Solve Separable Differential Equations **Name:** Answers

AP Calculus

Solve each differential equation below. Then verify your result by differentiating your answer.

1) Given $\dfrac{dy}{dx} = 4x^3y^2$ and $y(1) = -\frac{1}{2}$, solve the differential equation.

$$\frac{dy}{y^2} = 4x^3\,dx$$

$$\int y^{-2}\,dy = \int 4x^3\,dx$$

$$-1y^{-1} = x^4 + C_1$$

$$y^{-1} = -x^4 + C_2$$

$$\frac{1}{y} = -x^4 + C_2$$

$$1 = y(-x^4 + C_2)$$

$$\boxed{\frac{1}{-x^4+C} = y}$$ general solution

given: $(1, -\frac{1}{2})$

$$\frac{1}{-(1)^4+C} = -\frac{1}{2}$$

$$2 = -1(-1+C)$$

$$2 = 1 - C$$

$$1 = -C$$

$$\boxed{-1 = C}$$

ans: $\boxed{y = \dfrac{-1}{x^4+1}}$

2) Find a solution of the differential equation $\dfrac{dy}{dx} = x\sin(x^2)$ given $f(0) = -1$

$$\int dy = \int x\sin(x^2)\,dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$y = \frac{1}{2}\int \sin u\,du$$

$$y = -\frac{1}{2}\cos u + C$$

$$\boxed{y = -\frac{1}{2}\cos(x^2) + C}$$ general solution

given $(0, -1)$:

$$-1 = -\frac{1}{2}\cos(0^2) + C$$

$$-1 = -\frac{1}{2}(1) + C$$

$$-\frac{1}{2} = C$$

solution: $\boxed{y = -\dfrac{1}{2}\cos(x^2) - \dfrac{1}{2}}$

3) Find the general solution of $\frac{dy}{dx} = \frac{2xy}{x^2+1}$.

$$\int \frac{dy}{y} = \int \frac{2x}{x^2+1}\,dx \qquad u = x^2+1,\quad du = 2x\,dx$$

$$\ln|y| = \int \frac{du}{u}$$

$$\ln|y| = \ln|u| + C$$

$$\ln|y| = \ln|x^2+1| + C_1$$

$$e^{\ln|x^2+1| + C_1} = |y|$$

$$e^{\ln(|x^2+1|)} \cdot e^{C_1} = |y|$$

$$|x^2+1| \cdot C_2 = |y|$$

$$y = \pm C_2 (X^2+1)$$

No need for absolute value since this must be positive.

$$y = C_3 (x^2+1)$$

General Solution:

$$\boxed{y = C(x^2+1)}$$

4) Write an equation for the curve that passes through the point (3, 4) and has a slope at any point (x, y) as $\frac{dy}{dx} = \frac{x^2+1}{2y}$.

$$\int 2y\,dy = \int (x^2+1)\,dx$$

$$y^2 = \frac{1}{3}x^3 + x + C$$

general solution $y = \pm\sqrt{\frac{1}{3}x^3 + x + C}$

given (3, 4):

$$4 = \sqrt{\tfrac{1}{3}(3)^3 + (3) + C}$$

$$4 = \sqrt{12 + C}$$

$$16 = 12 + C$$

$$4 = C$$

This must be positive since point (3, 4) has a positive y-value.

$$\boxed{y = \sqrt{\frac{1}{3}x^3 + x + 4}}$$