Solving Systems of Equations

- You can solve a system of equations using different methods. The idea is to determine which method is easiest for that particular problem.
- These notes show how to solve the system algebraically using SUBSTITUTION.

Solving a system of equations by substitution

Step 1: Solve an equation for one variable.	Pick the easier equation. The goal is to get y= ; x= ; a= ; etc.
Step 2: Substitute	Put the equation solved in Step 1 into the other equation.
Step 3: Solve the equation.	Get the variable by itself.
Step 4: Plug back in to find the other variable.	Substitute the value of the variable into the equation.
Step 5: Check your solution.	Substitute your ordered pair into BOTH equations.

1) Solve the system using substitution

$$x + y = 5$$
$$y = 3 + x$$

Step 1: Solve an equation for one variable.	The second equation is already solved for y!
Step 2: Substitute	$x + y = 5$ $x + (3 + x) = 5$
Step 3: Solve the equation.	$2x + 3 = 5$ $2x = 2$ $x = 1$

1) Solve the system using substitution

$$x + y = 5$$
$$y = 3 + x$$

Step 4: **Plug back in to find the other variable.**

$x + y = 5$
$(1) + y = 5$
$y = 4$

Step 5: **Check your solution.**

(1, 4)
$(1) + (4) = 5$ ✓
$(4) = 3 + (1)$ ✓

The solution is (1, 4). What do you think the answer would be if you graphed the two equations?

Which answer checks correctly?

$$3x - y = 4$$
$$x = 4y - 17$$

1. (2, 2)
2. (5, 3)
3. (3, 5) ✓
4. (3, -5)

2) Solve the system using substitution

$$3y + x = 7$$
$$4x - 2y = 0$$

Step 1: Solve an equation for one variable.

It is easiest to solve the first equation for x.

$$\begin{array}{rcl} 3y + x &=& 7 \\ -3y & & -3y \\ \hline x &=& -3y + 7 \end{array}$$

Step 2: Substitute

$$4x - 2y = 0$$
$$4(-3y + 7) - 2y = 0$$

2) Solve the system using substitution

$$3y + x = 7$$

$$4x - 2y = 0$$

Step 3: Solve the equation.

$$-12y + 28 - 2y = 0$$
$$-14y + 28 = 0$$
$$-14y = -28$$
$$\mathbf{y = 2}$$

Step 4: Plug back in to find the other variable.

$$4x - 2y = 0$$
$$4x - 2(\mathbf{2}) = 0$$
$$4x - 4 = 0$$
$$4x = 4$$
$$\mathbf{x = 1}$$

2) Solve the system using substitution

$$3y + x = 7$$

$$4x - 2y = 0$$

Step 5: Check your solution.

(1, 2)

$3(2) + (1) = 7$ ✓

$4(1) - 2(2) = 0$ ✓

When is solving systems by substitution easier to do than graphing?

When only one of the equations has a variable already isolated (like in example #1).

If you solved the first equation for x, what would be substituted into the bottom equation.

$$2x + 4y = 4$$
$$3x + 2y = 22$$

1. $-4y + 4$
2. ✓ $-2y + 2$
3. $-2x + 4$
4. $-2y + 22$

3) Solve the system using substitution

$x = 3 - y$

$x + y = 7$

Step 1: **Solve an equation for one variable.**	The first equation is already solved for x!
Step 2: **Substitute**	$x + y = 7$ $(\mathbf{3 - y}) + y = 7$
Step 3: **Solve the equation.**	$3 = 7$ The variables were eliminated!! This is a special case. Does 3 = 7? FALSE!

When the result is FALSE, the answer is NO SOLUTIONS.

3) Solve the system using substitution

$$2x + y = 4$$
$$4x + 2y = 8$$

Step 1: Solve an equation for one variable.	The first equation is easiest to solved for y! $y = -2x + 4$
Step 2: Substitute	$4x + 2y = 8$ $4x + 2(-2x + 4) = 8$
Step 3: Solve the equation.	$4x – 4x + 8 = 8$ $8 = 8$ This is also a special case. Does 8 = 8? TRUE!

When the result is TRUE, the answer is **INFINITELY MANY SOLUTIONS**.

What does it mean if the result is “TRUE”?

1. The lines intersect
2. The lines are parallel
3. ✓ The lines are coinciding
4. The lines reciprocate
5. I can spell my name