

Probability

Lesson 4.3 Two-Way Tables and Venn Diagrams

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Learning Targets

After this lesson, you should be able to:

- Use a two-way table to find probabilities.
- Calculate probabilities with the general addition rule.
- Use a Venn diagram to find probabilities.



Mutually exclusive events A and B cannot both happen at the same time. For such events, "A or B" means that only event A happens or only event B happens.

You can find P(A or B) with the addition rule for mutually exclusive events: P(A or B) = P(A) + P(B)

How can we find P(A or B) when the two events are not mutually exclusive? Now we have to deal with the fact that "A or B" means one or the other or both.

When you're trying to find probabilities involving two events, like P(A or B), a two-way table can display the sample space in a way that makes probability calculations easier.

When you're trying to find probabilities involving two events, like P(A or B), a two-way table can display the sample space in a way that makes probability calculations easier. Consider the example about pierced ears in males and females.

		Gender			
		Male	Female	Total	
Pierced ear	Yes	19	84	103	
	No	71	4	75	
	Total	90	88	178	



We can't use the addition rule for mutually exclusive events unless events A and B have no outcomes in common. In the example, there are 19 outcomes that are shared by events A and B—the students who are male and have a pierced ear. If we did add the probabilities of A and B, we'd get 90/178 + 103/178 = 193/178. This is clearly wrong because the probability is bigger than 1!

We can fix the double-counting problem illustrated in the two-way table by subtracting the probability P(male and pierced ear) from the sum.

This result is known as the general addition rule.

The General Addition Rule for Two Events

If A and B are any two events resulting from some chance process, the **general addition rule** says that P(A or B) = P(A) + P(B) - P(A and B)

Two-way tables can be used to illustrate the sample space of a chance process involving two events. So can **Venn diagrams**.

Venn Diagram

A **Venn diagram** consists of one or more circles surrounded by a rectangle. Each circle represents an event. The region inside the rectangle represents the sample space of the chance process.



In the previous examples, our events of interest were A: is male and B: has a pierced ear. Here is the two-way table that summarizes the data.

		Gender			
		Male	Female	Total	
	Yes	19	84	103	
Pierced ear	No	71	4	75	
	Total	90	88	178	



Some standard vocabulary and notation have been developed to make our work with Venn diagrams a bit easier.

• The complement A^C contains the outcomes that are not in A.



• The event "A and B" is also called the **intersection** of A and B. The corresponding notation is A ∩ B.



The event "A or B" is also known as the union of A and B. The corresponding notation is A ∪ B.



Intersection, Union

- The event "A and B" is called the intersection of events A and B. It consists of all outcomes that are common to both events, and is denoted A ∩ B.
- The event "A or B" is called the **union** of events A and B. It consists of all outcomes that are in event A or event B, or both, and is denoted A ∪ B.

With this new notation, we can rewrite the general addition rule in symbols as:

$$P(\mathsf{A} \cup \mathsf{B}) = P(\mathsf{A}) + P(\mathsf{B}) - P(\mathsf{A} \cap \mathsf{B})$$

LESSON APP 4.3 Who owns a home?

What is the relationship between educational achievement and home ownership? A random sample of 500 U.S. adults was selected. Each member of the sample was identified as a high school graduate (or not) and as a homeowner (or not). The two-way table displays the data.

		High school graduate		
		Yes	No	
Homeowner	Yes	221	119	
	No	89	71	

Suppose we choose a member of the sample at random. Define events G: is a high school graduate and H: is a homeowner.

- 1. Explain why $P(G \text{ or } H) \neq P(G) + P(H)$. Then find P(G or H).
- 2. Make a Venn diagram to display the sample space of this chance process.
- 3. Write the event "is not a high school graduate but is a homeowner" in symbolic form.

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