4

Probability

Lesson 4.4
Conditional Probability and Independence

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Conditional Probability and Independence

Learning Targets

After this lesson, you should be able to:

- ✓ Find and interpret conditional probabilities using two-way tables.
- ✓ Use the conditional probability formula to calculate probabilities.
- ✓ Determine whether two events are independent.

Conditional Probability and Independence

The probability of an event can change if we know that some other event has occurred.

Conditional Probability

The probability that one event happens given that another event is known to have happened is called a **conditional probability**. The conditional probability that event B happens given that event A has happened is denoted by $P(B \mid A)$.

Conditional Probability and Independence

By exploring probabilities through a two-way table, we can determine a general formula for computing conditional probabilities.

Calculating Conditional Probabilities

To find the conditional probability P(A | B), use the formula

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A \cap B)}{P(B)} = \frac{P(\text{both events occur})}{P(\text{given event occurs})}$$

Conditional Probability and Independence

Suppose you toss a fair coin twice. Define events A: first toss is a head, and B: second toss is a head.

We know that $P(A) = 1/2$ and $P(B) = 1/2$.

- What's $P(B \mid A)$? It's the conditional probability that the second toss is a head given that the first toss was a head. The coin has no memory, so $P(B \mid A) = 1/2$.
- What's $P(B \mid A^C)$? It's the conditional probability that the second toss is a head given that the first toss was not a head. Getting a tail on the first toss does not change the probability of getting a head on the second toss, so $P(B \mid A^C) = 1/2$.

In this case, $P(B \mid A) = P(B \mid A^C) = P(B)$. Knowing whether or not the first toss was a head does not change the probability that the second toss is a head. We say that A and B are **independent events**.

Conditional Probability and Independence

Independent Events

A and B are **independent events** if knowing whether or not one event has occurred does not change the probability that the other event will happen.

In other words, events A and B are independent if

$$P(A \mid B) = P(A \mid B^C) = P(A) \text{ and } P(B \mid A) = P(B \mid A^C) = P(B)$$

LESSON APP 4.4 Who earns A's in college?

Students at the University of New Hampshire received 10,000 course grades in a recent semester. The two-way table breaks down these grades by which school of the university taught the course.

		School		
		Liberal Arts	Engineering and Physical Sciences	Health and Human Services
Grade	A	2142	368	882
	B	1890	432	630
	Lower than B	2268	800	588

The schools are Liberal Arts, Engineering and Physical Sciences (EPS), and Health and Human Services.

Choose a University of New Hampshire course grade at random. Consider the two events E: the grade comes from an EPS course, and L: the grade is lower than a B.

1. Find $P(L \mid E)$. Interpret this probability in context.
2. Are events L and E independent? Justify your answer.

Conditional Probability and Independence

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