4

Probability

Lesson 4.8
Combinations and Probability

Statistics and Probability with Applications, 3rd Edition
Starnes & Tabor

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Combinations and Probability

Learning Targets

After this lesson, you should be able to:

- ✓ Compute the number of combinations of *n* individuals taken *k* at a time.
- ✓ Use combinations to calculate probabilities.
- ✓ Use the multiplication counting principle and combinations to calculate probabilities.

Combinations and Probability

Recall that a permutation is a distinct arrangement of some group of individuals. With permutations, order matters. Sometimes, we're just interested in finding how many ways there are to choose some number of individuals from a group, but we don't care about the order in which the individuals are selected.

Combinations, ${}_nC_k$

A **combination** is a selection of individuals from some group in which the order of selection doesn't matter. If there are n individuals, then the notation ${}_nC_k$ represents the number of different combinations of k individuals chosen from the entire group of n.

Combinations and Probability

How to Compute Combinations

You can calculate the number of combinations of n individuals taken *k* at a time (where $k \le n$) using the multiplication counting principle, with the formula

$$ {}_nC_k = \frac{{}_nP_k}{k!} $$

or with the formula

$$ {}_nC_k = \frac{n!}{k!(n-k)!} $$

Combinations and Probability

The focus of this chapter is probability. Recall that when a chance process results in equally likely outcomes, the probability that event A occurs is

$$P(\text{A}) = \frac{\text{number of outcomes in event A}}{\text{total number of outcomes in sample space}}$$

You can use the multiplication counting principle and what you have learned about permutations and combinations to help count the number of outcomes.

Combinations and Probability

Consider New Jersey's "Pick Six" lotto game from the previous example. What's the probability that a player wins the jackpot by matching all 6 winning numbers?

Because the winning numbers are randomly selected, any set of 6 numbers from 1 to 49 is equally likely to be chosen.

So we can use the probability formula to calculate

$$P(\text{win the jackpot}) = \frac{\text{number of ways to choose all 6 winning numbers}}{\text{total number of ways to choose 6 numbers from 1 to 49}}$$

$$= \frac{{}_6C_6}{{}_{49}C_6}$$

$$= \frac{1}{13{,}983{,}816} \approx 0.0000000715$$

LESSON APP 4.8 How many ways can you set up an iPod playlist?

Janine wants to set up a play list with 8 songs on her iPod. She has 50 songs to choose from, including 15 songs by One Direction. Janine's iPod won't allow any song to appear more than once in a play list.

1. How many different sets of 8 songs are possible for Janine's play list? Assume that the order of the songs doesn't matter.
2. How many 8-song play lists contain no songs by One Direction?

Suppose Janine decides to let her iPod select an 8-song play list at random.

3. What's the probability that none of the 8 songs is by One Direction?
4. Find the probability that exactly 2 of the songs on the play list are by One Direction.

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