4

Probability

Lesson 4.5
The General Multiplication Rule and Tree Diagrams

Statistics and Probability with Applications, 3rd Edition
Starnes & Tabor

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The General Multiplication Rule and Tree Diagrams

Learning Targets

After this lesson, you should be able to:

- ✓ Use the general multiplication rule to calculate probabilities.
- ✓ Use a tree diagram to model a chance process involving a sequence of outcomes.
- ✓ Calculate conditional probabilities using tree diagrams.

The General Multiplication Rule and Tree Diagrams

Suppose that A and B are two events resulting from the same chance process. We can find the probability $P(\text{A or B})$ with the general addition rule:

$$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$$

How do we find the probability that both events happen, $P(\text{A and B})$?

General Multiplication Rule

For any chance process, the probability that events A and B both occur can be found using the **general multiplication rule**:

$$P(\text{A and B}) = P(\text{A}) \cdot P(\text{B} \mid \text{A})$$

The General Multiplication Rule and Tree Diagrams

The general multiplication rule says that for both of two events to occur, first one must occur. Then, given that the first event has occurred, the second must occur. To confirm that this result is correct, start with the conditional probability formula

$$P(\text{B} \mid \text{A}) = \frac{P(\text{B and A})}{P(\text{A})}$$

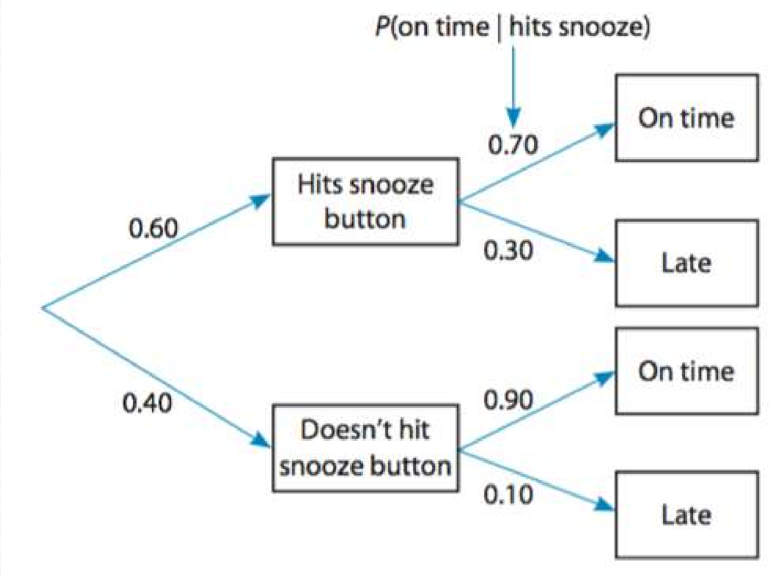
The numerator gives the probability we want because *P*(B and A) is the same as *P*(A and B). Multiply both sides of the above equation by *P*(A) to get

$$P(\text{A}) \cdot P(\text{B} \mid \text{A}) = P(\text{A and B})$$

The General Multiplication Rule and Tree Diagrams

Shannon hits the snooze button on her alarm on 60% of school days. If she hits snooze, there is a 0.70 probability that she makes it to her first class on time. If she doesn't hit snooze and gets up right away, there is a 0.90 probability that she makes it to class on time. Suppose we select a school day at random and record whether Shannon hits the snooze button and whether she arrives in class on time.

We can use a **tree diagram** to show this chance process.



The General Multiplication Rule and Tree Diagrams

Tree Diagram

A **tree diagram** shows the sample space of a chance process involving multiple stages. The probability of each outcome is shown on the corresponding branch of the tree. All probabilities after the first stage are conditional probabilities.

What is the probability that Shannon hits the snooze button and is late for class on a randomly selected school day?

The general multiplication rule provides the answer:

$$\begin{aligned} P(\text{hits snooze and late}) &= P(\text{hits snooze}) \cdot P(\text{late} \mid \text{hits snooze}) \\ &= (0.60)(0.30) \\ &= 0.18 \end{aligned}$$

The previous calculation amounts to multiplying probabilities along the branches of the tree diagram.

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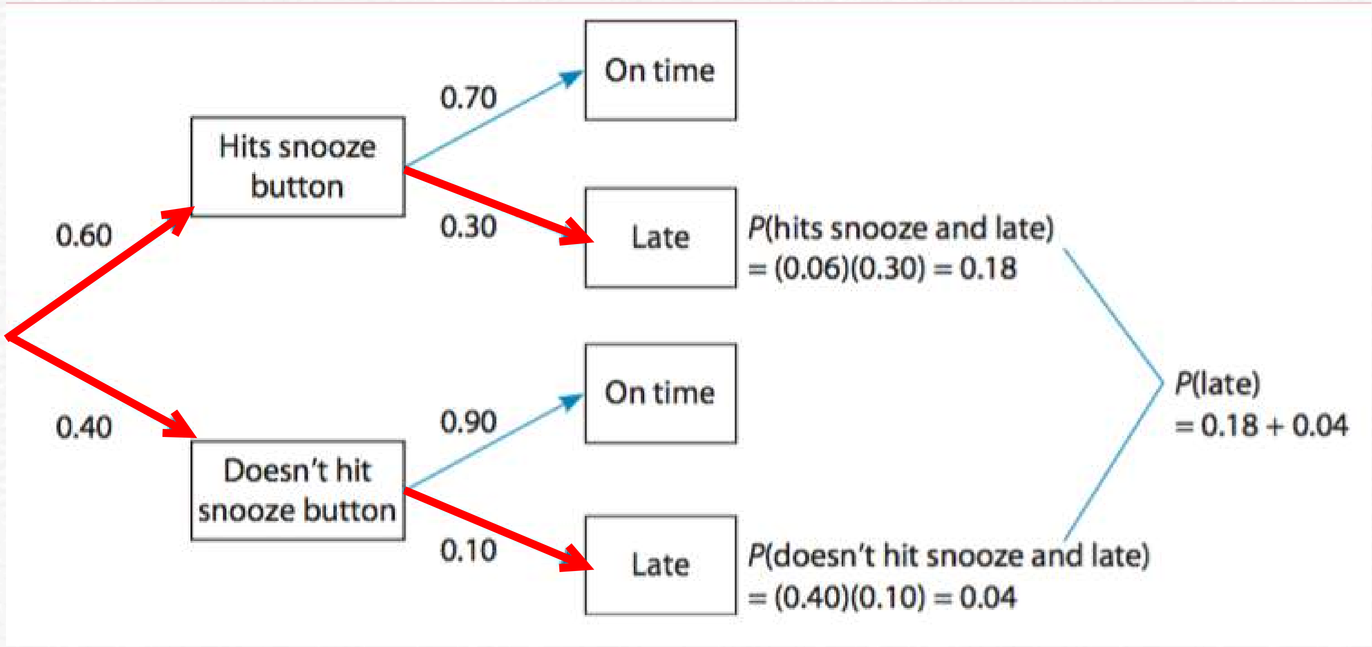
What's the probability that Shannon is late to class on a randomly selected school day? There are two ways this can happen: Shannon can hit the snooze button and be late or she cannot hit snooze and be late. Because these outcomes are mutually exclusive,

$$P(\text{late}) = P(\text{hits snooze and late}) + P(\text{doesn't hit snooze and late})$$

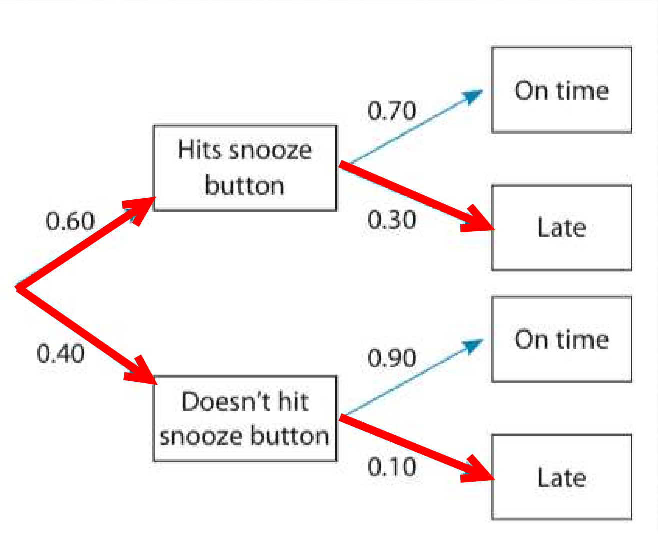


The General Multiplication Rule and Tree Diagrams

Some interesting conditional probability questions involve “going in reverse” on a tree diagram. For instance, suppose that Shannon is late for class on a randomly chosen school day. What is the probability that she hit the snooze button that morning?

We can use the information from the tree diagram and the conditional probability formula to do the required calculation:

P(late) = *P*(hits snooze and late) + *P*(doesn’t hit snooze and late)



$$P(\text{hit snooze button} \mid \text{late}) = \frac{P(\text{hit snooze button and late})}{P(\text{late})} = \frac{(0.60)(0.30)}{(0.60)(0.30) + (0.40)(0.10)} = \frac{0.18}{0.22} = 0.818$$

LESSON APP 4.5 Not Milk?

Lactose intolerance causes difficulty in digesting dairy products that contain lactose (milk sugar). It is particularly common among people of African and Asian ancestry. In the United States (not including other groups and people who consider themselves to belong to more than one race), 82% of the population is white, 14% is black, and 4% is Asian. Moreover, 15% of whites, 70% of blacks, and 90% of Asians are lactose intolerant. Suppose we select a U.S. person at random.

1. Construct a tree diagram to represent this situation.
2. Find the probability that the person is lactose intolerant.
3. Given that the chosen person is lactose intolerant, what is the probability that he or she is Asian?

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