} |x-1| & \text{if } x > -2 \\ 2x+7 & \text{if } x \le -2 \end{cases}?$$

(A) -3 (B) 1 (C) 3
(D) 11 (E) nonexistent

4. The graph of f' is shown in Figure 2T-1.

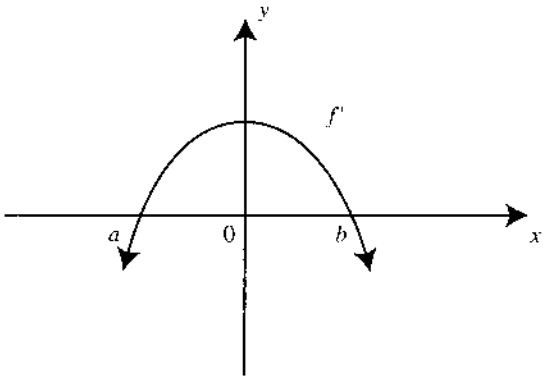

Figure 2T-1

Which of the graphs in Figure 2T-2 on page 352 is a possible graph of f?

5. $\int_{\pi/2}^{x} 2\cos t\,dt =$

(A) $2\cos x$ (B) $-2\cos x$ (C) $2\sin x$
(D) $-2\sin x + 2$ (E) $2\sin x - 2$

6. Given the equation $y = 3e^{-2x}$, what is an equation of the normal line to the graph at $x = \ln 2$?

(A) $y = \frac{2}{3}(x - \ln 2) + \frac{3}{4}$
(B) $y = \frac{2}{3}(x + \ln 2) - \frac{3}{4}$
(C) $y = -\frac{3}{2}(x - \ln 2) + \frac{3}{4}$
(D) $y = -\frac{3}{2}(x - \ln 2) - \frac{3}{4}$
(E) $y = 24(x - \ln 2) + 12$

7. What is the $\lim_{h\to0} \frac{\csc(\pi/4+h) - \csc(\pi/4)}{h}$?

(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) 0
(D) $-\frac{\sqrt{2}}{2}$ (E) undefined

8. If $f(x)$ is an antiderivative of $x^2\sqrt{x^3+1}$ and $f(2) = 0$, then $f(0) =$

(A) -6 (B) 6 (C) $\frac{2}{9}$
(D) $\frac{-52}{9}$ (E) $\frac{56}{9}$

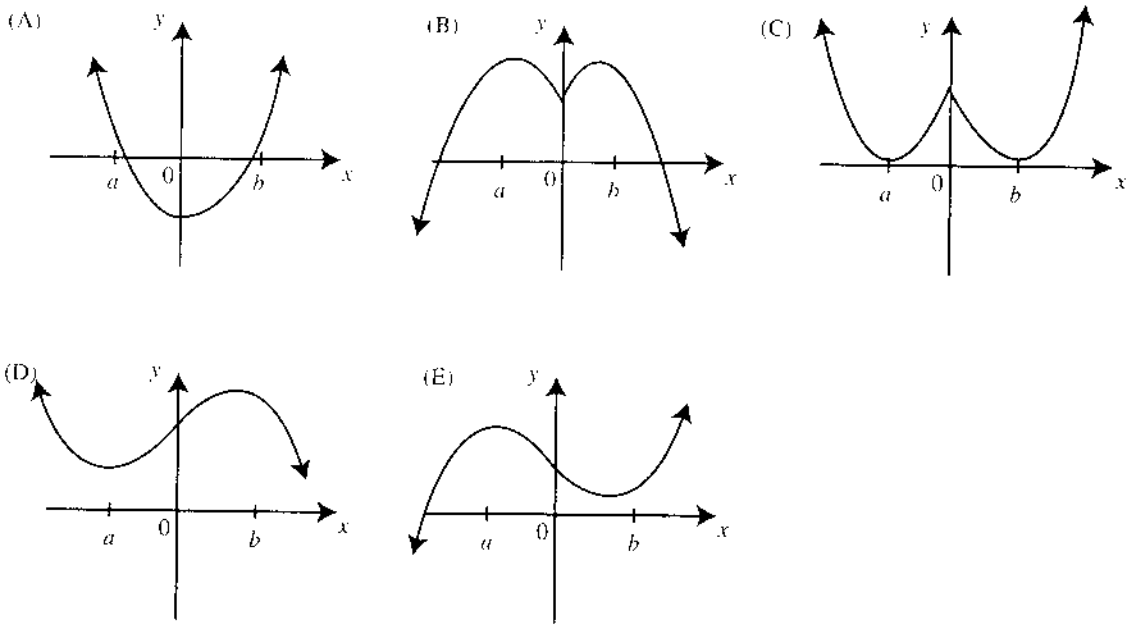

Figure 2T-2

9. If a function f is continuous for all values of x, which of the following statements is/are always true?

I. $2\int_a^b f(x)dx = \int_{2a}^{2b} f(x)dx$

II. $\int_a^b f(x)dx = \int_b^a -f(x)dx$

III. $\left|\int_a^b f(x)dx\right| = \int_a^b |f(x)|\,dx$

(A) I only
(B) I and II only
(C) II only
(D) II and III only
(E) I, II, and III

10. The graph of f is shown in Figure 2T-3 and f is twice differentiable. Which of the following has the largest value: $f(0), f'(0), f''(0)$?

(A) $f(0)$
(B) $f'(0)$
(C) $f''(0)$
(D) $f(0)$ and $f'(0)$
(E) $f'(0)$ and $f''(0)$

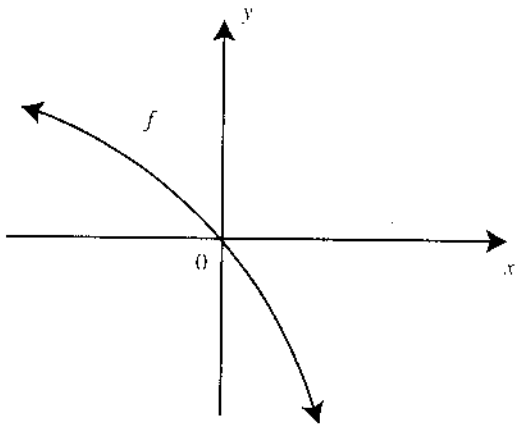

Figure 2T-3

11. $\int \frac{x^4 - 1}{x^2}dx =$

(A) $\frac{x^3}{3} + x + c$
(B) $\frac{x^3}{3} - x + c$
(C) $\frac{x^3}{3} + \frac{3}{x^3} + c$
(D) $\frac{x^3}{3} + \frac{1}{x} + c$
(E) $\frac{x^3}{3} - \frac{1}{x} + c$

12. If $p'(x) = q(x)$ and q is a continuous function for all values of x, then $\int_1^0 q(4x)dx$ is

(A) $p(0) - p(-4)$
(B) $4p(0) - 4p(-4)$
(C) $\frac{1}{4}p(0) - \frac{1}{4}p(-4)$
(D) $\frac{1}{4}p(0) + \frac{1}{4}p(-4)$
(E) $p(0) + p(-4)$

13. Water is leaking from a tank at a rate represented by $f(t)$ whose graph is shown in Figure 2T-4. Which of the following is the best approximation of the total amount of water leaked from the tank for $1 \le t \le 3$?

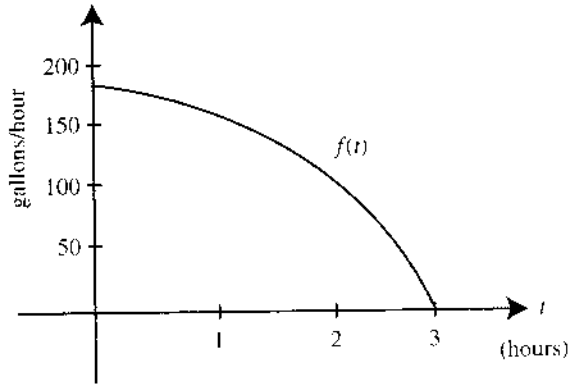

Figure 2T-4

(A) $\frac{9}{2}$ gallons
(B) 5 gallons
(C) 175 gallons
(D) 350 gallons
(E) 450 gallons

14. If $f(x) = 5\cos^2(\pi - x)$, then $f'\left(\frac{\pi}{2}\right)$ is

(A) 0 (B) −5 (C) 5
(D) −10 (E) 10

15. $g(x) = \int_1^x \frac{3t}{t^3+1}dt$, then $g'(2)$ is

(A) 0 (B) $-\frac{2}{3}$ (C) $\frac{2}{3}$
(D) $\frac{-5}{6}$ (E) $\frac{5}{6}$

16. If $\int_k^2 (2x-2)dx = -3$, a possible value of k is

(A) −2 (B) 0 (C) 1
(D) 2 (E) 3

17. If $\int_0^a f(x)dx = -\int_{-a}^0 f(x)dx$ for all positive values of a, then which of the following could be the graph of f? See Figure 2T-5.

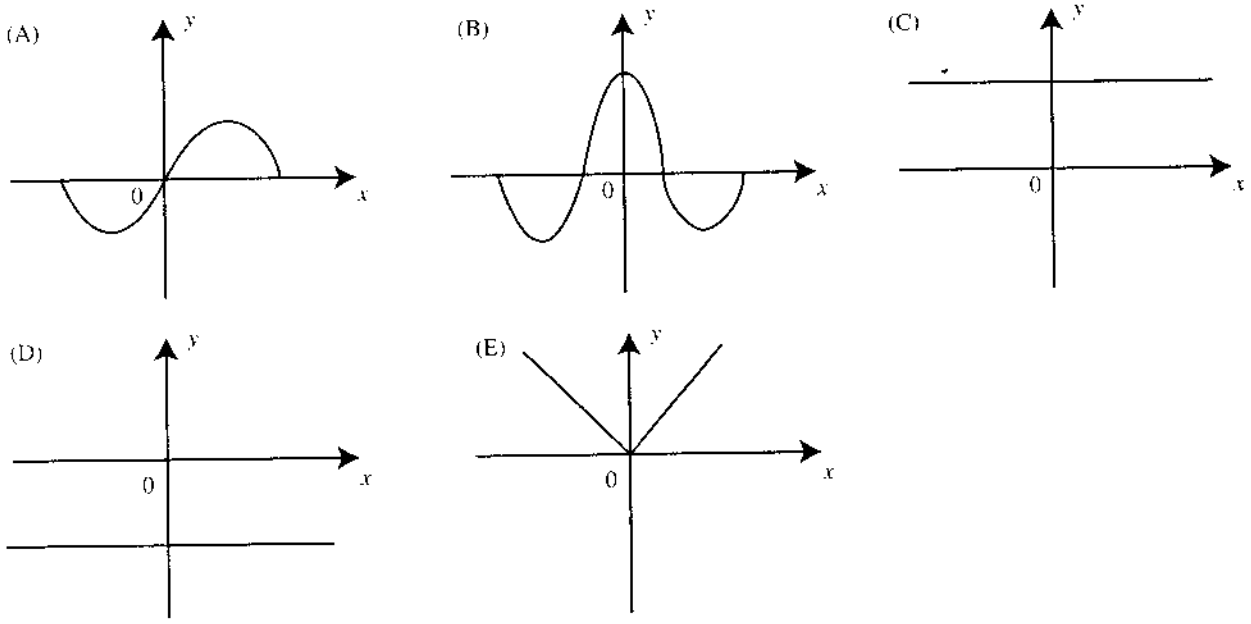

Figure 2T-5

18. A function f is continuous on $[1, 5]$ and some of the values of f are shown below:

x	1	3	5
$f(x)$	-2	b	-1

If f has only one root, r, on the closed interval $[1, 5]$, and $r \neq 3$, then a possible value of b is

(A) -1 (B) 0 (C) 1
(D) 3 (E) 5

19. Given the equation $V = \frac{1}{3}\pi r^2(5 - r)$, what is the instantaneous rate of change of V with respect to r at $r = 5$?

(A) $-\frac{25\pi}{3}$ (B) $\frac{25\pi}{3}$ (C) $\frac{50\pi}{3}$
(D) 25π (E) $\frac{125\pi}{3}$

20. What is the slope of the tangent to the curve $x^3 - y^2 = 1$ at $x = 1$?

(A) $-\frac{3}{2}$ (B) 0 (C) $\frac{3}{2\sqrt{2}}$
(D) $\frac{3}{2}$ (E) undefined

21. The graph of function f is shown in Figure 2T-6. Which of the following is true for f on the interval (a, b)?

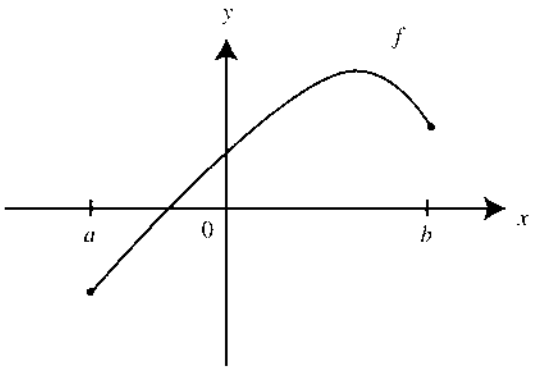

Figure 2T-6

I. The function f is differentiable on (a, b).
II. There exists a number k on (a, b) such that $f'(k) = 0$.
III. $f'' > 0$ on (a, b)

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II and III

22. The velocity function of a moving particle on the x-axis is given as $v(t) = t^2 - 3t - 10$. For what positive values of t is the particle's speed increasing?

(A) $0 < t < \frac{3}{2}$ only
(B) $t > \frac{3}{2}$ only
(C) $t > 5$ only
(D) $0 < t < \frac{3}{2}$ and $t > 5$ only
(E) $\frac{3}{2} < t < 5$ only

23. The graph of f consists of two line segments and a semicircle for $-2 \leq x \leq 2$ as shown in Figure 2T-7. What is the value of $\int_{-2}^{2} f(x)dx$?

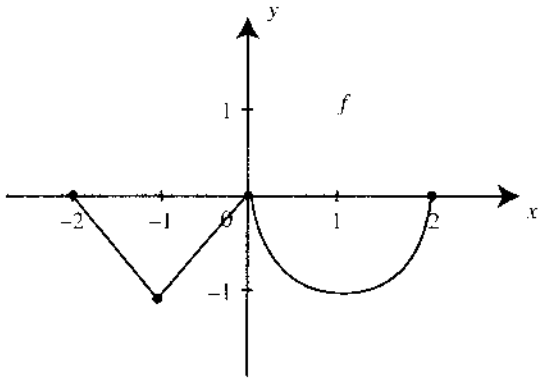

Figure 2T-7

(A) $-2 - 2\pi$ (B) $-2 - \pi$ (C) $-1 - \frac{\pi}{2}$
(D) $1 + \frac{\pi}{2}$ (E) $-1 - \pi$

24. What is the average value of the function $y = 3\cos(2x)$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$?

(A) -2 (B) $-\frac{2}{\pi}$ (C) 0
(D) $\frac{1}{\pi}$ (E) $\frac{3}{2\pi}$

25. If $f(x) = |x^3|$, what is the value of $\lim_{x \to 1} f'(x)$?

(A) -3 (B) 0 (C) 1
(D) 3 (E) undefined

26. A spherical balloon is being inflated. At the instant when the rate of increase of the volume of the sphere is four times the rate of increase of the radius, the radius of the sphere is

(A) $\frac{1}{4\sqrt{\pi}}$ (B) $\frac{1}{\sqrt{\pi}}$ (C) $\frac{1}{\pi}$

(D) $\frac{1}{16\pi}$ (E) π

27. If $\frac{dy}{dx} = \frac{x^2}{y}$ and at $x = 0, y = 4$, a solution to the differential equation is

(A) $y = \frac{x^3}{3}$

(B) $y = \frac{x^3}{3} + 4$

(C) $\frac{y^2}{2} = \frac{x^3}{3}$

(D) $\frac{y^2}{2} = \frac{x^3}{3} + 4$

(E) $\frac{y^2}{2} = \frac{x^3}{3} + 8$

28. The area of the region enclosed by the graph of $x = y^2 - 1$ and the y-axis is

(A) $-\frac{4}{3}$ (B) 0 (C) $\frac{2}{3}$

(D) $\frac{4}{3}$ (E) $\frac{8}{3}$

Section I—Part B

Number of Questions	Time	Use of Calculator
17	50 Minutes	Yes

Directions:

Use the same answer sheet from Part A. *Please note that the questions begin with number* 76. This is not an error. It is done to be consistent with the numbering system of the actual AP Calculus AB Exam. All questions are given equal weight. There is no penalty for unanswered questions. However, 1/4 of the number of incorrect answers will be subtracted from the number of correct answers. Unless otherwise indicated, the domain of a function f is the set of all real numbers. If the exact numerical value does not appear among the given choices, select the best approximate value. The use of a calculator is *permitted* in this part of the exam.

76. The graph of f', the derivative of f, is shown in Figure 2T-8. At which value of x does the graph f have a horizontal tangent?

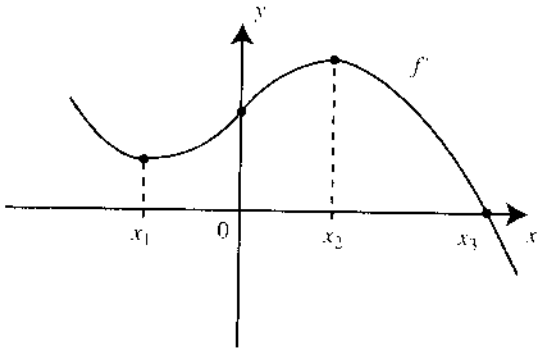

Figure 2T-8

(A) x_1 (B) 0 (C) x_2
(D) x_1 and x_2 (E) x_3

77. The position function of a moving particle is $s(t) = 5 + 4t - t^2$ for $0 \le t \le 10$ where s is in meters and t is measured in seconds. What is the maximum speed in m/sec of the particle on the interval $0 \le t \le 10$?

(A) −16 (B) 0 (C) 2
(D) 4 (E) 16

78. How many points of inflection does the graph of $y = \cos(x^2)$ have on the interval $(0, \pi)$?

(A) 0 (B) 1 (C) 2
(D) 3 (E) 4

79. Let f be a continuous function on [4, 10] and have selected values as shown below:

x	4	6	8	10
$f(x)$	2	2.4	2.8	3.2

Using three right endpoint rectangles of equal length, what is the approximate value of $\int_4^{10} f(x)dx$?

(A) 8.4 (B) 9.6 (C) 14.4
(D) 16.8 (E) 20.8

80. Given a differentiable function f with $f(-1) = 2$ and $f'(-1) = \frac{1}{2}$. Using a tangent line to the graph of f at $x = -1$, find an approximate value of $f(-1.1)$?

(A) −3.05 (B) −1.95 (C) 0.95
(D) 1.95 (E) 3.05

81. The area under the curve of $y = \frac{\ln x}{x}$ from $x = 1$ to $x = b$, where $b > 1$ is 0.66. Then the value of b is approximately,

(A) 1.93 (B) 2.25 (C) 3.15
(D) 3.74 (E) 5.71

82. The base of a solid is a region enclosed by the circle $x^2 + y^2 = 4$. What is the approximate volume of the solid if the cross sections of the solid perpendicular to the x-axis are semicircles?

(A) 8π (B) $\frac{16\pi}{3}$ (C) $\frac{32\pi}{3}$
(D) $\frac{64\pi}{3}$ (E) $\frac{512\pi}{15}$

83. The temperature of a cup of coffee is dropping at the rate of $f(t) = 4\sin\left(\frac{t}{4}\right)$ degrees for $0 \le t \le 5$, where f is measured in Fahrenheit and t in minutes. If initially, the coffee is 95°F, find its temperature to the nearest degree Fahrenheit 5 minutes later.

(A) 84 (B) 85 (C) 91
(D) 92 (E) 94

84. The graphs of f', g', p', and q' are shown in Figure 2T-9. Which of the functions f, g, p, or q have a relative minimum on (a, b)?

(A) f only (B) g only
(C) p only (D) q only
(E) q and p only

85. What is the volume of the solid obtained by revolving about the y-axis the region enclosed by the graphs of $x = y^2$ and $x = 9$?

(A) 36π (B) $\frac{81\pi}{2}$ (C) $\frac{486\pi}{2}$
(D) $\frac{1994}{5}$ (E) $\frac{1944\pi}{5}$

86. At what value(s) of x do the graphs of $y = e^x$ and $y = x^2 + 5x$ have parallel tangent lines?

(A) −2.5 (B) 0
(C) 0 and 5 (D) −5 and 0.24
(E) −2.45 and 2.25

87. Let y represent the population in a town. If y decreases according to the equation $\frac{dy}{dt} = ky$, t in years, and the population decreases by 25% in 6 years, then $k =$

(A) −8.318 (B) −1.726 (C) −0.231
(D) −0.120 (E) −0.048

88. If $h(x) = \int_4^x (t-5)^3 dt$ on [4, 8], then h has a local minimum at $x =$

(A) 4 (B) 5 (C) 6
(D) 7 (E) 8

89. If $p(x) = \int_a^x q(t)dt$ $a < x < b$ and the graph of q is shown in Figure 2T-10, which of the graphs shown in Figure 2T-11 is a possible graph of p?

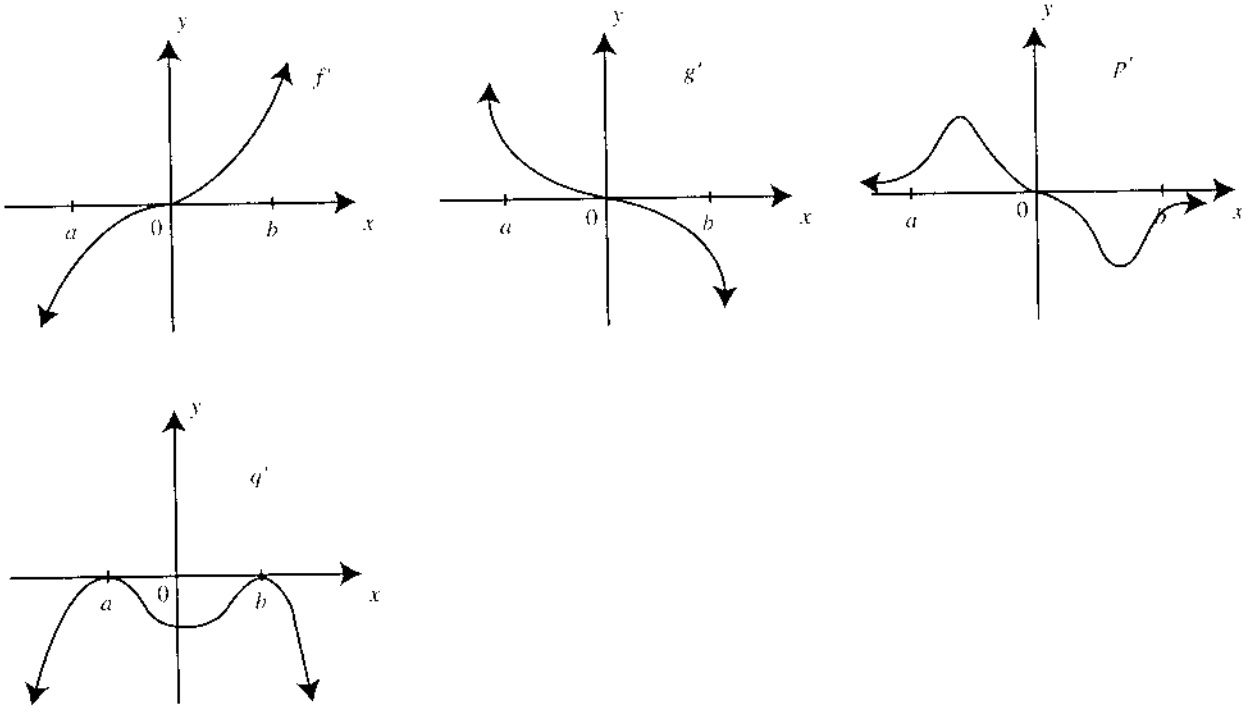

Figure 2T-9

90. The volume of the solid generated by revolving about the y-axis the region bounded by the graph of $y = x^3$, the line $y = 1$ and the y-axis is

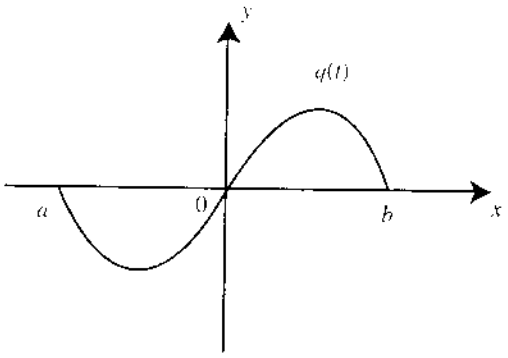

Figure 2T-10

(A) $\frac{\pi}{4}$ (B) $\frac{2\pi}{5}$ (C) $\frac{3\pi}{5}$
(D) $\frac{2\pi}{3}$ (E) $\frac{3\pi}{4}$

91. If $f(x) = -|x - 3|$, which of the following statement about f is true?

I. f is differentiable at $x = 3$.
II. f has an absolute minimum at $x = 3$.
III. f has a point of inflection at $x = 3$.

(A) II only
(B) III only
(C) II and III only
(D) I, II, and III
(E) none

92. The equation of the tangent line to the graph of $y = \sin x$ for $0 \le x \le \pi$ at the point where $\frac{dy}{dx} = \frac{1}{2}$ is

(A) $y = \frac{1}{2}\left(x - \frac{\pi}{3}\right) - \frac{\sqrt{3}}{2}$
(B) $y = \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$
(C) $y = \frac{1}{2}\left(x - \frac{1}{2}\right) + \frac{\pi}{3}$
(D) $y = \frac{1}{2}\left(x - \frac{1}{2}\right) - \frac{\pi}{3}$
(E) $y = \frac{1}{2}\left(x + \frac{1}{2}\right) - \frac{\pi}{3}$

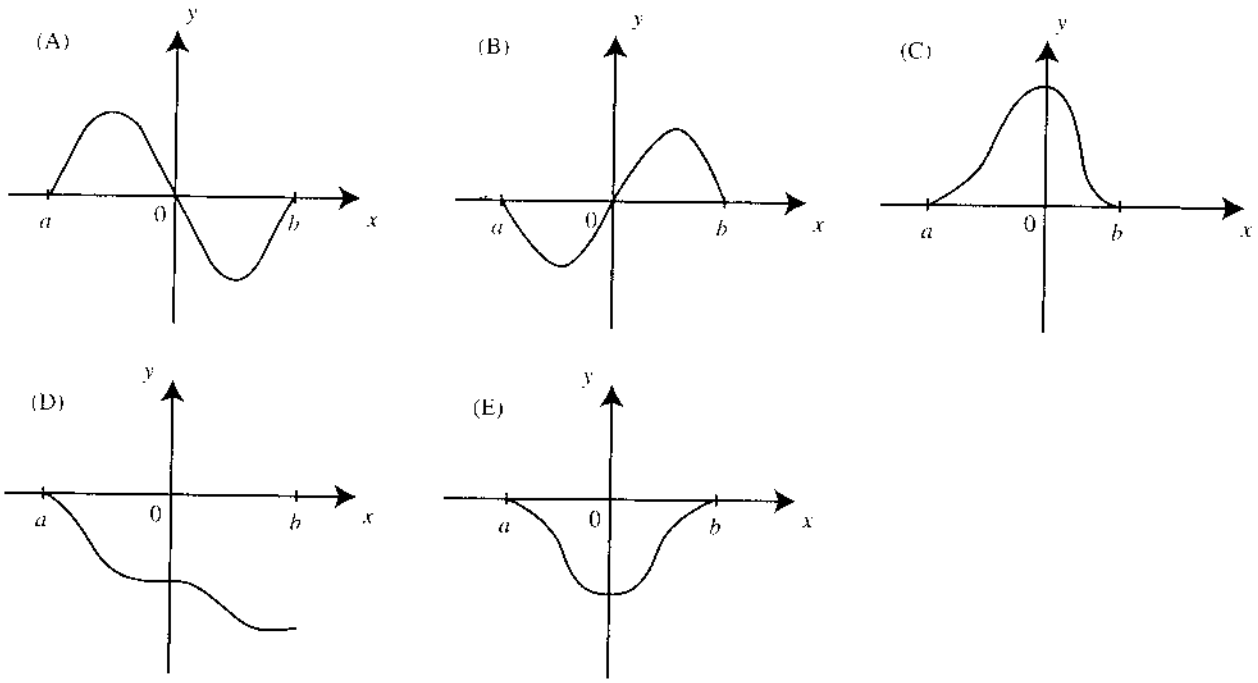

Figure 2T-11