4

Probability

Lesson 4.6
The Multiplication Rule for Independent Events

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The Multiplication Rule for Independent Events

Learning Targets

After this lesson, you should be able to:

- ✓ Use the multiplication rule for independent events to calculate probabilities.
- ✓ Calculate *P*(at least one) using the complement rule and the multiplication rule for independent events.
- ✓ Determine if it is appropriate to use the multiplication rule for independent events in a given setting.

The Multiplication Rule for Independent Events

What happens to the general multiplication rule in the special case when events A and B are independent?

In that case, $P(B \mid A) = P(B)$ because knowing that event A occurred doesn't change the probability that event B occurs.

We can simplify the general multiplication rule as follows:

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A) = P(A) \cdot P(B)$$

This result is known as the **multiplication rule for independent events**.

Multiplication Rule for Independent Events

If A and B are independent events, the probability that A and B both occur is

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

The Multiplication Rule for Independent Events

The multiplication rule $P(\text{A and B}) = P(\text{A}) \cdot P(\text{B})$ holds if A and B are independent but not otherwise.

The addition rule $P(\text{A or B}) = P(\text{A}) + P(\text{B})$ holds if A and B are mutually exclusive but not otherwise.

Resist the temptation to use these simple rules when the conditions that justify them are not met.

The Multiplication Rule for Independent Events

The multiplication rule $P(\text{A and B}) = P(\text{A}) \cdot P(\text{B})$ gives us another way to determine if two events are independent. Let's return to the pierced-ear example from earlier in the chapter. The following two-way table summarizes data from a college statistics class.

		Gender		
		Male	Female	Total
Pierced ear	Yes	19	84	103
	No	71	4	75
	Total	90	88	178

Our events of interest were A: is male and B: has a pierced ear. Are these two events independent? No, because

$$P(\text{A and B}) = P(\text{male and pierced ear}) = \frac{19}{178} \approx 0.107$$

is not equal to

$$P(\text{A}) \cdot P(\text{B}) = P(\text{male}) \cdot P(\text{pierced ear}) = \frac{90}{178} \cdot \frac{103}{178} \approx 0.293$$

LESSON APP 4.6 How should we interpret genetic screening?

The First Trimester Screen is a test given during the first trimester of pregnancy to determine if there are specific chromosomal abnormalities in the fetus. According to a study published in the New England Journal of Medicine, approximately 5% of normal pregnancies will receive a positive result. Assume that test results for individual women are independent.

1. Suppose that two unrelated women who are having normal pregnancies, Devondra and Miranda, are given the First Trimester Screen. What is the probability that Devondra gets a positive result and Miranda gets a negative result?

2. If 100 unrelated women with normal pregnancies are tested with the First Trimester Screen, what is the probability that at least 1 woman will receive a positive result?

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