4

Probability

Lesson 4.7
The Multiplication Counting Principle and Permutations

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Starnes & Tabor

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The Multiplication Counting Principle and Permutations

Learning Targets

After this lesson, you should be able to:

- ✓ Use the multiplication counting principle to determine the number of ways to complete a process involving several steps.
- ✓ Use factorials to count the number of permutations of a group of individuals.
- ✓ Compute the number of permutations of *n* individuals taken *k* at a time.

The Multiplication Counting Principle and Permutations

Finding the probability of an event often involves counting the number of possible outcomes of some chance process.

The Agricola Restaurant offers a three-course dinner menu. Customers who order from this menu must choose one appetizer, one main dish, and one dessert. Here are the options for each course.

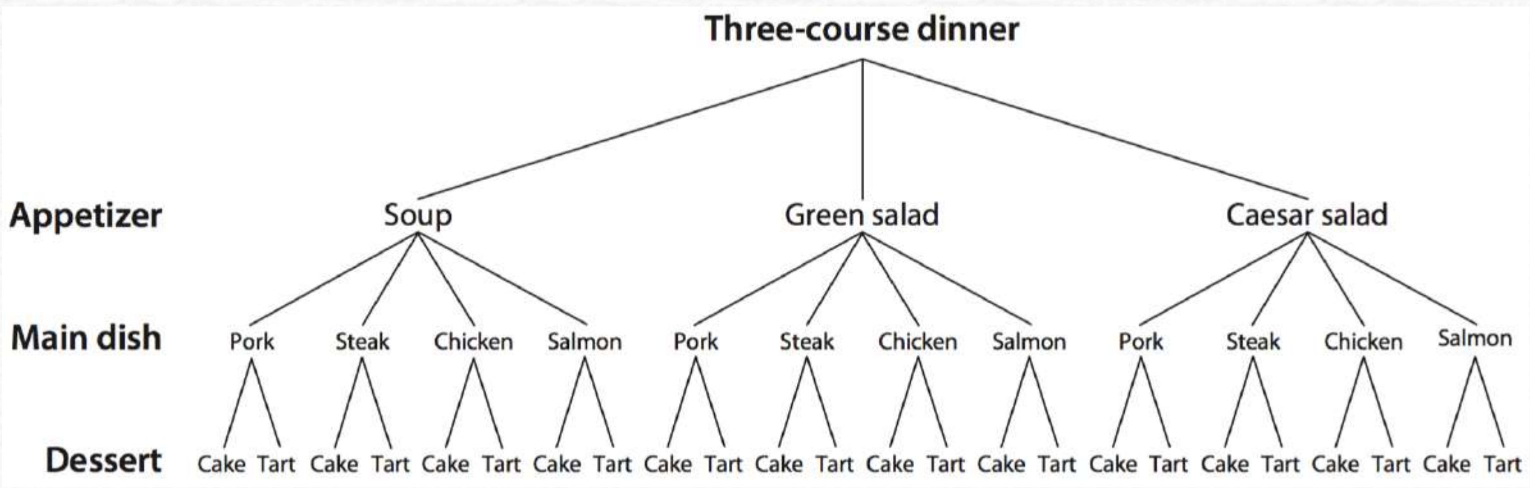
Appetizer	Main dish	Dessert
Butternut squash soup	Grilled pork chop	Chocolate cake
Green salad	Ribeye steak	Apple and cranberry tart
Caesar salad	Roasted chicken breast	
	Poached salmon	

The Multiplication Counting Principle and Permutations

How many different meals can be ordered from this menu? We could try to list all possible orders:

Soup—Pork—Cake, Soup—Pork—Tart, Soup—Steak—Cake, . . .

It might be easier to display all of the options in a diagram:



The Multiplication Counting Principle and Permutations

From the diagram, we can see that for each of the three choices of appetizer, there are four choices of main dish, and for each of those main dish choices, there are two dessert choices. So there are

$$3 \cdot 4 \cdot 2 = 24$$

different meals that can be ordered from the three-course dinner menu. This is an example of the **multiplication counting principle**.

Multiplication Counting Principle

For a process involving multiple (k) steps, suppose that there are n_1 ways to do Step 1, n_2 ways to do Step 2, ..., and n_k ways to do Step k. The total number of different ways to complete the process is

$$n_1 \cdot n_2 \cdot \ldots \cdot n_k$$

This result is called the **multiplication counting principle**.

The Multiplication Counting Principle and Permutations

The multiplication counting principle can also help us determine how many ways there are to arrange a group of people, animals, or things.

We call arrangements where the order matters **permutations**.

Permutation

A **permutation** is a distinct arrangement of some group of individuals.

Suppose you have 5 framed photographs of different family members that you want to arrange in a line on top of your dresser. In how many ways can you do this? Let's count the options moving from left to right across the dresser. There are 5 options for the first photo, 4 options for the next photo, and so on.

By the multiplication counting principle, there are

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

different photo arrangements.

The Multiplication Counting Principle and Permutations

Expressions like $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ occur often enough in counting problems that mathematicians invented a special name and notation for them. We write $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$, read as "5 **factorial**."

Factorial

For any positive integer n, we define $n!$ (read "*n* **factorial**") as

$$n! = n(n-1)(n-2) \ldots \cdot 3 \cdot 2 \cdot 1$$

That is, n factorial is the product of the numbers starting with n and going down to 1.

The Multiplication Counting Principle and Permutations

So far, we have shown how to count the number of distinct arrangements of *all* the individuals in a group of people, animals, or things. Sometimes, we want to determine how many ways there are to select and arrange only *some* of the individuals in a group.

Denoting Permutations: ${}_nP_k$

If there are n individuals, the notation ${}_nP_k$ represents the number of different permutations of k individuals selected from the entire group of n.

Two Ways to Compute Permutations

You can calculate the number of permutations of n individuals taken k at a time (where $k \leq n$) using the multiplication counting principle or with the formula

$${}_nP_k = \frac{n!}{(n-k)!}$$

By definition, $0! = 1$.

LESSON APP 4.7 Do you scream for ice cream?

The local ice cream shop in Dontrelle's town is called 21 Choices. Why? Because they offer 21 different flavors of ice cream. Dontrelle likes all but three of the flavors that 21 Choices offers: bubble gum, butter pecan, and pistachio.

1. A 21 Choices "basic sundae" comes in three sizes— small, medium, or large—and includes one flavor of ice cream and one of 12 toppings. Dontrelle has enough money for a small or medium basic sundae. How many different sundaes could Dontrelle order that include only flavors that he likes?

2. Dontrelle could order a cone with three scoops of ice cream instead of a sundae. He prefers to have three different flavors (for variety) and he considers the order of the flavors on his cone to be important. How many three-scoop cones with three different flavors that Dontrelle likes are possible at 21 Choices? Give your answer as a number and using ${}_nP_k$ notation.

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