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Progressive Mathematics Initiative

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Algebra II

Sequences and Series

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Arithmetic Sequences

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Goals and Objectives

Students will be able to understand how the common difference leads to the next term of an arithmetic sequence, the explicit form for an Arithmetic sequence, and how to use the explicit formula to find missing data.

Why Do We Need This?

Arithmetic sequences are used to model patterns and form predictions for events based on these patterns, such as in loan payments, sales, and revenue.

Vocabulary

An Arithmetic sequence is the set of numbers found by adding the same value to get from one term to the next.

Example:

1, 3, 5, 7,...

10, 20, 30,...

10, 5, 0, -5,..

Vocabulary

The common difference for an arithmetic sequence is the value being added between terms, and is represented by the variable d.

Example:

1, 3, 5, 7,... ⟶ d=2

10, 20, 30,... ⟶ d=10

10, 5, 0, -5,.. ⟶ d=-5

Notation

As we study sequences we need a way of naming the terms.
a_1 to represent the first term,
a_2 to represent the second term,
a_3 to represent the third term,
and so on in this manner.

If we were talking about the 8^{th} term we would use a_8.

When we want to talk about general term call it the n^{th} term
and use a_n.

Finding the Common Difference

1. Find two subsequent terms such as a_1 and a_2

2. Subtract $a_2 - a_1$

Find d:

4, 10, 16, ...

Solution

Find the common difference:

1, 4, 7, 10, . . .

5, 11, 17, 23, . . .

9, 5, 1, -3, . . .

$\frac{1}{2}, 3, \frac{11}{2}, 8, \ldots$

Solutions

NOTE: You can find the common difference using ANY set of consecutive terms

For the sequence 10, 4, -2, -8, ...

Find the common difference using a_1 and a_2:

Find the common difference using a_3 and a_4:

What do you notice?

To find the next term:

1. Find the common difference

2. Add the common difference to the last term of the sequence

3. Continue adding for the specified number of terms

Example: Find the next three terms

1, 5, 9, 13, ...

Solution

Find the next three terms:

1, 4, 7, 10, . . .

5, 11, 17, 23, . . .

9, 5, 1, -3, . . .

$\frac{1}{2}, 3, \frac{11}{2}, 8, \ldots$

1 **Find the next term in the arithmetic sequence:**
3, 9, 15, 21, . . .

Solution

2 **Find the next term in the arithmetic sequence:**
-8, -4, 0, 4, . . .

Solution

3 **Find the next term in the arithmetic sequence:**
2.3, 4.5, 6.7, 8.9, . . .

Solution

4 **Find the value of d in the arithmetic sequence:**
10, -2, -14, -26, . . .

Solution

5 **Find the value of d in the arithmetic sequence:**
-8, 3, 14, 25, . . .

Solution

Write the first four terms of the arithmetic sequence that is described.

1. Add d to a_1

2. Continue to add d to each subsequent terms

Solution

Example:

Write the first four terms of the sequence:

a_1=3, d= 7

Find the first three terms for the arithmetic sequence described:

a_1 = 4; d = 6

a_1 = 3; d = -3

Solution

a_1 = 0.5; d = 2.3

a_2 = 7; d = 5

6 Which sequence matches the description?

$$a_1 = 2; d = 4$$

- **A 4, 6, 8, 10**
- **B 2, 6,10, 14**
- **C 2, 8, 32, 128**
- **D 4, 8, 16, 32**

Solution

7 Which sequence matches the description?

$$a_1 = -3; d = -4$$

- A -3, -7, -10, -14
- B -4, -7, -10, -13
- C -3, -7, -11, -15
- D -3, 1, 5, 9

Solution

8 Which sequence matches the description?

$$a_3 = 7; d = 3$$

- A 7, 10, 13, 16
- B 4, 7, 10, 13
- C 1, 4, 7,10
- D 3, 5, 7, 9

Solution

Recursive Formula

To write the recursive formula for an arithmetic sequence:

1. Find a_1
2. Find d
3. Write the recursive formula: $\begin{cases} a_1 = initial \\ a_n = a_{n-1} + d \end{cases}$

Example:

Write the recursive formula for 1, 7, 13, ...

Solution

Write the recursive formula for the following sequences:

$a_1 = 3$; d = -3

$a_1 = 0.5$; d = 2.3

Solution

1, 4, 7, 10, . . .

5, 11, 17, 23, . . .

9 Which sequence is described by the recursive formula?

$$\begin{cases} a_1 = -2 \\ a_n = a_{n-1} + 4 \end{cases}$$

- **A** -2, -8, -16, ...
- **B** -2, 2, 6, ...
- **C** 2, 6, 10, ...
- **D** 4, 2, 0, ...

10 A recursive formula is called recursive because it uses the previous term.

- True
- False

11 Which sequence matches the recursive formula?

$$\begin{cases} a_1 = -5 \\ a_n = a_{n-1} + 2.5 \end{cases}$$

- **A** -2.5, 0, 2.5, ...
- **B** -5, -7.5, -9, ...
- **C** -5, -2.5, 0, ...
- **D** -5, -12.5, -31.25, ...

Arithmetic Sequence

To find a specific term,say the 5^{th} or a_5, you could write out all of the terms.

But what about the 100^{th} term(or a_{100})?

We need to find a formula to get there directly without writing out the whole list.

DISCUSS:
Does a recursive formula help us solve this problem?

Arithmetic Sequence

Consider: 3, 9, 15, 21, 27, 33, 39,. . .

a_1	3
a_2	9 = 3+6
a_3	15 = 3+12 = 3+2(6)
a_4	21 = 3+18 = 3+3(6)
a_5	27 = 3+24 = 3+ 4(6)
a_6	33 = 3+30 = 3+5(6)
a_7	39 = 3+36 = 3+6(6)

Do you see a pattern that relates the term number to its value?

This formula is called the explicit formula.

It is called explicit because it does not depend on the previous term

The explicit formula for an arithmetic sequence is:

$$a_n = a_1 + (n-1)d$$

To find the explicit formula:

1. Find a_1
2. Find d
3. Plug a_1 and d into $a_n = a_1 + (n-1)d$
4. Simplify

Example: Write the explicit formula for 4, -1, -6, ...

Solution

Write the explicit formula for the sequences:

1) 3, 9, 15, ...

2) -4, -2.5, -1, ...

3) 2, 0, -2, ...

Solution

12 The explicit formula for an arithmetic sequence requires knowledge of the previous term

- True
- False

Solution

13 Find the explicit formula for 7, 3.5, 0, ...

- A $a_n = 7 + (n-1)(-3.5)$
- B $a_n = 10.5 - 3.5n$
- C $a_n = 7 - 3.5n$
- D $a_n = 3.5 + 3.5n$

Solution

14 Write the explicit formula for -2, 2, 6, ...

- A $a_n = -2 + (n-1)4$
- B $a_n = -2 + 4n$
- C $a_n = 6 - 4n$
- D $a_n = -6 + 4n$

Solution

15 Which sequence is described by:

$$a_n = 5 + 2n$$

- A 7, 9, 11, ...
- B 5, 7, 9, ...
- C 5, 3, 1, ...
- D 7, 5, 3, ...

Solution

16 Find the explicit formula for -2.5, 3, 8.5, ...

- A $a_n = -2.5 + 5.5n$
- B $a_n = 8.5 + 5.5n$
- C $a_n = -2.5 + (n-1)5.5$
- D $a_n = 5.5n - 7.5$

Solution

17 What is the initial term for the sequence described by:

$$a_n = -3.5 - 4n$$

Solution

Finding a Specified Term

1. Find the explicit formula for the sequence.
2. Plug the number of the desired term in for n
3. Evaluate

Example: Find the 31st term of the sequence described by

$$a_n = 3 + 2n$$

Solution

Example Find the 21st term of the arithmetic sequence with $a_1 = 4$ and $d = 3$.

Solution

Example Find the 12th term of the arithmetic sequence with $a_1 = 6$ and $d = -5$.

Solution

Finding the Initial Term or Common Difference

1. Plug the given information into $a_n=a_1+(n-1)d$
2. Solve for a_1, d, or n

Example: Find a_1 for the sequence described by $a_{13}=16$ and $d=-4$

Solution

Example Find the 1st term of the arithmetic sequence with $a_{15} = 30$ and $d = 7$.

Solution

Example Find the 1st term of the arithmetic sequence with $a_{17} = 4$ and $d = -2$.

Solution

Example Find d of the arithmetic sequence with $a_{15} = 45$ and $a_1 = 3$.

Solution

Example Find the term number n of the arithmetic sequence with $a_n = 6$, $a_1 = -34$ and $d = 4$.

Solution

18 Find a_{11} when $a_1 = 13$ and $d = 6$.

Solution

19 Find a_{17} when $a_1 = 12$ and $d = -0.5$

Solution

20 Find a_{17} for the sequence 2, 4.5, 7, 9.5, ...

Solution

21 Find the common difference d when $a_1 = 12$ and $a_{14} = 6$.

Solution

22 Find n such $a_1 = 12$, $a_n = -20$, and $d = -2$.

Solution

23 Tom works at a car dealership selling cars. He is paid $4000 a month plus a $300 commission for every car he sells. How much did he make in April if he sold 14 cars?

Solution

24 Suppose you participate in a bikeathon for charity. The charity starts with $1100 in donations. Each participant must raise at least $35 in pledges. What is the minimum amount of money raised if there are 75 participants?

Solution

25 Elliot borrowed $370 from his parents. He will pay them back at the rate of $60 per month. How long will it take for him to pay his parents back?

Solution

Geometric Sequences

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Goals and Objectives

Students will be able to understand how the common ratio leads to the next term and the explicit form for an Geometric sequence, and use the explicit formula to find missing data.

Why Do We Need This?

Geometric sequences are used to model patterns and form predictions for events based on these patterns, such as in loan payments, sales, and revenue.

Vocabulary

An Geometric sequence is the set of numbers found by multiplying by the same value to get from one term to the next.

Example:

1, 0.5, 0.25,...

2, 4, 8, 16, ...

.2, .6, 1.8, ...

Vocabulary

The ratio between every consecutive term in the geometric sequence is called the common ratio.

This is the value each term is multiplied by to find the next term.

Example:

1, 0.5, 0.25,... $\longrightarrow$ r = 0.5

2, 4, 8, 16, ... $\longrightarrow$ r = 2

0.2, 0.6, 1.8, ... $\longrightarrow$ r = 3

Write the first four terms of the geometric sequence described.

1. Multiply a_1 by the common ratio r.

2. Continue to multiply by r to find each subsequent term.

Example: Find the first four terms:

$a_1=3$ and r = 4

Solution

To Find the Common Ratio

1. Choose two consecutive terms

2. Divide a_n by a_{n-1}

Example:

Find r for the sequence:

4, 2, 1, 0.5, ...

Solution

Find the next 3 terms in the geometric sequence

3, 6, 12, 24, . . .

5, 15, 45, 135, . . .

32, -16, 8, -4, . . .

16, 24, 36, 54, . . .

Solution

26 **Find the next term in geometric sequence:**
6, -12, 24, -48, 96, . . .

Solution

27 **Find the next term in geometric sequence:**
64, 16, 4, 1, . . .

Solution

28 **Find the next term in geometric sequence:**
6, 15, 37.5, 93.75, . . .

Solution

Verifying Sequences

To verify that a sequence is geometric:

1. Verify that the common ratio is common to all terms by dividing each consecutive pair of terms.

Example:

Is the following sequence geometric?

3, 6, 12, 18, ...

Solution

29 Is the following sequence geometric?
48, 24, 12, 8, 4, 2, 1

- Yes
- No

Solution

Examples: Find the first five terms of the geometric sequence described.

1) $a_1 = 6$ and $r = 3$

click

2) $a_1 = 8$ and $r = -.5$

click

3) $a_1 = -24$ and $r = 1.5$

click

4) $a_1 = 12$ and $r = {}^{2}/_{3}$

click

30 Find the first four terms of the geometric sequence described: $a_1 = 6$ and $r = 4$.

- A 6, 24, 96, 384
- B 4, 24, 144, 864
- C 6, 10, 14, 18
- D 4, 10, 16, 22

Solution

31 Find the first four terms of the geometric sequence described: $a_1 = 12$ and $r = -1/2$.

- A 12, -6, 3, -.75
- B 12, -6, 3, -1.5
- C 6, -3, 1.5, -.75
- D -6, 3, -1.5, .75

Solution

32 Find the first four terms of the geometric sequence described: $a_1 = 7$ and $r = -2$.

- A 14, 28, 56, 112
- B -14, 28, -56, 112
- C 7, -14, 28, -56
- D -7, 14, -28, 56

Solution

Recursive Formula

To write the recursive formula for an geometric sequence:

1. Find a_1
2. Find r
3. Write the recursive formula: $\begin{cases} a_1 = initial \\ a_n = a_{n-1} \times r \end{cases}$

Example: Find the recursive formula for the sequence

0.5, -2, 8, -32, ...

Solution

Write the recursive formula for each sequence:

1) 3, 6, 12, 24, . . .

2) 5, 15, 45, 135, . . .

3) $a_1 = -24$ and $r = 1.5$

4) $a_1 = 12$ and $r = {}^{2}/_{3}$

33 **Which sequence does the recursive formula represent?**

$$a_n = \begin{cases} a_1 = \frac{1}{4} \\ a_n = (a_{n-1})\left(\frac{-3}{2}\right) \end{cases}$$

- A 1/4, 3/8, 9/16, ...
- B -1/4, 3/8, -9/16, ...
- C 1/4, -3/8, 9/16, ...
- D -3/2, 3/8, -3/32, ...

Solution

34 **Which sequence matches the recursive formula?**

$$a_n = \begin{cases} a_1 = -2 \\ a_n = (a_{n-1})(4) \end{cases}$$

- A -2, 8, -16, ...
- B -2, 8, -32, ...
- C 4, -8, 16, ...
- D -4, 8, -16, ...

Solution

35 **Which sequence is described by the recursive formula?**

$$a_n = \begin{cases} a_1 = 10 \\ a_n = (a_{n-1})(-0.5) \end{cases}$$

- A 10, -5, 2.5, ...
- B -10, 5, -2.5, ...
- C -0.5, -5, -50, ...
- D 0.5, 5, 50, ...

Solution

Consider the sequence: 3, 6, 12, 24, 48, 96, . . .
To find the seventh term, just multiply the sixth term by 2.
But what if I want to find the 20th term?

Look for a pattern:

a_1	3
a_2	$6 = 3(2)$
a_3	$12 = 3(4) = 3(2)^2$
a_4	$24 = 3(8) = 3(2)^3$
a_5	$48 = 3(16) = 3(2)^4$
a_6	$96 = 3(32) = 3(2)^5$
a_7	$192 = 3(64) = 3(2)^6$

Do you see a pattern?

click

This formula is called the explicit formula.

It is called explicit because it does not depend on the previous term

The explicit formula for an geometric sequence is:

$$a_n = a_1 \cdot r^{n-1}$$

To find the explicit formula:

1. Find a_1
2. Find r
3. Plug a_1 and r into $a_n = a_1 \cdot r^{n-1}$
4. Simplify if possible

Example: Write the explicit formula for 2, -1, 1/2, ...

Solution

Write the explicit formula for the sequence

1) 3, 6, 12, 24, . . .

2) 5, 15, 45, 135, . . .

3) $a_1 = -24$ and $r = 1.5$

4) $a_1 = 12$ and $r = {}^{2}/_{3}$

Solution

36 Which explicit formula describes the geometric sequence 3, -6.6, 14.52, -31.944, ...

A $a_n = 3 \cdot 2.2^{n-1}$

B $a_n = -3 \cdot 2.2^{n-1}$

C $a_n = 3(-2.2)^{n-1}$

D $a_n = -2.2(3)^{n-1}$

Solution

37 What is the common ratio for the geometric sequence described by

$$a_n = (-2)(3/2)^{n-1}$$

Solution

38 What is the initial term for the geometric sequence described by

$$a_n = -\frac{7}{3}(3)^{n-1}$$

Solution

39 Which explicit formula describes the sequence 1.5, 4. 13.5, ...

- A $a_n = 3(1.5)^{n-1}$
- B $a_n = 1.5(3)^{n-1}$
- C $a_n = 4.5(1.5)^{n-1}$
- D $a_n = 3(13.5)^{n-1}$

Solution

40 What is the explicit formula for the geometric sequence -8, 4, -2, 1,...

- A $a_n = (8)(-2)^{n-1}$
- B $a_n = (-8)(2)^{n-1}$
- C $a_n = (-8)(0.5)^{n-1}$
- D $a_n = (-8)(-0.5)^{n-1}$

Solution

Finding a Specified Term

1. Find the explicit formula for the sequence.
2. Plug the number of the desired term in for n
3. Evaluate

Example: Find the 10th term of the sequence described by

$$a_n = 3(-5)^{n-1}$$

Solution

Find the indicated term.

Example: a_{20} given a_1 =3 and r = 2.

Solution

Example: a_{10} for 2187, 729, 243, 81

Solution

41 Find a_{12} in a geometric sequence where $a_1 = 5$ and $r = 3$.

Solution

42 Find a_7 in a geometric sequence where $a_1 = 10$ and $r = -{}^1/_2$.

Solution

43 Find a_{10} in a geometric sequence where $a_1 = 7$ and $r = -2$.

Solution

Finding the Initial Term, Common Ratio, or Term

1. Plug the given information into $a_n=a_1(r)^{n-1}$
2. Solve for a_1, r, or n

Example: Find r if $a_6 = 0.2$ and $a_1 = 625$

Solution

Example: Find n if $a_1 = 6$, $a_n = 98{,}304$ and $r = 4$.

Solution

44 Find r of a geometric sequence where $a_1 = 3$ and $a_{10}=59049$.

Solution

45 Find n of a geometric sequence where $a_1 = 72$, $r = .5$, and $a_n = 2.25$

Solution

46 Suppose you want to reduce a copy of a photograph. The original length of the photograph is 8 in. The smallest size the copier can make is 58% of the original.

Find the length of the photograph after five reductions.

Solution

47 The deer population in an area is increasing. This year, the population was 1.025 times last year's population of 2537. How many deer will there be in the year 2022?

Solution

Geometric Series

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Goals and Objectives

Students will be able to understand the difference between a sequence and a series, and how to find the sum of a geometric series.

Why Do We Need This?

Geometric series are used to model summation events such as radioactive decay or interest payments.

Vocabulary

A geometric series is the sum of the terms in a geometric sequence.

Example:

1+2+4+8+...

a_1=1

r=2

The sum of a geometric series can be found using the formula:

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

To find the sum of the first n terms:

1. Plug in the values for a_1, n, and r

2. Evaluate

Example: Find the sum of the first 11 terms

$a_1 = -3$, r = 1.5

Solution

Examples: Find S_n

$a_1 = 5$, r= 3, n= 6

Solution

Example: Find S_n

$a_1 = -3$, $r = -2$, $n = 7$

Solution

48 Find the indicated sum of the geometric series described: $a_1 = 10$, $n = 6$, and $r = 6$

Solution

49 Find the indicated sum of the geometric series described: $a_1 = 8$, $n = 6$, and $r = -2$

Solution

50 Find the indicated sum of the geometric series described: $a_1 = -2$, $n = 5$, and $r = {}^1/_4$

Solution

Sometimes information will be missing, so that using $S_n = \frac{a_1(1-r^n)}{1-r}$ isn't possible to start.

Look to use $a_n = a_1 \cdot r^{n-1}$ to find missing information.

To find the sum with missing information:
1. Plug the given information into $a_n = a_1 \cdot r^{n-1}$
2. Solve for missing information
3. Plug information into $S_n = \frac{a_1(1-r^n)}{1-r}$
4. Evaluate

Example: $a_1 = 16$ and $a_5 = 243$, find S_5

Solution

51 Find the indicated sum of the geometric series described: 8 - 12 + 18 - . . . find S_7

Solution

52 Find the indicated sum of the geometric series described: $a_1 = 8$, $n = 5$, and $a_6 = 8192$

Solution

53 Find the indicated sum of the geometric series described: $r = 6$, $n = 4$, and $a_4 = 2592$

Solution

Sigma ($\sum$)can be used to describe the sum of a geometric series.

We can still use $S_n = \frac{a_1(1-r^n)}{1-r}$, but to do so we must examine sigma notation.

Examples:

$$\sum_{n=1}^{4} 6(3)^{n-1} = 6(3)^0 + 6(3)^1 + 6(3)^2 + 6(3)^3 = 6 + 18 + 54 + 162 = 240$$

n = 4 Why? [ ]

a_1 = 6 Why? [ ]

r = 3 Why? [ ]

54 Find the sum: $\sum_{n=1}^{5} 4(2)^{n-1}$

Solution

55 Find the sum: $\sum_{n=1}^{6} 5(-3)^{n-1}$

Solution

56 **Find the sum:** $\sum_{n=1}^{7}(5)^{n-1}$

Solution

Special Sequences

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A recursive formula is one in which to find a term you need to know the preceding term.

So to know term 8 you need the value of term 7, and to know the n^{th} term you need term n-1

In each example, find the first 5 terms

$a_1 = 6, a_n = a_{n-1} + 7$		$a_1 = 10, a_n = 4a_{n-1}$		$a_1 = 12, a_n = 2a_{n-1} + 3$
6	a_1	10	a_1	12
13	a_2	40	a_2	27
20	a_3	160	a_3	57
27	a_4	640	a_4	117
34	a_5	2560	a_5	237

57 Find the first four terms of the sequence:

$a_1 = 6$ and $a_n = a_{n-1} - 3$

- A 6, 3, 0, -3
- B 6, -18, 54, -162
- C -3, 3, 9, 15
- D -3, 18, 108, 648

Solution

58 Find the first four terms of the sequence:

$a_1 = 6$ and $a_n = -3a_{n-1}$

- A 6, 3, 0, -3
- B 6, -18, 54, -162
- C -3, 3, 9, 15
- D -3, 18, 108, 648

Solution

59 Find the first four terms of the sequence:

$a_1 = 6$ and $a_n = -3a_{n-1} + 4$

- A 6, -22, 70, -216
- B 6, -22, 70, -214
- C 6, -14, 46, -134
- D 6, -14, 46, -142

Solution

$a_1 = 6, a_n = a_{n-1} + 7$		$a_1 = 10, a_n = 4a_{n-1}$		$a_1 = 12, a_n = 2a_{n-1} + 3$
6	a_1	10	a_1	12
13	a_2	40	a_2	27
20	a_3	160	a_3	57
27	a_4	640	a_4	117
34	a_5	2560	a_5	237

The recursive formula in the first column represents an Arithmetic Sequence.
We can write this formula so that we find a_n directly.

Recall: $a_n = a_1 + (n-1)d$

We will need a_1 and d,they can be found both from the table and the recursive formula.

click

$a_1 = 6, a_n = a_{n-1} + 7$		$a_1 = 10, a_n = 4a_{n-1}$		$a_1 = 12, a_n = 2a_{n-1} + 3$
6	a_1	10	a_1	12
13	a_2	40	a_2	27
20	a_3	160	a_3	57
27	a_4	640	a_4	117
34	a_5	2560	a_5	237

The recursive formula in the second column represents a Geometric Sequence.
We can write this formula so that we find a_n directly.

Recall: $a_n = a_1 (r)^{n-1}$

We will need a_1 and r,they can be found both from the table and the recursive formula.

click

$a_1 = 6, a_n = a_{n-1} + 7$		$a_1 = 10, a_n = 4a_{n-1}$		$a_1 = 12, a_n = 2a_{n-1} + 3$
6	a_1	10	a_1	12
13	a_2	40	a_2	27
20	a_3	160	a_3	57
27	a_4	640	a_4	117
34	a_5	2560	a_5	237

The recursive formula in the third column represents neither an Arithmetic or Geometric Sequence.

This observation comes from the formula where you have both multiply and add from one term to the next.

60 Identify the sequence as arithmetic, geometric,or neither.

$$a_1 = 12 \text{ , } a_n = 2a_{n-1} + 7$$

- A Arithmetic
- B Geometric
- C Neither

Solution

61 Identify the sequence as arithmetic, geometric,or neither.

$$a_1 = 20 \text{ , } a_n = 5a_{n-1}$$

- A Arithmetic
- B Geometric
- C Neither

Solution

62 Which equation could be used to find the n^{th} term of the recursive formula directly?

$$a_1 = 20 \text{ , } a_n = 5a_{n-1}$$

- A $a_n = 20 + (n-1)5$
- B $a_n = 20(5)^{n-1}$
- C $a_n = 5 + (n-1)20$
- D $a_n = 5(20)^{n-1}$

Solution

63 Identify the sequence as arithmetic, geometric,or neither.

$$a_1 = -12 \text{ , } a_n = a_{n-1} - 8$$

- A Arithmetic
- B Geometric
- C Neither

Solution

64 Which equation could be used to find the n^{th} term of the recursive formula directly?

$$a_1 = -12 \text{ , } a_n = a_{n-1} - 8$$

- A $a_n = -12 + (n-1)(-8)$
- B $a_n = -12(-8)^{n-1}$
- C $a_n = -8 + (n-1)(-12)$
- D $a_n = -8(-12)^{n-1}$

Solution

65 Identify the sequence as arithmetic, geometric,or neither.

$$a_1 = 10 \text{ , } a_n = a_{n-1} + 8$$

- A Arithmetic
- B Geometric
- C Neither

Solution

66 Which equation could be used to find the n^{th} term of the recursive formula directly?

$$a_1 = 10 \text{ , } a_n = a_{n-1} + 8$$

- A $a_n = 10 + (n-1)(8)$
- B $a_n = 10(8)^{n-1}$
- C $a_n = 8 + (n-1)(10)$
- D $a_n = 8(10)^{n-1}$

Solution

67 Identify the sequence as arithmetic, geometric,or neither.

$$a_1 = 24 \text{ , } a_n = (1/2)a_{n-1}$$

- A Arithmetic
- B Geometric
- C Neither

Solution

68 Which equation could be used to find the n^{th} term of the recursive formula directly?

$$a_1 = 24 \text{ , } a_n = (1/2)a_{n-1}$$

- A $a_n = 24 + (n-1)(1/2)$
- B $a_n = 24(1/2)^{n-1}$
- C $a_n = (1/2) + (n-1)24$
- D $a_n = (1/2)(24)^{n-1}$

Solution

Special Recursive Sequences

Some recursive sequences not only rely on the preceding term, but on the two preceding terms.

Find the first five terms of the sequence:
$a_1 = 4$, $a_2 = 7$, and $a_n = a_{n-1} + a_{n-2}$

4
7
7 + 4 = 11
11 + 7 = 18
18 + 11 =29

Find the first five terms of the sequence:
$a_1 = 6$, $a_2 = 8$, and $a_n = 2a_{n-1} + 3a_{n-2}$

6
8
2(8) + 3(6) = 34
2(34) + 3(8) = 92
2(92) + 3(34) = 286

Find the first five terms of the sequence:
$a_1 = 10$, $a_2 = 6$, and $a_n = 2a_{n-1} - a_{n-2}$

10
6
2(6) -10 = 2
2(2) - 6 = -2
2(-2) - 2= -6

Find the first five terms of the sequence:
$a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$

1
1
1+1 = 2
1 + 2 = 3
2 + 3 =5

The sequence in the preceding example is called

The Fibonacci Sequence
1, 1, 2, 3, 5, 8, 13, 21, . . .

where the first 2 terms are 1's and any term there after is the sum of preceding two terms.

This is as famous as a sequence can get an is worth remembering.

69 **Find the first four terms of sequence:** $a_1 = 5$, $a_2 = 7$, and $a_n = a_1 + a_2$

- A 7, 5, 12, 19
- B 5, 7, 35, 165
- C 5, 7, 12, 19
- D 5, 7, 13, 20

Solution

70 Find the first four terms of sequence:

$a_1 = 4$, $a_2 = 12$, and $a_n = 2a_{n-1} - a_{n-2}$

- A 4, 12, -4, -20
- B 4, 12, 4, 12
- C 4, 12, 20, 28
- D 4, 12, 20, 36

Solution

71 Find the first four terms of sequence:

$a_1 = 3$, $a_2 = 3$, and $a_n = 3a_{n-1} + a_{n-2}$

- A 3, 3, 6, 9
- B 3, 3, 12, 39
- C 3, 3, 12, 36
- D 3, 3, 6, 21

Solution

Writing Sequences as Functions

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Discuss:

Do you think that a sequence is a function?

What do you think the domain of that function be?

Recall:

A function is a relation where each value in the domain has exactly one output value.

A sequence is a function, which is sometimes defined recursively with the domain of integers.

To write a sequence as a function:

1. Write the explicit or recursive formula using function notation.

Example: Write the following sequence as a function

1, 2, 4, 8, ...

Solution

Example: Write the sequence as a function

1, 1, 2, 3, 5, 8, ...

Solution

Example: Write the sequence as a function and state the domain

1, 3, 3, 9, 27, ...

Solution

72 All functions that represent sequences have a domai positive integers

- True
- False

Solution

73 Write the sequence as a function -10, -5, 0 , 5, . . .

- A $f(x) = -10 + 5x$
- B $f(x) = -10 + 5(x - 1)$
- C $f(x) = 5 - 10x$
- D $f(x) = -15 + 5x$

Solution

74 Find f(10) for -2, 4, -8, 16, . . .

Solution

Discuss:

Is it possible to find f(-3) for a function describing a sequence? Why or why not?