Curriculum Management System

MONROE TOWNSHIP SCHOOLS

Course Name: **Senior Mathematics Essentials**
Grade: **12**

For adoption by all regular education programs as specified and for adoption or adaptation by all Special Education Programs in accordance with Board of Education Policy # 2220.

Board Approved: September 2012

Table of Contents	

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MATHEMATICS CURRICULUM INCHARGE (9-12)

Dr. Manjit K. Sran

Mission, Vision, Beliefs, and Goals

Mission Statement

The Monroe Public Schools in collaboration with the members of the community shall ensure that all children receive an exemplary education by well-trained committed staff in a safe and orderly environment.

Vision Statement

The Monroe Township Board of Education commits itself to all children by preparing them to reach their full potential and to function in a global society through a preeminent education.

Beliefs

1. All decisions are made on the premise that children must come first.
2. All district decisions are made to ensure that practices and policies are developed to be inclusive, sensitive and meaningful to our diverse population.
3. We believe there is a sense of urgency about improving rigor and student achievement.
4. All members of our community are responsible for building capacity to reach excellence.
5. We are committed to a process for continuous improvement based on collecting, analyzing, and reflecting on data to guide our decisions.
6. We believe that collaboration maximizes the potential for improved outcomes.
7. We act with integrity, respect, and honesty with recognition that the schools serve as the social core of the community.
8. We believe that resources must be committed to address the population expansion in the community.
9. We believe that there are no disposable students in our community and every child means every child.

Board of Education Goals

1. Raise achievement for all students paying particular attention to disparities between subgroups.
2. Systematically collect, analyze, and evaluate available data to inform all decisions.
3. Improve business efficiencies where possible to reduce overall operating costs.
4. Provide support programs for students across the continuum of academic achievement with an emphasis on those who are in the middle.
5. Provide early interventions for all students who are at risk of not reaching their full potential.
6. To Create a 21st Century Environment of Learning that Promotes Inspiration, Motivation, Exploration, and Innovation.

Philosophy

Monroe Township Schools are committed to providing all students with a quality education resulting in life-long learners who can succeed in a global society. The mathematics program, grades K - 12, is predicated on that belief and is guided by the following six principles as stated by the National Council of Teachers of Mathematics (NCTM) in the *Principles and Standards for School Mathematics, 2000.* First, a mathematics education requires equity. All students will be given worthwhile opportunities and strong support to meet high mathematical expectations. Second, a coherent mathematics curriculum will effectively organize, integrate, and articulate important mathematical ideas across the grades. Third, effective mathematics teaching requires the following: **a)** knowing and understanding mathematics, students as learners, and pedagogical strategies **b)** having a challenging and supportive classroom environment and **c)** continually reflecting on and refining instructional practice. Fourth, students must learn mathematics with understanding. A student's prior experiences and knowledge will actively build new knowledge. Fifth, assessment should support the learning of important mathematics and provide useful information to both teachers and students. Lastly, technology enhances mathematics learning, supports effective mathematics teaching, and influences what mathematics is taught.

As students begin their mathematics education in Monroe Township, classroom instruction will reflect the best thinking of the day. Children will engage in a wide variety of learning activities designed to develop their ability to reason and solve complex problems. Calculators, computers, manipulatives, technology, and the Internet will be used as tools to enhance learning and assist in problem solving. Group work, projects, literature, and interdisciplinary activities will make mathematics more meaningful and aid understanding. Classroom instruction will be designed to meet the learning needs of all children and will reflect a variety of learning styles.

In this changing world those who have a good understanding of mathematics will have many opportunities and doors open to them throughout their lives. Mathematics is not for the select few but rather is for everyone. Monroe Township Schools are committed to providing all students with the opportunity and the support necessary to learn significant mathematics with depth and understanding.

Common Core State Standards (CSSS)

The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy.

Links:

1. CCSS Home Page: http://www.corestandards.org
2. CCSS FAQ: http://www.corestandards.org/frequently-asked-questions
3. CCSS The Standards: http://www.corestandards.org/the-standards
4. NJDOE Link to CCSS: http://www.state.nj.us/education/sca
5. Partnership for Assessment of Readiness for College and Careers (PARCC): http://parcconline.org

Quarter 1

Performance Assessment tasks

I. Equivalence

a. Real numbers
b. Rational numbers
c. Irrational numbers
d. Powers, roots, and exponents
e. Absolute value
f. Scientific notation
g. Properties of equivalence relations (e.g., reflexive, symmetric, transitive)
h. Properties of arithmetic operations (e.g., associative, commutative)
i. Primes, factors, and multiples
j. Ratio
k. Proportion
l. Percent (less than, greater than, or equal to 100%)

II. Transformation

a. Geometric terms (e.g. point, ray, line, angle, plane, side, vertices, Polygon, face, polyhedron, circle, sphere)
b. Standard notations used in geometry
c. Properties of geometric figures
d. Fundamental relationships between geometric figures (*e.g. Parallelism, perpendicularity, intersection, congruence, similarity)*
e. Inductive and deductive reasoning
f. Spatial relationships (e.g., direction, orientation, and perspective of objects in space)
g. Congruence
h. Similarity
i. Symmetry
j. Transformations
    1. Rotations
    2. Reflections
    3. Translations
    4. Dilations
k. The rectangular coordinate system
l. Matrices
m. Tessellations
n. Vectors

III. Measurement

a. Measurable attributes (e.g., perimeter, circumference, area, surface, area, volume, angle measure)
b. Standard and non-standard units of measure
c. Dimensions, shapes, and properties of figures and objects
d. Right triangle relationships
e. The Pythagorean Theorem
f. Basic trigonometric ratios.

Quarter 2 AHSA TESTING CYCLE 1	
Unit Topic(s)	
IV. Risk a. Expected value of a probability-based games b. Determining whether the game is fair c. Calculate geometric probabilities d. Model situations involving probability with simulations and theoretical models e. Probability Models f. Determine probabilities in complex situations g. Conditional events h. Complementary events i. Dependent and independent events j. Theoretical probabilities k. Predictions based on experimental and theoretical probabilities. l. "Law of Large numbers" **V. Patterns** a. Combinations with replacement b. Combinations without replacement c. Multiplication rule of counting in complex situations d. Replacement e. Without replacement f. Ordered counting situations g. Unordered counting situations h. Justify solutions to counting problems. i. Pascal's Triangle j. Sequences and Series k. Explicit formulas for n_{th} terms l. Sums of finite arithmetic series m. Sums of finite and infinite geometric series n. Informal notation of limit o. Inductive reasoning to form generalizations	**VI. Data Analysis** a. Surveys and sampling techniques b. Evaluate the use of data in real-world contexts. c. Statistical experiments d. Lines of best fit or curves of best fit e. Analyze data using technology f. Use statistical terminology to describe conclusions.

Quarter 3

Unit Topic(s)

VII. Relationships

- a. Relations and Functions
- b. Representations of relations and functions
- c. Equations
  - i. Inequalities
  - ii. Graphs.
  - iii. Properties and behavior of functions of one variable
- d. Slope of a line or curve
- e. Domain and range
- f. Intercepts
- g. Continuity
  - i. Maximum/minimum
  - ii. Estimating roots of equations
  - iii. Intersecting points as solutions of systems of equations
- h. Rates of change
- i. Transformations
- j. Translations
- k. Reflections
- l. Dilations
- m. Parameters of linear and quadratic graphs
- n. Complex functions using technology
- o. Classes of functions
  - i. Exponential
  - ii. Polynomial
  - iii. Rational
  - iv. Trigonometric functions
  - v. Properties
  - vi. Linear vs. non-linear
  - vii. Symmetry
  - viii. Increasing/decreasing on an interval

VIII. Equivalence

- a. Evaluate and simplify expressions.
  - i. Add and subtract polynomials
  - ii. Multiply a polynomial by a monomial or binomial
  - iii. Divide a polynomial by a monomial
- b. Select and use appropriate methods to solve equations and inequalities.
  - i. Linear equations - algebraically
  - ii. Quadratic equations - factoring (when the coefficient of x^2 is 1) and using the quadratic formula
  - iii. All types of equations using graphing, computer, and graphing calculator techniques
  - iv. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology

Quarter 4: AHSA TESTING CYCLE 2	
Unit Topic(s)	
IX. Mathematical Modeling a. Model real-world phenomena ▪ Applications of Linear functions in business ▪ Use area of right triangles to deduce areas of other shapes ▪ Application problems involving quadratics in one variable ▪ Application problems involving 2D and 3D representations of objects ▪ Applications of distance-time graphs and relating speed to slopes of graphs ▪ How estimation is used in real life situations ▪ Applications of percent in real life situations ▪ Applications of trigonometric rations in real life ▪ Optimization ▪ Simple Interest ▪ Compound interest ▪ Applications of scale models ▪ Other Applications ▪ Application problems involving patterns	

Unit I – Equivalence
Stage 1 Desired Results

ESTABLISHED GOALS

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- **N-RN.1.**

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

- **N-RN.2.**

Rewrite expressions involving radicals and rational exponents using the properties of exponents.

- **N-Q.1.**

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- **N-Q.2.**

Define appropriate quantities for the purpose of descriptive modeling.

Transfer

Students will be able to independently use their learning to …

- Solve problems by simplifying them, using equivalent statements based on the properties of real numbers and the order of operations.
- Analyze when any rule in any system (language, law, math) is an essential principle or merely conventional.

Meaning

UNDERSTANDINGS

Students will understand that…

- Mathematical ideas interconnect and build on one another resulting in a coherent whole.
- Understand types of numbers, our numeration system, and the different ways they are applied and used in real life situations

ESSENTIAL QUESTIONS

1. How do Mathematical ideas interconnect and build on one another resulting in a coherent whole?
2. How are numbers applied in real-world situations?
3. Does this make sense?

Acquisition

Students will know…

- Real numbers are all values that are found on a number line
- Rational numbers can be written in the form a/b where a and b are both integers and $b \neq 0$
- Irrational numbers cannot be written in the form a/b.
- A terminating decimal has finite number of digits
- A repeating decimal has a digit or sequence of digits that repeats indefinitely

Students will be skilled at…

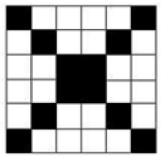
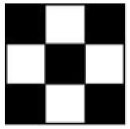
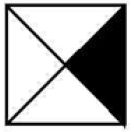
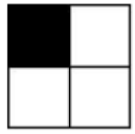
- Extending understanding of the number System
- Comparing and ordering rational and irrational numbers
- Extending understanding and use of operations to include real numbers and algebraic procedures
- Developing, applying and explaining various methods for solving problems involving exponents including rational and negative exponents
- Distinguishing between terminating and repeating decimal forms of rational numbers
- Recognizing that mathematics is used in

▪ **N-Q.3.** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ▪ **A-APR.1.** Understand that polynomials form a system analogous to the integers namely; they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. ▪ **A-APR.7.** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **Modeling** Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).	▪ Integers are the set of positive and negative whole numbers ▪ Square root is one of the two equal factors of a number ▪ To multiply powers having the same base, the exponents are added. ▪ To find the power of a power, multiply the exponents. To find a power of product, find the power of each factor and multiply. ▪ A non-zero number to the zero power is 1. ▪ $a^{-n} = \frac{1}{a^n}; a \neq 0$ ▪ Exponential function is of the form: $y = a \cdot b^x$ ▪ Quotient of powers property states to divide powers having the same base, subtract exponents. ▪ Power of a quotient property states to find the power of the quotient, find the power of the numerator and the power of the denominator and divide. ▪ All positive real numbers have two square roots: a positive and negative square root. The positive square root is called the principle square root. ▪ The number or expression inside a radical symbol $(\sqrt{\ })$ is the radicand. ▪ The square root of a negative number is undefined. ▪ Numbers whose square roots are integers or quotients of integers are called perfect squares. ▪ Absolute value of a number is the distance of a value from zero on the number line. ▪ Scientific notation is of the form $c \times 10^n$ where $1 \leq c < 10$ and n is an integer. ▪ The **reflexive** property of equality	a variety of contexts ▪ Determining whether or not properties of equivalence relations and arithmetic operations apply to different relations and operations ▪ Evaluating and writing expressions containing exponents ▪ Recognizing that mathematical facts, procedures, and claims have to be justified ▪ Using verbal and algebraic models to represent real-life situations. ▪ Solving simple quadratic equations ▪ Determining exponential growth and decay factors ▪ Using technology to gather, analyze, and communicate mathematical information ▪ Recognizing that absolute value of a number is the distance of a value from zero on the number line. ▪ Using absolute values, exponents, and approximations for roots of numbers in real-life situations ▪ Distinguishing between rational and irrational numbers from their decimal representations ▪ Applying the reflexive property of equality ▪ Applying the symmetric property ▪ Applying approximation techniques to situations involving initial portions of infinite decimal ▪ Applying associative, commutative, and distributive properties to simplify algebraic expressions ▪ Using primes, factors and multiples in real-world situations ▪ Understanding how mathematical ideas

	states that any quantity is equal to itself ▪ The **symmetric** property of equality states that if first quantity equals the second, then the second quantity also equals the first ▪ The **transitive** property of equality states that when one quantity equals the second quantity and the second quantity equals a third quantity, then the first and the third quantity are equal ▪ The **substitution** property of equality states that a quantity may be substituted for its equal in a given expression ▪ **Commutative property** of addition and multiplication states that the order in which numbers are added/multiplied does not change the sum/product ▪ **Associative property** of addition/multiplication states that the way numbers/factors are grouped does not change the sum/product ▪ **Distributive property** states that the product of a factor and a sum or difference is the same as the sum or difference of two products ▪ A factor is a whole number that can divide another number evenly ▪ A Multiple is the product of a given whole number and another number ▪ A prime number is a whole number greater than 1 that has exactly two factors, 1 and itself ▪ A composite number is a whole number greater than 1 that has more than two factors	interconnect and build on one another to complete a whole ▪ Solving a proportion by multiplying the numerator of each ratio by the denominator of the other ratio using cross products ▪ Using reasoning to support mathematical conclusions and problem solutions ▪ Selecting and applying a variety of appropriate problem-solving strategies ▪ Changing from a fraction or decimal to a percent and from a percent to a fraction or decimal ▪ Illustrating and modeling ratios, proportions, and percentages in real-life situations ▪ Comparing effects of percent decrease and percent increase in price of objects with and without sale tax ▪ Solving a variety of problems using proportions and percentages

	▪ Prime factorization is an expression that shows the given number as a product of factors that are all prime numbers ▪ Ratio is a comparison of two numbers ▪ A proportion is an equation stating that the two ratios are equal ▪ Cross products is a way of solving a proportion by multiplying the numerator of each ratio by the denominator of the other ratio ▪ Apportionment is the division of something proportionally according to numbers or population	

Unit I – Equivalence
Stage 2 - Evidence

Evaluative Criteria	Assessment Evidence
RUBRIC/SCALE **3 - POINT RESPONSE** The response shows complete understanding of the problem's essential mathematical concepts. The student executes procedures completely and gives relevant responses to all parts of the task. The response contains few minor errors, if any. The response contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made. **2 - POINT RESPONSE** The response shows nearly complete understanding of the problem's essential mathematical concepts. The student executes nearly all procedures and gives relevant responses to most parts of the task. The response may have minor errors. The explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences.	PERFORMANCE TASK (S): **SAMPLE MULTIPLE CHOICE RESPONSE ITEM (WWW.STATE.NJ.US)** Which of the following figures has the same fractional part of the area shaded as the sample below?  A.  B.  C.  D. 

1 - POINT RESPONSE

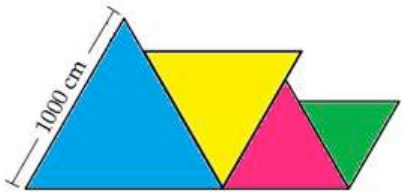
The response shows limited understanding of the problem's essential mathematical concepts. The response and procedures may be incomplete and/or may contain major errors. An incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made.

0 - POINT RESPONSE

The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.

SAMPLE SHORT CONSTRUCTED RESPONSE ITEM (ADAPTED FROM: FCAT Released Test Item)

- John is painting a mural using equilateral triangles. The first triangles sides measure 1000 cm each. The sides of each subsequent triangle will measure 20% smaller than the previous triangle. What will be the length of the fourth triangle's sides? Show all your work!



- In the compound interest formula $A = P(1 + r/n)^{nt}$, A represents the value of the investment in the future, P is the amount of the original investment, r is the annual interest rate, t is the number of years of the investment, and n is the number of times the interest is compounded each year.

You have $2500 to invest in an account.
→ Account A offers a timeframe to be invested for 18 years at a rate of 6%, compounded quarterly.
→ Account B offers a timeframe to be invested for 10 years at a rate of 6%, compounded quarterly.
→ Account C offers a timeframe to be invested for 10 years at a rate of 10%, compounded quarterly.
→ Account D offers a timeframe to be invested for 10 years at a rate of 10%, compounded annually.

Which account do you feel would be the most beneficial? What change (time, rate, or frequency) has the most significant impact on the outcome?

(Adapted from www.state.nj.us by Jaclyn E. Varacallo)

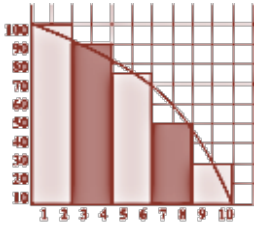
SAMPLE OPEN ENDED RESPONSE (NJDOE TEST SPECIFICATIONS LINK)

Every Tuesday, at the Dog Deli, the manager gives away free hot dogs and soda. Every sixth

customer gets a free soda, and every eighth customer gets a free hot dog. The Dog Deli served 73 customers last Tuesday.

- How many free sodas did the Dog Deli give away last Tuesday? How many hot dogs were given away?
- Did any customers receive both a free hot dog and a free soda? If so, how many customers?
- If a soda sells for 99¢ and a hot dog sells for $1.99, how much did the Dog Deli lose in income by giving away these items?

SAMPLE OPEN ENDED RESPONSE (NJDOE TEST SPECIFICATIONS LINK)



- Find the length, width, and area of each of the 5 shaded rectangles.
- What is the total area represented by the 5 rectangles?
- How do you think the area of the 5 rectangles compares to the area of the region under the curve? Explain your reasoning.
  Justify your answers.

SAMPLE OPEN ENDED RESPONSE (NJDOE TEST SPECIFICATIONS LINK)

Monthly Salary Options

Plan A: $1000 + 5% of monthly sales
Plan B: 10% of monthly sales

Sales for the prior salesperson were:

January: $40,000

	February: $30,000 **March:** $50,000 **April:** 60,000 Based on this information, a. Which salary option would Bob select if he decides to accept the job? Explain your answer **SAMPLE SHORT CONSTRUCTED RESPONSE** Kira and her brother drove to their grandparent's house. Kira drove for an hour at 40 MPH and her brother drove for 3 hours at 55 MPH. Find the average speed for the entire trip? **SAMPLE SHORT CONSTRUCTED RESPONSE** Sam paid $95.20 in sales tax on a TV purchase. If the sales tax rate was 7%, what was the original price of the TV?
Student Responses should be: ➢ ***Accurate*** ➢ ***Clear*** ➢ ***Effective*** ➢ ***Organized*** ➢ ***Thorough*** ➢ ***Thoughtful***	OTHER EVIDENCE: ***Students will show they have achieved Stage 1 goals by . . .*** • Providing written or oral response to one of the essential questions. • Developing a journal of Rules for Success. The students will keep an ongoing journal all year of accumulating insight about which rules and properties. Include examples that show the rule or property correctly applied, as well as common mistakes. • Passing all quizzes and tests relating to the unit. • Braingenie.com ➢ **Diagnostic/Pre – Assessment:** ✓ Pre-test (5-10 open ended questions) will be given covering multiple concepts. ➢ **Open-Ended (Formative) Assessment:** ✓ Homework is assigned daily, from the textbook, Chapter Resource Practice

	Workbook, or other sources. *(Synthesis, Analysis, Evaluation)* ✓ Introductory and Closing Activities will be done every day to pre-assess student knowledge and assess understanding of topics.*(Synthesis, Analysis, Evaluation)* ✓ Excerpts from previous HSPA exams including multiple choice and open-ended problems should be given every class to assess student understanding and measure their individual skills. ➢ **Summative Assessment:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Synthesis, Analysis, Evaluation)* ✓ Students will be given quizzes that provide a brief review of the concepts and skills in the previous lessons. ✓ Students will be given a unit test that provides a review of the concepts and skills in the unit. ✓ Students will take the HSPA ✓ Students will work on Performance Assessment process similar to the ASHA process

Unit I – Equivalence
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

- *Throughout the unit teacher made open-ended performance assessment tasks (PATs) will be used as instructional materials with a stress on communicating reasoning and justification for solutions through well reasoned and well written explanations.*
- ***Whole group instruction*** *to introduce new topics*
- ***Small group instruction*** *(based on student needs as identified by ongoing formative assessments)*
- ***Individual instruction*** *(As needed based on student needs and formative assessment results)*

TECHNOLOGY USE *in daily teaching as formative assessment tool and as independent practice tool for practicing and deepening knowledge*

- ***Class Wiki*** *for distributing materials and resources to students*
- ***Using iPads:*** ***www.socrative.com*** *(for formative assessments and classroom surveys)*
- ***Using iPads:*** ***www.****Braingenie.com*
- ***Using iPads:*** ***www.****Interactmath.com*
- ***Using Internet:*** *www.Khanacademy.org*

LESSON ACTIVITY: To Use properties of rational and irrational numbers

- *This lesson is designed to assess how well students are able to distinguish between rational and irrational numbers and teaching how to move*

between different representations of rational and irrational numbers
    - *Students will be able to construct viable arguments*
    - *Critique the reasoning of their classmates*

LESSON ACTIVITY: A formative lesson designed to assess
- *How well students can translate between repeating decimals and fractions*
- *Using linear equations to find fractional equivalents of repeating decimals*
- *Understand how multiplying a decimal with powers of ten changes the decimal*

LESSON ACTIVITY: Mathematical goals of the lesson
- *Use estimation for finding approximate lengths of everyday objects*
- *Convert between scientific notation and decimal notation*
- *Compare numbers expresses in scientific and decimal notation*

LESSON ACTIVITY: Increasing and decreasing quantities by a percent
- *This lesson unit is designed to assess student's ability to interpret percent increase and decrease and to teach and clarify the following:*
    - *Translating between fractions, percents, and decimals*
    - *To use multiplication to present percent increase and decrease*
    - *To recognize the relationship between increases and decreases*

Discuss multiple strategies to solve a given problem
- **Concept applications**: Attendance *(created by Dr. M. K. Sran)*
- **Concept applications**: Taxes and deductions *(created by Dr. M. K. Sran)*
- **Concept applications:** Commission *(created by Dr. M. K. Sran)*
- **Concept applications:** Car Maintenance *(created by Dr. M. K. Sran)*
- **Released PAT:** Keeping Records at the Gas Station
- **Released PAT**: Pizza Party
- **Released PAT:** Credit Card Calculations

Unit II – Transformation
Stage 1 Desired Results

ESTABLISHED GOALS

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- **G-SRT.1.**

Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

- **G-SRT.2.**

Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

- **G-SRT.3.**

Use the properties of similarity transformations

Transfer

Students will be able to independently use their learning to…

- Solve problems by classifying geometric figures, simplifying the problem, using appropriate statements based on the properties of the figures and algebraic concepts.
- Analyze when any rule in any system (language, law, math) is an essential principle or merely conventional.

Meaning

UNDERSTANDINGS

Students will understand that…

- The effect of change in the dimensions of an object on its surface area, volume, and perimeter can be expressed mathematically

ESSENTIAL QUESTIONS

1. How can properties of three-dimensional objects be understood using two-dimensional representations?
2. How can geometric models be used to solve real-life problems?
3. How can you explain the impact of change in an object's dimensions on its surface area, volume, and perimeter?

Acquisition

Students will know…

- Two points are needed to make a line and it extends in either direction indefinitely
- A ray has a starting point and direction in which it extends indefinitely
- An angle has a vertex and two sides
- A polygon is a closed figure with 3 or more sides that are segments
- Vertex is appoint where the sides of a polygon or the edges of a solid meet
- A plane is a flat surface with no thickness that extends indefinitely in all directions
- A circle is a set of points in a plane that are equidistant from a center point

Students will be skilled at…

- Using properties, definitions, and relationships to identify, classify, and describe two-dimensional and three-dimensional geometric figures
- Drawing two-dimensional representations of three-dimensional objects by sketching shadows, projections, perspectives, and map views
- Recognizing, identifying, and describing geometric relationships and properties as they exist in nature, art, and other real-world settings
- Applying concepts of symmetry, similarity,

to establish the AA criterion for two triangles to be similar. ▪ **G-SRT.4.** Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. ▪ **G-SRT.5.** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. ▪ **G-GPE.5.** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). ▪ **G-GPE.6.** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. ▪ **G-GMD.4.** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. ▪ **G-MG.3.** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*	▪ The symbol for ray AB is $\overrightarrow{AB}$ ▪ The symbol for line AB is $\overleftrightarrow{AB}$ ▪ The symbol for angle ABC is $\angle ABC \text{ or } \measuredangle ABC \text{ or } \sphericalangle ABC$ ▪ Symbol for perpendicular lines is given by $\perp$ ▪ Squares have four right angles and congruent sides ▪ The sum of all angles of a ▪ Quadrilateral equals 360 degrees ▪ Three angles of a triangle add up to 180 degrees ▪ All sides and angles are congruent in an equilateral triangle ▪ Two consecutive angles in a parallelogram are supplementary ▪ A trapezoid has exactly two sides parallel ▪ Right triangle has a 90 degree angle ▪ Isosceles triangle has two congruent legs and angles ▪ 2 is a parallelogram with congruent sides ▪ Two parallel lines cut by a transversal form corresponding angles ▪ Two intersecting lines form two pairs of vertical angles ▪ Congruent figures have same shape(same corresponding angles) and size ▪ Similar figures have same shape and all corresponding sides in the same ratio. ▪ Dilation is a transformation in which a figure is proportionally made smaller or larger ▪ The new image after performing a transformation is called an image where as the original is called the preimage ▪ The line of reflection is a line in which a figure is flipped in a reflection ▪ The point of rotation is the point around which a figure is turned	and congruence to problem solving ▪ Recognizing and using symbols appropriately ▪ Using properties of perpendicular lines to solve problems ▪ Using inductive and deductive reasoning to solve real-life problems and justify solutions ▪ Using properties, definitions, and relationships for identifying, classifying, and describing two-dimensional and three-dimensional geometric figures ▪ Recognizing, identifying, and describing geometric relationships and properties that exist in real-world settings ▪ Analyzing properties of three-dimensional geometric figures by using models and by drawing and interpreting two-dimensional representations of them ▪ Applying concepts of symmetry, similarity, and congruence to problem solving ▪ Solving real-world and mathematical problems using geometric models ▪ Finding the image and vice versa when given the pre-image and transformation, ▪ Determining the transformation given the pre-image & image ▪ Determining the sequence of transformations needed to map one figure onto another ▪ Solving problems in geometry using transformations, coordinates, and vectors ▪ Relating the concepts of symmetry, similarity, and congruence to transformations ▪ Predicting and representing resulting figures when combining, subdividing, and changing figure ▪ Recognizing regular and semi-regular tessellations

▪ **N-VM.1.** (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, v). ▪ **N-VM.2.** (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. ▪ **N-VM.3.** (+) Solve problems involving velocity and other quantities that can be represented by vectors. ▪ **N-VM.6.** (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. ▪ **N-VM.7.** (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **Modeling** Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).	▪ Transformation is a way of changing the position or size of a geometric figure without altering the shape ▪ Translation is a form of transformation in which a figure is slid horizontally, vertically, or both ▪ Tessellation results from the use of congruent geometric figures to cover a plane without overlapping and without gaps ▪ A regular tessellation is made up of one type of polygon ▪ A semi-regular tessellation uses more than one type of regular polygon ▪ In a tessellation, the angles of polygons at a vertex add up to $360°$ ▪ Matrix is a rectangular array of numbers ▪ The dimensions of matrix are given by $n \times m$ the number of rows (n) by the number of columns (m) ▪ An individual entry in a matrix is called an elements ▪ Vectors are used to represent motion ▪ A vector is a directed line segment that is shown with an arrow ▪ A vector has both length and direction ▪ The length of a vector can be found using the distance formula ▪ Equivalent vectors have same direction and same magnitude ▪ Two vectors are opposite if they have the same length but opposite direction	▪ Creating an original tessellation ▪ Drawing a figure & tessellating it ▪ Solving problems in geometry using transformations, coordinates, and vectors ▪ Performing scalar multiplication on matrices

Unit II – Transformation
Stage 2 - Evidence

Evaluative Criteria	Assessment Evidence

RUBRIC/SCALE

3 - POINT RESPONSE
The response shows complete understanding of the problem's essential mathematical concepts. The student executes procedures completely and gives relevant responses to all parts of the task. The response contains few minor errors, if any. The response contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made.

2 - POINT RESPONSE
The response shows nearly complete understanding of the problem's essential mathematical concepts. The student executes nearly all procedures and gives relevant responses to most parts of the task. The response may have minor errors. The explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences.

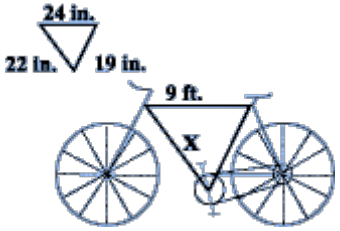
1 - POINT RESPONSE
The response shows limited understanding of the problem's essential mathematical concepts. The response and procedures may be incomplete and/or may contain major errors. An incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made.

0 - POINT RESPONSE
The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to

PERFORMANCE TASK (S):

You are a billboard designer. Your job is to enlarge this picture of a bicycle to fit on a large outdoor sign.

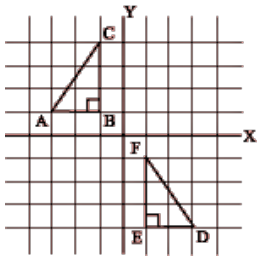
The actual dimensions of the seat-pedal-handle bar triangle are shown below. The bar connecting the seat with the handle bars, for example, is 24 inches long. The corresponding part on the billboard would be 9 feet long.



Find the missing billboard length, ***x***, as shown. *(www.state.nj.us)*

SAMPLE SHORT CONSTRUCTED RESPONSE

What transformation will map triangle ABC to triangle DEF ?



understand how and why decisions were made.

SAMPLE SHORT CONSTRUCTED RESPONSE

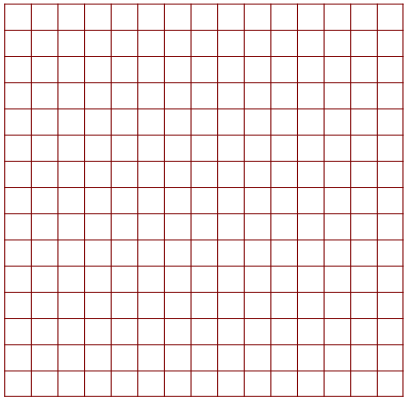
Given the following isometric drawing:

Sketch the top, side, and front view of the solid figure.

SAMPLE OPEN ENDED RESPONSE

A boat starts at point A and travels 8 miles east, and then turns south and travels 6 miles to a point B located on the shore of the lake.

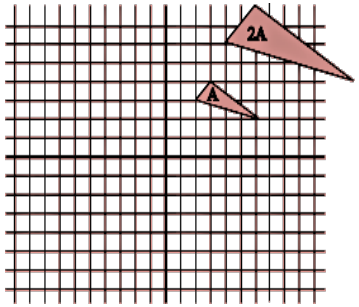
- Using the grid paper, make a scale drawing using vectors to show the boat's movement, starting from point A.
- Draw a vector that would show the direct path from point A to point B.
- What would be the approximate number of miles the boat could have traveled along this path?
- Approximately how many degrees from North would this path be? Explain how you arrived at your answer.



SAMPLE OPEN-ENDED RESPONSE (NJDOE TEST SPECIFICATIONS LINK)

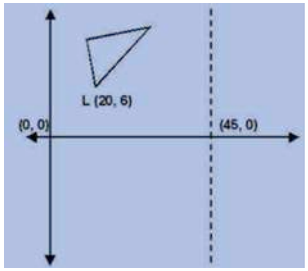
Figure A below is the original. Figure 2A is an expansion of A with magnitude 2.

- Draw a figure with a magnitude of 1/2A.
- Draw figure -A by multiplying the coordinates of points on A by a negative 1 (the multiplication by negatives reverses directions).
- Give the coordinates of the vertices of triangle -A.



SAMPLE SHORT CONSTRUCTED RESPONSE (WWW.STATE.NJ.US)

A graphic artist designing a company's logo wants to reflect triangle LMN across the dotted line to form triangle PQR. What will the vertex of point P be?



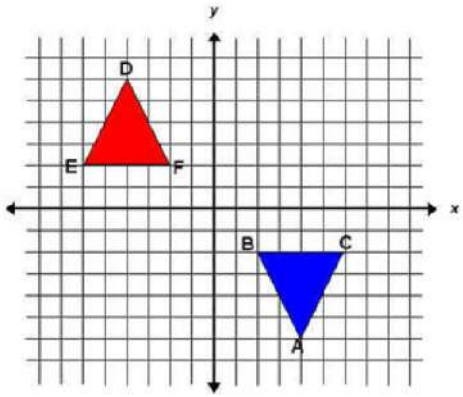
SAMPLE MULTIPLE CHOICE (NJDOE TEST SPECIFICATIONS LINK)

If a tessellation is made from regular polygons, what is the sum of the measures of the angles that meet at a vertex if the polygons do not overlap?

a. $60°$
b. $90°$
c. $180°$
d. $360°$

SAMPLE MULTIPLE CHOICE (WWW.STATE.NJ.US)

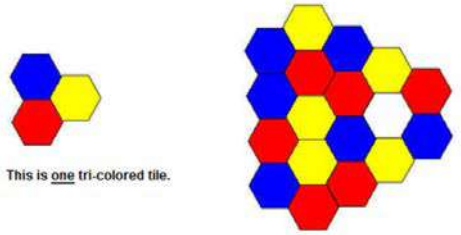
What must happen to triangle ABC to create triangle DEF?



A. Triangle ABC is reflected over X-axis and translated 8 spaces to the left and 4 spaces up
B. Triangle ABC is reflected over Y-axis and translated 4 spaces up.
C. Triangle ABC is reflected over Y-axis and translated 8 spaces to the left
D. Triangle ABC is reflected over X-axis and translated 8 spaces to the left

SAMPLE MULTIPLE CHOICE (WWW.STATE.NJ.US)

A stone mason is repaving a school's courtyard with the following 3-color tiles. Given that he is only using these tiles, what color will the white stone in the large area be?



A. The white space may be any of the three colors
B. The white space should be blue
C. The white space should be red
D. The white space should be yellow

SAMPLE MULTIPLE CHOICE (NJDOE TEST SPECIFICATIONS LINK)

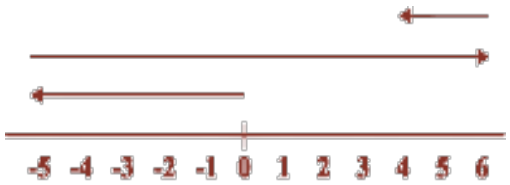
The given vector diagram represents an airplane flying with an air speed of 200 mph directly into a headwind of 30 mph.

What resulting speed and direction of the plane does the vector AC represent?

a. 230 mph East
b. 170 mph East
c. 230 mph West
d. 170 mph West

SAMPLE MULTIPLE CHOICE (NJDOE TEST SPECIFICATIONS LINK)

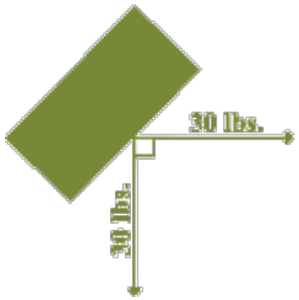
The vector diagram below models which addition problem? (Assume the origin is the starting point.)



a. $(-5)+11+(-2)$*
b. $(-5)+6+4$
c. $5+(-11)+2$
d. $5+(-6)+(-4)$

SAMPLE OPEN-ENDED RESPONSE (NJDOE TEST SPECIFICATIONS LINK)

You may use the grid, a ruler, and protractor to solve this problem

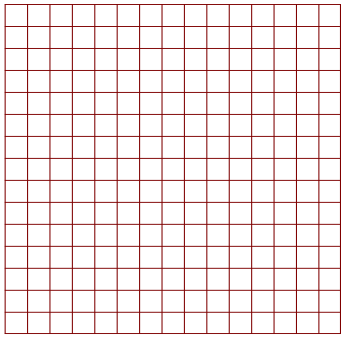


Juan and Carlos need to move a heavy crate which is on a loading platform. The ropes which they attach to it form a right angle. Each boy pulls on his rope with a force of 30 lb.

Make a vector drawing to present the forces pulling on the crate and the resulting path along which the crate would move.

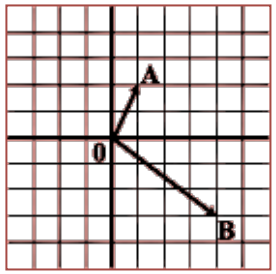
Juan and Carlos know that a single force could have the same result in moving the crate? Approximately how many pounds would this force be, and in what direction? Explain how

you arrived at your answer.



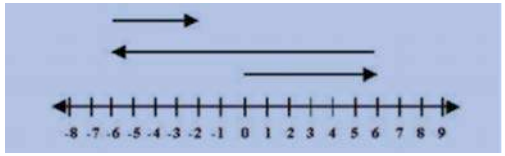
SAMPLE MULTIPLE CHOICE (NJDOE TEST SPECIFICATIONS LINK)

Let vector OA be represented by the ordered pair (1, 2). If vector OB is represented by (4, -3), what ordered pair represents vector AB?



Four answer choices given.

a. (-3.5)
b. $(5,-3)$
c. $(-5,3)$
d. $(3,-5)$*

	SAMPLE MULTIPLE CHOICE (WWW.STATE.NJ.US) Which addition problem is represented by the following vector diagram? (Assume that the origin is the starting point.  a. $6+(-12)+4$ b. $6+(-6)+(-2)$ c. $6+(-12)+(-2)$ d. $6+(-6)+4$
Student Responses should be: ➢ ***Accurate*** ➢ ***Clear*** ➢ ***Effective*** ➢ ***Organized*** ➢ ***Thorough*** ➢ ***Thoughtful***	OTHER EVIDENCE: ***Students will show they have achieved Stage 1 goals by . . .*** • Providing written or oral response to one of the essential questions. • Developing a journal of Rules for Success. The students will keep an ongoing journal all year of accumulating insight about which rules and properties. Include examples that show the rule or property correctly applied, as well as common mistakes. • Passing all quizzes and tests relating to the unit. • Braingenie.com ➢ **Diagnostic/Pre – Assessment:** ✓ Pre-test (5-10 open ended questions) will be given covering multiple concepts. ➢ **Open-Ended (Formative) Assessment:** ✓ Homework is assigned daily, from the textbook, Chapter Resource Practice Workbook, or other sources. *(Synthesis, Analysis, Evaluation)* ✓ Introductory and Closing Activities will be done every day to pre-assess student knowledge and assess understanding of topics.*(Synthesis, Analysis, Evaluation)* ✓ Excerpts from previous HSPA exams including multiple choice and open-ended

problems should be given every class to assess student understanding and measure their individual skills.

- **Summative Assessment:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Synthesis, Analysis, Evaluation)*
  - ✓ Students will be given quizzes that provide a brief review of the concepts and skills in the previous lessons.
  - ✓ Students will be given a unit test that provides a review of the concepts and skills in the unit.
  - ✓ Students will take the HSPA
  - ✓ Students will work on Performance Assessment process similar to the ASHA process

Unit II – Transformation
Stage 3 – Learning Plan

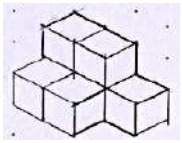
Summary of Key Learning Events and Instruction

- *Throughout the unit teacher made open-ended performance assessment tasks (PATs) will be used as instructional materials with a stress on communicating reasoning and justification for solutions through well reasoned and well written explanations.*
- ***Whole group** instruction to introduce new topics*
- ***Small group instruction** (based on student needs as identified by ongoing formative assessments)*
- ***Individual instruction** (As needed based on student needs and formative assessment results)*

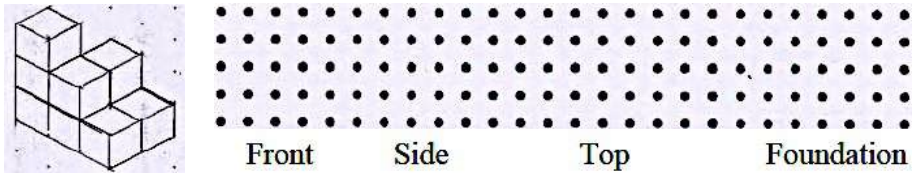
- ***Transformations** (Adapted from http://www.doe.state.la.us/topics/comprehensive_curriculum.html)*
  - ✓ ***Activity 1:Translations***
    - *Start with a given triangle and record the coordinates for all three vertices*
    - *Each group of students will translate the triangles x-units along the x-axis and y-units along the y axis*
    - *Record the new vertices of the translated triangle*
    - *Describe the effect of the translation on the coordinates of the vertices*
    - *Draw conclusions based on the data from all groups during class discussion*
    - *Does this "rule" apply to other geometric shapes? Explain.*
  - ✓ ***Activity 2: Reflections (Students will use a transparency)***
    - *Start with a triangle with its vertices labeled with ordered pairs*
    - *Reflect the triangle over the y-axis and record the new ordered pars for the vertices*
    - *Return the triangle to original position and then reflect over the line x=1 and record the new vertices*
    - *Draw and test a conclusion about the rule that can be used to reflect a polygon over a vertical line or y- axis*
    - *What would change if the figure is reflected over a horizontal line or the x-axis*
    - *What happens if the polygon is not a triangle?*

- ✓ ***Activity 3: Rotations (Use a table to record your results)***
  - *Start with a new triangle and record the vertices*
  - *Rotate the triangle 90 degrees, 180 degrees, 270 degrees respectively and recording the new vertices each time*
  - *Can you see a pattern in the coordinates of these triangles? If so what conclusions can you make?*
  - *How will a rotation of 360 degrees affect the coordinates of the vertices? Explain.*
  - *Does your rule hold for other shapes?*
  - *Students will share and justify their "rules" and agree on a single rule as a group*

➢ ***How Many blocks?*** *(Adapted from http://www.doe.state.la.us/topics/comprehensive_curriculum.html)*
- ✓ *Given a three dimensional drawing students will use spatial sense to identify the number of blocks in each diagram*
  - *Example:*



➢ ***Orthographic Drawing*** *(Adapted from http://www.doe.state.la.us/topics/comprehensive_curriculum.html)*
- ✓ *Given various 3D drawing students will draw three primary views and the foundational drawing of each object*
  - ***Front view***
  - ***Side view***
  - ***Top view***
  - ***Foundation***
- ✓ ***EXAMPLE:***



➢ ***Isometric Drawings:***
- o *Given the three views draw the 3D image*

TECHNOLOGY USE *in daily teaching as formative assessment tool and as independent practice tool for practicing and deepening knowledge*
- ➢ ***Class Wiki*** *for distributing materials and resources to students*
- ➢ ***Using iPads: www.socrative.com*** *(for formative assessments and classroom surveys)*
- ➢ ***Using iPads: www.****Braingenie.com*
- ➢ ***Using iPads: www.****Interactmath.com*
- ➢ ***Using Internet:*** *www.Khanacademy.org*

▪ ***Discuss multiple strategies to solve a given problem***

- ***Concept Application:*** *Location (Created by Dr. M. K. Sran)*
- ***Concept Application:*** *Vectors (Created by Dr. M. K. Sran)*
- ***Concept Application:*** *Going Places (Created by Dr. M. K. Sran)*

Unit III – Measurement
Stage 1 Desired Results

ESTABLISHED GOALS

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- **G-SRT.6.**

Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

- **G-SRT.7.**

Explain and use the relationship between the sine and cosine of complementary angles.

- **G-SRT.8.**

Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems*

- **G-C.2.**

Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius

Transfer

Students will be able to independently use their learning to…

1. Analyze when any rule in any system (language, law, math) is an essential principle or merely conventional.
2. Acquire an understanding of the concepts to apply to a variety of real-world learning experiences in new, diverse situations.

Meaning

UNDERSTANDINGS

Students will understand that…

- The properties of a geometric figure are dependent on the number of sides in the figure.
- Area, perimeter, surface area, and volume are interrelated with each other and with the properties of a geometric figure.

ESSENTIAL QUESTIONS

1. How can properties of three-dimensional objects be understood using two-dimensional representations?
2. How can geometric models be used to solve real-life problems?
3. How can you explain the impact of change in an object's dimensions on its surface area, volume, and perimeter?
4. Does this **make sense**?

Acquisition

Students will know…

- Area is the measure, in square units, of the inside of a two-dimensional surface or shape of a three-dimensional object
- Area of a parallelogram is given by the formula: $A = bh$
- Area of a triangle is given by the formula: $A = \frac{1}{2}bh$
- Area of a trapezoid is given by the formula: $A = \frac{1}{2}h\left(b_1 + b_2\right)$
- Area of a circle is given by the formula: $A = \pi r^2$

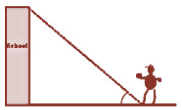
Students will be skilled at…

- Utilizing appropriate formulas and label answers with appropriate units of measure
- Measuring geometric objects and determine the degree of accuracy needed when measuring them
- Choosing the appropriate techniques, tools, and units to measure quantities to achieve the desired level of accuracy
- Developing and applying a variety of strategies for determining perimeter, circumference, area, surface area, volume, and angle measures

intersects the circle. ▪ **G-C.5.** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. ▪ **G-GPE.4.** Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2). ▪ **G-GPE.7.** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.* ▪ **G-GMD.3.** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems* ▪ **G-MG.1.** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder)* ▪ **G-MG.2.** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot)* ▪ **G-MG.3.** Apply geometric methods to solve design	▪ Volume represents the amount of space enclosed by a three-dimensional shape ▪ Volume of a cone is given by the formula: $V = \frac{1}{3}Bh$ where B is the area of the base and h is the height of the solid ▪ Volume of a pyramid is given by the formula: $V = \frac{1}{3}Bh$ where B is the area of the base and h is the height of the solid ▪ Volume of a cylinder is given by the formula: $V = Bh$ *or* $\pi r^2 h$ where B is the area of the base and h is the height of the solid ▪ Volume of a prism is given by the formula: $V = Bh$ where B is the area of the base and h is the height of the solid ▪ Volume of a sphere is given by the formula: $V = \frac{4}{3}\pi r^3$ where r is the radius ▪ All non-zero digits are always significant ▪ Zeros between two significant digits are significant (e.g. 5.03 has three significant digits) ▪ Zeros to the right of the decimal point and a significant digit are significant (e.g. 0.0030 has two significant digits) ▪ Placeholders are not significant (e.g. 66,000 has two significant digits) ▪ Triangle Inequality Theorem states that the sum of the lengths of two sides of any triangle is greater than the length of the third side ▪ Mid-segment of a triangle connects the midpoints of two sides of a triangle and is parallel to the third side ▪ Hypotenuse is the side of a right triangle that is opposite the right angle ▪ Leg is the side of the right triangle adjacent to	▪ Solving problems using the Pythagorean Theorem ▪ Developing informal ways of approximating the measures of familiar objects ▪ Expressing mathematically and explaining the impact of change in an object's dimensions on its surface area, volume, and/or perimeter ▪ Measuring geometric objects and determining the degree of accuracy needed when measuring them ▪ Utilizing appropriate formulas and labeling answers with appropriate units of measure ▪ Correctly identifying all parts of a right triangle ▪ Solving problems using Pythagorean Theorem ▪ Using basic trigonometric ratios to solve problems involving indirect measurement ▪ Developing and applying a variety of strategies for determining perimeter, circumference, area, surface area, and angle measure

problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).* **Modeling** Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).	the hypotenuse ▪ The legs of a right triangle are always perpendicular to each other ▪ Pythagorean Theorem states that the sum of the squares of the lengths of two legs is equal to the square of the hypotenuse ▪ Trigonometric ratios are the ratios made up of the lengths of the sides of the right triangle ▪ Sine Ratio is the ratio of the length of the side opposite an acute angle to the length of the hypotenuse in a given right triangle ▪ Cosine Ratio is the ratio of the length of the side adjacent to an acute angle to the length of the hypotenuse in a given right triangle ▪ Tangent Ratio is the ratio of the length of the side opposite an acute angle to the length of the adjacent side in a given right triangle	

Unit III – Measurement
Stage 2 - Evidence

Evaluative Criteria	Assessment Evidence
RUBRIC/SCALE **3 - POINT RESPONSE** The response shows complete understanding of the problem's essential mathematical concepts. The student executes procedures completely and gives relevant responses to all parts of the task. The response contains few minor errors, if any. The response contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made. **2 - POINT RESPONSE** The response shows nearly complete understanding of the problem's essential mathematical concepts. The student executes	PERFORMANCE TASK (S): **SAMPLE OPEN-ENDED RESPONSE ITEM** (NJDOE TEST SPECIFICATIONS LINK) Describe in detail how you could use a calculator with trigonometric functions to help find the height of your school building if you knew the distance you were standing from the building and the angle from the ground at your feet to the top of the building.  **SAMPLE SHORT CONSTRUCTED RESPONSE ITEM** (NJDOE TEST SPECIFICATIONS

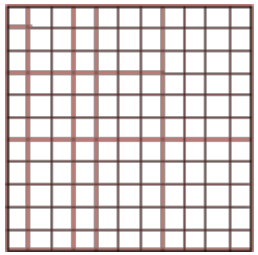
nearly all procedures and gives relevant responses to most parts of the task. The response may have minor errors. The explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences.

1 - POINT RESPONSE
The response shows limited understanding of the problem's essential mathematical concepts. The response and procedures may be incomplete and/or may contain major errors. An incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made.

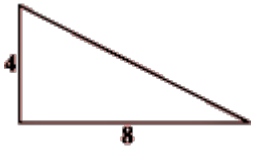
0 - POINT RESPONSE
The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.

LINK)

On the unit-grid in your answer folder, draw a right triangle with an area of 16 square units.



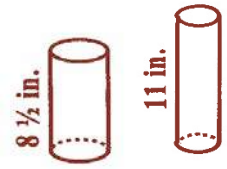
One possible answer:



SAMPLE OPEN_ENDED RESPONSE ITEM (NJDOE TEST SPECIFICATIONS LINK)

A standard 8 1/2" x 11" sheet of paper is rolled along its short side to form a cylinder as shown.

A second sheet of standard 8 1/2" x 11" paper is rolled along its longer side to form a second cylinder. There is no overlap.

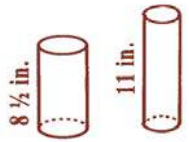


- Will the taller cylinder have the name volume, greater volume, or less volume than that of the short cylinder?
- Explain your answer

SAMPLE MULTIPLE CHOICE RESPONSE (NJDOE TEST SPECIFICATIONS LINK)

A standard 8 1/2" x 11" sheet of paper is rolled along its short side to form a cylinder as shown.

A second sheet of standard 8 1/2" x 11'' paper is rolled along its longer side to form a second cylinder.
There is no overlap.

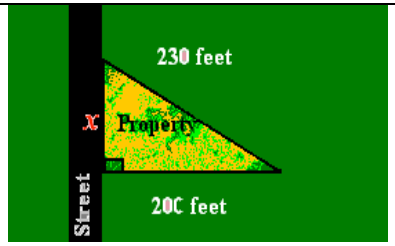


How will the volumes of the two cylinders be related?

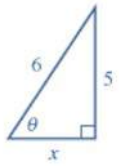
a. The shorter cylinder will have the greater volume.
b. The taller cylinder will have the greater volume.
c. The two cylinders will have the same volume, but the shorter cylinder will have the greater surface area.
d. The two cylinders will have the same volume, but the taller cylinder will have the greater surface area.

SAMPLE SHORT CONSTRUCTED RESPONSE ITEM

You are surveying a triangular-shaped piece of land. You have measured and recorded two lengths on a plot plan. What is the length of the property along the street?



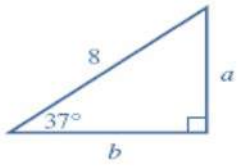
SAMPLE SHORT CONSTRUCTED RESPONSE ITEM



- Find the length of the missing side
- What is the measure of the acute angle?
- Find all six trigonometric ratios

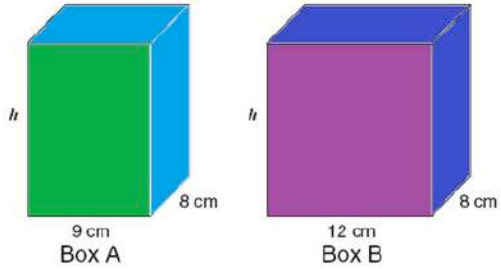
SAMPLE SHORT CONSTRUCTED RESPONSE ITEM

Find the lengths of the two missing sides and show all your work.



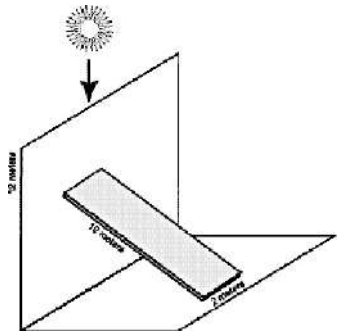
SAMPLE SHORT CONSTRUCTED RESPONSE ITEM (ADAPTED FROM: FCAT Released Test Item)

Two rectangular boxes have same height and length, but different width as shown in the figure below. The difference in the volume of box B and box A is 360 cubic cms. What is the height of both boxes in cms?



SAMPLE OPEN_ENDED RESPONSE ITEM (NJDOE TEST SPECIFICATIONS LINK

A rectangular board that measures 2 m wide and 10 m long is leaning against a wall, as shown below. The sun is shining directly above the board.



A. Sketch the shape of the shadow that the board makes on the ground.
B. If the sun's rays are vertical, describe how moving the bottom edge of the board closer to the wall would affect the width and length of the shadow on the ground.
C. How close to the wall should the bottom edge of the board be positioned so that the shadow of the board forms a square? Explain.
D. Find the height at which the top of the board touches the wall when the shadow on the ground is a square. Explain how you found your answer.

Student Responses should be:

- ***Accurate***
- ***Clear***

OTHER EVIDENCE:

➢ ***Effective*** ➢ ***Organized*** ➢ ***Thorough*** ➢ ***Thoughtful***	***Students will show they have achieved Stage 1 goals by . . .*** • Providing written or oral response to one of the essential questions. • Developing a journal of Rules for Success. The students will keep an ongoing journal all year of accumulating insight about which rules and properties. Include examples that show the rule or property correctly applied, as well as common mistakes. • Passing all quizzes and tests relating to the unit. • Braingenie.com **Diagnostic/Pre – Assessment:** ✓ Pre-test (5-10 open ended questions) will be given covering multiple concepts. **Open-Ended (Formative) Assessment:** ✓ Homework is assigned daily, from the textbook, Chapter Resource Practice Workbook, or other sources. *(Synthesis, Analysis, Evaluation)* ✓ Introductory and Closing Activities will be done every day to pre-assess student knowledge and assess understanding of topics.*(Synthesis, Analysis, Evaluation)* ✓ Excerpts from previous HSPA exams including multiple choice and open-ended problems should be given every class to assess student understanding and measure their individual skills. **Summative Assessment:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Synthesis, Analysis, Evaluation)* ✓ Students will be given quizzes that provide a brief review of the concepts and skills in the previous lessons. ✓ Students will be given a unit test that provides a review of the concepts and skills in the unit. ✓ Students will take the HSPA ✓ Students will work on Performance Assessment process similar to the ASHA process

Unit III – Measurement
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

- *Throughout the unit teacher made open-ended performance assessment tasks (PATs) will be used as instructional materials with a stress on communicating reasoning and justification for solutions through well reasoned and well written explanations.*
- ***Whole group instruction*** *to introduce new topics*
- ***Small group instruction*** *(based on student needs as identified by ongoing formative assessments)*

- ***Individual instruction*** *(As needed based on student needs and formative assessment results)*

- **Group Activity: Right Triangle Trigonometry** *(Adapted from http://www.doe.state.la.us/topics/comprehensive_curriculum.html)*

    - *Students will solve problems related to building and construction using trigonometry*
        - *To make decisions about building stairs to reach second floor given the angle and height to the second floor*
        - *Determine the length and height of the roof given the angle the roof makes with the ceiling beam*
        - *Given the slope length and the base angle of a concrete trapezoidal blocks used for guardrails during construction, find its height*

- ***Activity: Exploring surface area, and volume of solids*** *(Adapted from http://www.doe.state.la.us/topics/comprehensive_curriculum.html)*
    - ***Rectangular prisms***
    - ***Cylinders***
    - ***Pyramids***
    - ***Cones***

TECHNOLOGY USE *in daily teaching as formative assessment tool and as independent practice tool for practicing and deepening knowledge*

- ***Class Wiki*** *for distributing materials and resources to students*
- ***Using iPads: www.socrative.com*** *(for formative assessments and classroom surveys)*
- ***Using iPads: www.****Braingenie.com*
- ***Using iPads: www.****Interactmath.com*
- ***Using Internet:*** *www.Khanacademy.org*

Discuss multiple strategies to solve a given problem and have students use the state rubric to grade the responses for open-ended questions
Concept applications: *Gardening choices (Created by Dr. M. K. Sran)*
Concept applications: *Draining the pool (Created by Dr. M. K. Sran)*
Concept applications: *Container (Created by Dr. M. K. Sran)*
Concept applications: *House Pets (Created by Dr. M. K. Sran)*
Concept applications: *Storage (Created by Dr. M. K. Sran)*
Concept applications: *Boxes and More Boxes (Created by Dr. M. K. Sran)*
Released PAT: *Sand and Salt Storage*
Released PAT*: Fencing the field*
Released PAT: *Cake Cutting*

Unit IV - Risk
Stage 1 Desired Results

ESTABLISHED GOALS

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- **S-CP.1.**

Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

- **S-CP.2.**

Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

- **S-CP.3.**

Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

Transfer

Students will be able to independently use their learning to…

- Analyze when any rule in any system (language, law, math) is an essential principle or merely conventional.
- Acquire an understanding of the concepts to apply to a variety of real-world learning experiences in new, diverse situations.

Meaning

UNDERSTANDINGS

Students will understand that…

- **Probability** is a way of **predicting** outcomes, but does not assure outcomes.
- **Risk** and **fairness** are interrelated.
- **Profit** is affected by the delicate **balance** of human perception of the odds and underlying probability of the product.

ESSENTIAL QUESTIONS

1. How are theoretical and empirical probability related?
2. How can you use probability and expected value to help determine whether a game is fair?

Acquisition

Students will know…

- **Expected Value** is the average of a probability distribution. It is the sum of the products of the outcomes of an event and their associated probabilities. $E(X) = \sum X \cdot P(X)$
- **Geometric probability** uses the concept of space and area to calculate probability of an event.
- **Mutually exclusive** events are two events that cannot occur at the same time.
- **Independent events** are two events where one event does not affect the outcome of the second event.
- **Dependent events** are two events where one event affects the outcome of the second

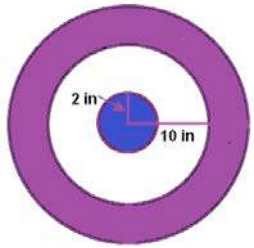
Students will be skilled at…

- Calculating the expected value of a probability-based game, given the probabilities and payoffs of the various outcomes, and determine whether the game is fair.
- Using concepts and formulas of area to calculate geometric probabilities.
- Modeling situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models, and solve problems using these models.
- Determining probabilities in complex situations such as conditional, complementary, and

▪ **S-CP.4.** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. ▪ **S-CP.5.** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. ▪ **S-CP.6.** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. ▪ **S-CP.7.** Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. ▪ **S-CP.8.** (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	event. ▪ The **"Law of Large Numbers"** states that empirical probability will approach the theoretical probability of an event after an infinite number of trials.	independent/dependent events. ▪ Estimating probabilities and making predictions based on experimental and theoretical probabilities. ▪ Understanding and using the "law of large numbers".

▪ **A-APR.5.** (+) Know and apply the Binomial Theorem for the expansion of (x + y)^ n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.		

Unit IV - Risk
Stage 2 - Evidence

Evaluative Criteria	Assessment Evidence
RUBRIC/SCALE **3 - POINT RESPONSE** The response shows complete understanding of the problem's essential mathematical concepts. The student executes procedures completely and gives relevant responses to all parts of the task. The response contains few minor errors, if any. The response contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made. **2 - POINT RESPONSE** The response shows nearly complete understanding of the problem's essential mathematical concepts. The student executes nearly all procedures and gives relevant responses to most parts of the task. The response may have minor errors. The explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences. **1 - POINT RESPONSE** The response shows limited understanding of the problem's essential mathematical concepts.	PERFORMANCE TASK (S): Create a game based on probability. Who is in favor, the player or the owner? Explain how you know. If you were to put a price tag on your game, what would it be? Do you think you would make money in the long run? Why or why not? **SAMPLE SHORT CONSTRUCTED RESPONSE ITEM** What is the probability of hitting the bull's eye on the target below?  **SAMPLE MC RESPONSE ITEM** (NJDOE TEST SPECIFICATIONS LINK) Weatherpersons predict tomorrow's weather based on what has happened in the past on the days following days just like today. During the past 50 years, there have been 380 days that have been just like today, and of those, 200 have been followed by a clear day. Which of the following is the approximate probability of a clear day tomorrow that would be given by a weatherperson using the prediction rule described in this problem?

The response and procedures may be incomplete and/or may contain major errors. An incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made.

0 - POINT RESPONSE

The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.

a. 13%
b. 34%
c. 53% *
d. 66%

SAMPLE OPEN-ENDED RESPONSE ITEM (NJDOE TEST SPECIFICATIONS LINK)

Two number cubes, numbered 1-6, are rolled.

- What sum is more likely to occur than any other?
- Explain your answer.

SAMPLE OPEN-ENDED RESPONSE ITEM (NJDOE TEST SPECIFICATIONS LINK)

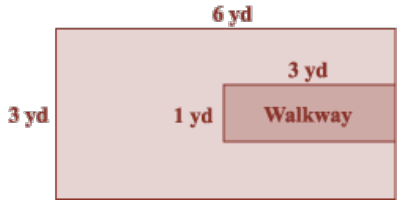
A computer simulated tossing 3 coins 400 times. The results are shown in this table.

HHH	**41**	**TTH**	**50**
HHT	**54**	**THT**	**53**
HTH	**48**	**HTT**	**45**
THH	**57**	**TTT**	**52**

- Calculate the experimental probability as shown by this simulation.
- Determine the theoretical probability of tossing 2 heads and 1 tail.
- Compare the two probabilities and explain any differences.

SAMPLE MC RESPONSE ITEM (NJDOE TEST SPECIFICATIONS LINK)

The rectangular garden shown contains a rectangular, brick walkway.

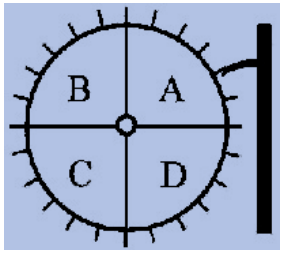


What is the probability that a seed tossed randomly into the garden will land on the walkway?

a. 1/6 *
b. 1/5
c. 1/4
d. 1/3

SAMPLE MC RESPONSE ITEM (NJDOE TEST SPECIFICATIONS LINK)

A fair spinner has four congruent regions (with equal areas) on its face. They are labeled A, B, C, and D. The spinner is spun 20 times by each member of a class of 24 students. The results are tallied (counted) and then combined



Assume that the class obtained the expected results when they conducted the experiment.

- Make a bar graph illustrating the combined class results.
- Explain why an individual student's results might be different from the class results.
- If the experiment were conducted again, with the regions A, B, C, and D having central angles of 45°, 90°, 90°, and 135° respectively, what might you expect the results to be?

SAMPLE SHORT CONSTRUCTED RESPONSE ITEM

Jack and Jill are playing a game. One option is to flip a coin 10 times and the other is to flip a coin 100 times and count the number of tails that pop up. The object of the game is to have the greater empirical probability of flipping "tails". Which option would you choose and why?

SAMPLE SHORT CONSTRUCTED RESPONSE ITEM

	What is the probability of rolling a 6 on a standard die and picking a jack of clubs? **SAMPLE SHORT CONSTRUCTED RESPONSE ITEM** A card is randomly selected from a standard deck of 52 cards. a. What is the probability that it is an ace *or* a face card? b. What is the probability that the card is a heart *or* a face card?
Student Responses should be: ➢ ***Accurate*** ➢ ***Clear*** ➢ ***Effective*** ➢ ***Organized*** ➢ ***Thorough*** ➢ ***Thoughtful***	OTHER EVIDENCE: ***Students will show they have achieved Stage 1 goals by . . .*** • Providing written or oral response to one of the essential questions. • Developing a journal of Rules for Success. The students will keep an ongoing journal all year of accumulating insight about which rules and properties. Include examples that show the rule or property correctly applied, as well as common mistakes. • Passing all quizzes and tests relating to the unit. • Braingenie.com ➢ **Diagnostic/Pre – Assessment:** ✓ Pre-test (5-10 open ended questions) will be given covering multiple concepts. ➢ **Open-Ended (Formative) Assessment:** ✓ Homework is assigned daily, from the textbook, Chapter Resource Practice Workbook, or other sources. *(Synthesis, Analysis, Evaluation)* ✓ Introductory and Closing Activities will be done every day to pre-assess student knowledge and assess understanding of topics.*(Synthesis, Analysis, Evaluation)* ✓ Excerpts from previous HSPA exams including multiple choice and open-ended problems should be given every class to assess student understanding and measure their individual skills. ➢ **Summative Assessment:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Synthesis, Analysis, Evaluation)* ✓ Students will be given quizzes that provide a brief review of the concepts and skills in the previous lessons. ✓ Students will be given a unit test that provides a review of the concepts and skills in the unit. ✓ Students will take the HSPA

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Unit IV - Risk
Stage 3 – Learning Plan

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- ***Using Internet:*** *www.Khanacademy.org*

- ***Cooperative Learning Activity*** (*McDougal Littell,* Algebra 1*, 2004, Chapter 1 Resource Books*, p.90)
- ***Activity Lesson Opener*** (*McDougal Littell,* Algebra 1*, 2004, Chapter 6 Resource Books*, p.94)
- *11.4 Activity Lesson Opener* (*McDougal Littell,* Algebra 1*, 2004, Chapter 11 Resource Books*, p.56
- *11.6 Activity Lesson Opener* (*McDougal Littell,* Algebra 1*, 2004, Chapter 11 Resource Books*, p.81)
- *11.3 Graphing Calculator Activity* (*McDougal Littell,* Algebra 1*, 2004, Chapter 11 Resource Books*, p.40)
- **Exploration Activities**:
  - ✓ *Remove One (Probability and Statistics)*
  - ✓ *Declaration of Dependence? (Probability and Statistics)*
  - ✓ *Addition Rules Discovery (Probability and Statistics)*

Unit V – Patterns
Stage 1 Desired Results

ESTABLISHED GOALS

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- **S-CP.5.**

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

- **S-CP.6.**

Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

- **S-CP.8.** (+)

Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

- **S-CP.9.** (+)

Use permutations and combinations to compute probabilities of compound events and solve

Transfer

Students will be able to independently use their learning to…

- Analyze when any rule in any system (language, law, math) is an essential principle or merely conventional.
- Acquire an understanding of the concepts to apply to a variety of real-world learning experiences in new, diverse situations.

Meaning

UNDERSTANDINGS

Students will understand that…

- **Theoretical probability** is dependent on the **sample space** of an event.

ESSENTIAL QUESTIONS

1. How can systematic listing and counting be useful in the organization of outcomes?
2. How can we utilize a small strand of information to help give us information in the grand scheme?

Acquisition

Students will know…

- **Tree Diagrams** are a useful tool in listing the outcomes to an event.
- **Permutations** and combinations can be used to calculate combinations without replacement.
- The **Fundamental Counting Rule** can be used to calculate combinations with replacement.
- The multiplication rule for **independent** events is: $P(A\ and\ B) = P(A) \cdot P(B)$
- The multiplication rule for **dependent** events is: $P(A\ and\ B) = P(A) \cdot P(B|A)$
- A **sequence** is an ordered list of numbers.
- The sum of terms in a sequence is called a **series**. There are two types of series: arithmetic and geometric series.
- An **arithmetic series** is when a constant or

Students will be skilled at…

- Calculating combinations with replacement.
- Calculating combinations without replacement.
- Applying the multiplication rule of counting in complex situations.
- Recognizing the difference between situations with replacement and without replacement.
- Recognizing the difference between ordered and unordered counting situations.
- Justifying solutions to counting problems.
- Recognizing and explaining relationships involving combinations and Pascal's Triangle and apply those methods to situations involving probability.
- Using models and algebraic formulas to represent and analyze sequences and series.

problems. • **F-IF.3.** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by *f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)* for $n \geq 1$. • **F-BF.2.** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **Modeling** Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).	common difference is being added to each preceding term. ▪ To find any term in an arithmetic series we use the formula: ▪ $a_n = a_1 + (n-1)d$ ▪ Where a_1 is the first term in a sequence, d is the common difference; n is the number of the term to find. ▪ To find the sum of a certain number of terms of an arithmetic series we use the formula: ▪ $S_n = \frac{n(a_1+a_n)}{2}$ ▪ Where S_n is the sum of the first n terms, a_1 is the first term in a sequence, and a_n is the n^{th} term to find. ▪ A **geometric series** is when a constant or common ratio is being multiplied to each preceding term. ▪ To find any term in a geometric sequence we use the formula: ▪ $a_n = a_1 \cdot r^{n-1}$ ▪ where a_1 is the first term of the sequence, r is the common ratio, n is the number of the term to find. ▪ To find the sum of a certain number of terms of a **(finite) geometric sequence**: ▪ $S_n = \frac{a_1(1-r^n)}{1-r}$ ▪ Where $S_n$is the sum of n terms (n^{th} partial sum), a_1 is the first term, r is the common ratio. ▪ To find the sum of an **infinite geometric sequence**: ▪ $S = \frac{a_1}{1-r}$ ▪ Where a_1 is the first term, r is the common ratio.	▪ Evaluating explicit formulas for nth terms. ▪ Evaluating sums of finite arithmetic series. ▪ Evaluating sums of finite and infinite geometric series. ▪ Developing an informal notion of limit. ▪ Using inductive reasoning to form generalizations.

Unit V – Patterns
Stage 2 - Evidence

Evaluative Criteria	Assessment Evidence
RUBRIC/SCALE **3 - POINT RESPONSE** The response shows complete understanding of the problem's essential mathematical concepts. The student executes procedures completely and gives relevant responses to all parts of the task. The response contains few minor errors, if any. The response contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made. **2 - POINT RESPONSE** The response shows nearly complete understanding of the problem's essential mathematical concepts. The student executes nearly all procedures and gives relevant responses to most parts of the task. The response may have minor errors. The explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences. **1 - POINT RESPONSE** The response shows limited understanding of the problem's essential mathematical concepts. The response and procedures may be incomplete and/or may contain major errors. An incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made. **0 - POINT RESPONSE** The response shows insufficient understanding	PERFORMANCE TASK (S): At Johnny's Burger Place, a customer can get a customized meal by ordering either: a turkey burger, chicken burger, hamburger, or garden burger with a side order of potato chips or French fries with a choice of either: juice, milk, or soda. a. Use a tree diagram to list all the different combinations of a burger, side order and a drink. b. Describe ways and give examples of how Johnny could change his menu so that a customer would have 30 different choices. c. Research an online menu from your favorite place to eat. How many combinations do they offer? What are pros and cons of a restaurant offering many combinations? *(Adapted from www.state.nj.us by Jaclyn E. Varacallo)* **SAMPLE CONCEPTUAL UNDERSTANDING ITEM** (WWW.STATE.NJ.US) How many different ways can you put 6 books in a row so that one specific book is always in the 4th position? **SAMPLE SHORT CONSTRUCTED RESPONSE ITEM** You and a group of 9 friends are playing pickup basketball in a local park. At the end of the game, if each player shakes hands with every other player, how many handshakes will there be? **SAMPLE SHORT CONSTRUCTED RESPONSE ITEM** The Orchid Orchard has 5,000 orchid plants to sell. Each month the orchard plans to sell 12% of its orchids and start 600 new plants. Which of the following statements is true?

of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.

SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

A garden center is growing bean plants using a new fertilizer. At the end of each week, they record the number of new sprouts.

Week	1	2	3	4	5	6
New Sprouts	10	12	16	24	40	72

Using the information above, how many sprouts should the center expect at the end of the seventh week?

SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

Jessie's business partner set a computer password on the office computer and went on vacation. Jessie remembered that the password is a simple pattern. The first few letters of the password are

A Z D Y G X ___

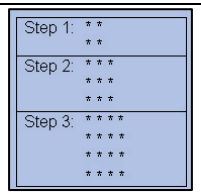
Assuming that the pattern continues, what is the next and final letter of the password?

SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

Kylee and Robin are racing their pet turtles. Kylee's turtle starts at ½ foot from the starting line and moves at 4 inches per minute. Robin's turtle starts at the starting line and moves at 6 inches per minute. Assuming they move in the same direction, where are the turtles after 5 minutes?

SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

Belinda wants to determine the number of dots in the 30th step of the following pattern, but she does not want to actually draw all 30 steps.



Explain how Belinda could find the number of dots in Step 30 without actually drawing them.
What would be the number of dots in the 30th step?
Write an algebraic expression for the number of dots in the nth step.

SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

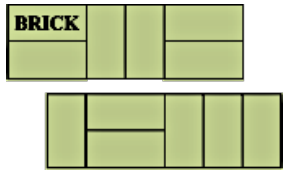
Analyze the pattern below. What letter will be in the 76th position? Show your work and explain your answer.

M A T H M A T H M A T H M A …

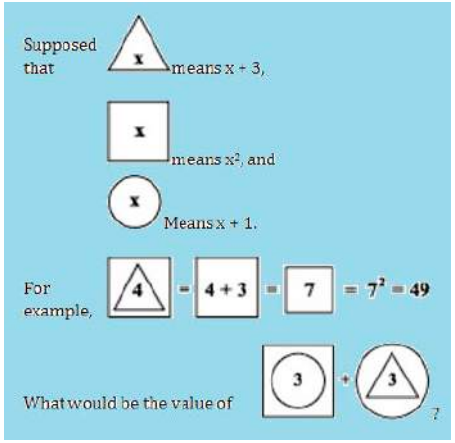
SAMPLE OPEN-ENDED RESPONSE ITEM (NJDOE TEST SPECIFICATIONS LINK)

Assume that you are planning to construct a walkway in front of a house. You have bricks that are two units by one unit in size. The walkway is to be two units wide, as shown in the diagram.

It is possible to arrange the bricks vertically or horizontally, as you see in the two different walkway patterns shown below.



- Find out how many different front walk patterns it is possible to construct with 6 bricks, including the two examples above.

	• Show all of your work. **SAMPLE SHORT CONSTRUCTED RESPONSE ITEM** (NJDOE TEST SPECIFICATIONS LINK) 
Student Responses should be: ➢ ***Accurate*** ➢ ***Clear*** ➢ ***Effective*** ➢ ***Organized*** ➢ ***Thorough*** ➢ ***Thoughtful***	OTHER EVIDENCE: ***Students will show they have achieved Stage 1 goals by . . .*** • Providing written or oral response to one of the essential questions. • Developing a journal of Rules for Success. The students will keep an ongoing journal all year of accumulating insight about which rules and properties. Include examples that show the rule or property correctly applied, as well as common mistakes. • Passing all quizzes and tests relating to the unit. • Braingenie.com ➢ **Diagnostic/Pre – Assessment:** ✓ Pre-test (5-10 open ended questions) will be given covering multiple concepts. ➢ **Open-Ended (Formative) Assessment:** ✓ Homework is assigned daily, from the textbook, Chapter Resource Practice Workbook, or other sources. *(Synthesis, Analysis, Evaluation)* ✓ Introductory and Closing Activities will be done every day to pre-assess student

	knowledge and assess understanding of topics.*(Synthesis, Analysis, Evaluation)* ✓ Excerpts from previous HSPA exams including multiple choice and open-ended problems should be given every class to assess student understanding and measure their individual skills. ➢ **Summative Assessment:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Synthesis, Analysis, Evaluation)* ✓ Students will be given quizzes that provide a brief review of the concepts and skills in the previous lessons. ✓ Students will be given a unit test that provides a review of the concepts and skills in the unit. ✓ Students will take the HSPA ✓ Students will work on Performance Assessment process similar to the ASHA process

Unit V – Patterns
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

- *Throughout the unit teacher made open-ended performance assessment tasks (PATs) will be used as instructional materials with a stress on communicating reasoning and justification for solutions through well reasoned and well written explanations.*
- ***Whole group instruction*** *to introduce new topics*
- ***Small group instruction*** *(based on student needs as identified by ongoing formative assessments)*
- ***Individual instruction*** *(As needed based on student needs and formative assessment results)*

TECHNOLOGY USE *in daily teaching as formative assessment tool and as independent practice tool for practicing and deepening knowledge*

- ***Class Wiki*** *for distributing materials and resources to students*
- ***Using iPads:*** ***www.socrative.com*** *(for formative assessments and classroom surveys)*
- ***Using iPads:*** ***www.****Braingenie.com*
- ***Using iPads:*** ***www.****Interactmath.com*
- ***Using Internet:*** *www.Khanacademy.org*

Discuss multiple strategies to solve a given problem:
Concept Application: *Senior Class Meeting (Created by Dr. M. K. Sran)*
Concept Application: *Triangular Numbers (Created by Dr. M. K. Sran)*
Concept Application: *Display of Dots (Created by Dr. M. K. Sran)*

Concept Application: *Design (Created by Dr. M. K. Sran)*
Concept Application: *Exam Scores (Created by Dr. M. K. Sran)*
Concept Application: *Patterns (Created by Dr. M. K. Sran)*

The following projects also will contribute to learning:

- *Towers (Rutgers Longitudinal study)*
- *Tower of Hanoi (Rutgers Longitudinal study)*
- *A "Numbers" Puzzle (Rutgers Longitudinal study)*
- *Pizza Halves Problem (Rutgers Longitudinal study)*
- *Pizza Problem (Rutgers Longitudinal study)*

Unit VI – Data Analysis
Stage 1 Desired Results

ESTABLISHED GOALS

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- **S-ID.1.**

Represent data with plots on the real number line (dot plots, histograms, and box plots).

- **S-ID.2.**

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

- **S-ID.3.**

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

- **S-ID.4.**

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal

Transfer

Students will be able to independently use their learning to…

- Analyze when any rule in any system (language, law, math) is an essential principle or merely conventional.
- Acquire an understanding of the concepts to apply to a variety of real-world learning experiences in new, diverse situations.

Meaning

UNDERSTANDINGS

Students will understand that…

- Visual **representations** of data are essential in **analyzing** data.
- An effective analysis is comprised of the mean, median, mode, and range of the data.
- **Trends** and strengths of **relationships** are apparent in the visual representations of data.

ESSENTIAL QUESTIONS

1. How does the type of data influence the choice of display?
2. How can various statistical techniques be used to organize, display, and compare sets of data?
3. How can advertisers intentionally create data displays that mislead consumers?

Acquisition

Students will know…

- **Sample selection** methods include convenience sampling, responses to survey, random sampling.
- **Histograms** and **box-and-whisker plots** are used to represent numerical data.
- **Bar graphs** and **pie graphs** are used to represent categorical or qualitative variables.
- The **mean** of data is computed by dividing the sum of the data values by the number of data values.
- The **median** of a set of data is the middle

Students will be skilled at…

- Using surveys and sampling techniques to generate data and draw conclusions about large groups.
- Identifying advantages/disadvantages of sample selection methods.
- Evaluating the use of data in real-world contexts.
- Evaluating the accuracy and reasonableness of conclusions drawn.
- Evaluating the bias in conclusions drawn (e.g., influence of how data is displayed)
- Evaluating statistical claims based on

curve. • **S-ID.6.** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. • **S-ID.7.** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. • **S-ID.8.** Compute (using technology) and interpret the correlation coefficient of a linear fit. **Modeling** Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).	value when the data is placed in ascending order. ▪ The **mode** of a set of data is the value that occurs most often in a data set. ▪ The **range** of a data set is the minimum value subtracted from the maximum value in the data set. ▪ *Range = maximum value – minimum value* ▪ Measures of **dispersion** include: variance, standard deviation, outliers ▪ The **Normal distribution** states that approximately 95% of the sample lies between two standard deviations on either side of the mean.	sampling. ▪ Designing a statistical experiment, conducting the experiment, and interpreting and communicating the outcome. ▪ Using data to draw bar graphs, line graphs, and double line graphs. ▪ Using a scatter plot to identify the correlation by a set of data; approximate the line of best fit for a set of data. ▪ Estimating or determining lines of best fit (or curves of best fit if appropriate) with technology, and use them to interpolate within the range of the data. ▪ Analyzing data using technology, and use statistical terminology to describe conclusions.

Unit VI – Data Analysis
Stage 2 - Evidence

Evaluative Criteria	Assessment Evidence
RUBRIC/SCALE	PERFORMANCE TASK (S):

3 - POINT RESPONSE
The response shows complete understanding of the problem's essential mathematical concepts. The student executes procedures completely and gives relevant responses to all parts of the task. The response contains few minor errors, if any. The response contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made.

2 - POINT RESPONSE
The response shows nearly complete understanding of the problem's essential mathematical concepts. The student executes nearly all procedures and gives relevant responses to most parts of the task. The response may have minor errors. The explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences.

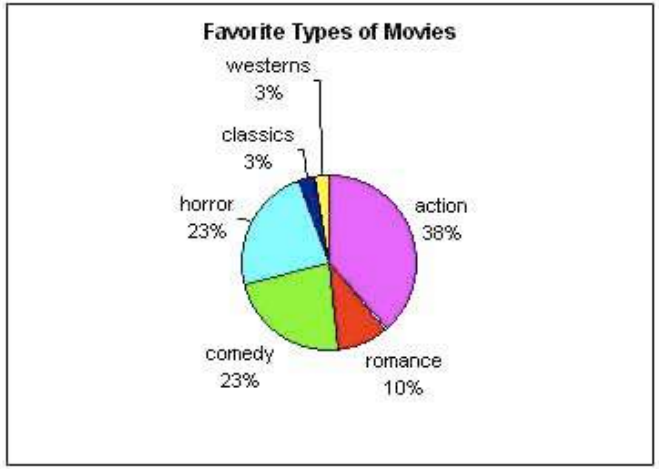
1 - POINT RESPONSE
The response shows limited understanding of the problem's essential mathematical concepts. The response and procedures may be incomplete and/or may contain major errors. An incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made.

0 - POINT RESPONSE
The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions

Classroom Survey: Have students write as many **x**'s on a sheet of paper as they can in 60 seconds, all students must use their right hand. Then repeat, this time all students must use their left hand. Plot results on board to demonstrate scatter plot and trend line.

SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

The 315 members of the Smithfield High School Sophomore class voted to see what kinds of movies the class liked. Their results are below. If they had an "Action Movie Night" fundraiser and sell tickets for $4 each, approximately how much money should they expect to raise? Assume that students who like the type of movie will be the only ones in attendance.



SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

A liquid substance in a science experiment doubles in volume every two minutes. The experiment begins at 1:00PM with a small amount of the substance being placed in a large beaker. At 2:00PM the container is full. At what time was the container one-eighth filled?

SAMPLE SHORT CONSTRUCTED RESPONSE ITEM

Researchers decided to see if there was a relationship between the number of flu shots each year

<table>
<tr>
<td>were made.

Copyright © State of New Jersey, 2006
NJ Department of Education
http://www.nj.gov/education/njpep/assessment/TestSpecs/MathNJASK/rubrics.html</td>
<td>and the number of cases of the flu that year.

Sketch a data display of how you might display this data and what type of relationship the data would demonstrate.

Describe the relationship between the variables in words

SAMPLE OPEN-ENDED ITEM (NJDOE TEST SPECIFICATIONS LINK)

<table>
<tr><td>HOURS</td><td>1.0</td><td>1.25</td><td>1.5</td><td>1.75</td><td>2.0</td><td>2.25</td></tr>
<tr><td>SCORE</td><td>60</td><td>70</td><td>68</td><td>85</td><td>90</td><td>98</td></tr>
<tr><td>HOURS</td><td>2.5</td><td>2.75</td><td>3.0</td><td>3.25</td><td>3.5</td><td>3.75</td></tr>
<tr><td>SCORE</td><td>85</td><td>92</td><td>91</td><td>87</td><td>85</td><td>72</td></tr>
</table>

The data provided shows test scores for twelve students and the number of hours they studied for the test during the three days prior to taking it.

• Make a scatter plot of this data.
• Does there appear to be a relationship between a student's test score and the time spent studying? Use the scatter plot to support your answer.
Do any of the points appear to be outliers? Explain</td>
</tr>
<tr>
<td>Student Responses should be:
➢ Accurate
➢ Clear
➢ Effective
➢ Organized
➢ Thorough
➢ Thoughtful</td>
<td>OTHER EVIDENCE:

Students will show they have achieved Stage 1 goals by . . .

• Providing written or oral response to one of the essential questions.
• Developing a journal of Rules for Success. The students will keep an ongoing journal all year of accumulating insight about which rules and properties. Include examples that show the rule or property correctly applied, as well as common mistakes.
• Passing all quizzes and tests relating to the unit.
• Braingenie.com

➢ Diagnostic/Pre – Assessment:
✓ Pre-test (5-10 open ended questions) will be given covering multiple concepts.
➢ Open-Ended (Formative) Assessment:</td>
</tr>
</table>

	✓ Homework is assigned daily, from the textbook, Chapter Resource Practice Workbook, or other sources. *(Synthesis, Analysis, Evaluation)* ✓ Introductory and Closing Activities will be done every day to pre-assess student knowledge and assess understanding of topics.*(Synthesis, Analysis, Evaluation)* ✓ Excerpts from previous HSPA exams including multiple choice and open-ended problems should be given every class to assess student understanding and measure their individual skills. ➢ **Summative Assessment:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Synthesis, Analysis, Evaluation)* ✓ Students will be given quizzes that provide a brief review of the concepts and skills in the previous lessons. ✓ Students will be given a unit test that provides a review of the concepts and skills in the unit. ✓ Students will take the HSPA ✓ Students will work on Performance Assessment process similar to the ASHA process

Unit VI – Data Analysis
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

- *Throughout the unit teacher made open-ended performance assessment tasks (PATs) will be used as instructional materials with a stress on communicating reasoning and justification for solutions through well reasoned and well written explanations.*
- ***Whole group instruction*** *to introduce new topics*
- ***Small group instruction*** *(based on student needs as identified by ongoing formative assessments)*
- ***Individual instruction*** *(As needed based on student needs and formative assessment results)*

- **Centers Activity**: Have students go to different centers and collect different types of data. At each center, they should create an appropriate data display and describe the data using measures of center and dispersion.
- **Sampling Techniques** Have students compare and contrast the different surveying techniques by having them survey topics around the school. Assign students to use two different sampling techniques and compare and contrast the effectiveness/bias of each.

TECHNOLOGY USE *in daily teaching as formative assessment tool and as independent practice tool for practicing and deepening knowledge*

- ***Class Wiki*** *for distributing materials and resources to students*

- ***Using iPads:*** ***www.socrative.com*** *(for formative assessments and classroom surveys)*
- ***Using iPads:*** ***www.****Braingenie.com*
- ***Using iPads:*** ***www.****Interactmath.com*
- ***Using Internet:*** *www.Khanacademy.org*

Discuss multiple strategies to solve a given real life problem:
Concept Application: *Scatterplot (Created by Dr. M. K. Sran)*
Concept Application: *Record Times (Created by Dr. M. K. Sran)*
Concept Application: *Taxes (Created by Dr. M. K. Sran)*

Unit VII – Relationships
Stage 1 Desired Results

ESTABLISHED GOALS

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- **A-APR.3.**

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

- **A-REI.1.**

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

- **A-REI.3.**

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- **A-REI.5.**

Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces

Transfer

Students will be able to independently use their learning to…

- Utilize patterns both, graphically and algebraically, to anticipate outcomes of real life events.

Meaning

UNDERSTANDINGS

Students will understand that…

- Equations **model** patterns that occur in real life problems and are used to **solve** for unknown quantities.
- A graph and its equation are in an **interdependent** relationship.
- Formulas are direct **representations** of real life applications that help to **solve** for an unknown quantity.
- A solution of a system of equations **models** a unique **outcome** for two or more real-life situations.

ESSENTIAL QUESTIONS

1. How is a linear model used to approximate a real life situation?
2. Explain how to use a linear model to make predictions from given data.
3. How do the different forms of linear functions and the concept of slope help solve real world situations?
4. How can transformations be used in architecture and various types of art?

Acquisition

Students will know…

- The **slope** of a line is defined as the rate of change over an interval. It is the ration of vertical change to horizontal change.
- $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$
- The **domain** of a function is the set of possible x values. The **range** of a function is the set of possible y values.
- The **intercepts** of a function is where the graph has a value where $y = 0$.
- **Continuity** of a function is described as not

Students will be skilled at…

- Understanding relations and functions; and selecting, converting flexibly among and using various representations for them, including equations or inequalities, tables, and graphs.
- Analyzing and explaining the general properties and behavior of functions of one variable, using appropriate graphing technologies.
- Understanding and performing

a system with the same solutions. • **A-REI.6.** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. • **A-REI.7.** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. • **A-REI.10.** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). • **A-REI.11.** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* • **A-REI.12.** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. • **F-IF.1.**	having to lift your pencil off the paper to draw it; for instance, it has no jumps or breaks. ▪ Finding the x-intercepts of the equations can approximate the root (s) of equations. ▪ A **solution** to system of equations is a point where the graphs of all the equations in the system intersect (i.e. a common point). ▪ **Slope** represents rate of change. ▪ The **translation** of a graph is a horizontal and/or vertical shift of a figure or graph on a coordinate plane. ▪ The **reflection** of a graph is when a figure is reflected over an axis or line. ▪ The **dilation** of a graph is a transformation in which all distances on the coordinate plane are lengthened by multiplying either all X-coordinates (horizontal dilation) or all Y-coordinates (vertical dilation) by a common factor greater than 1. *(http://www.mathwords.com)*	translations, reflections, and dilations on commonly used functions. ▪ Understanding and performing transformations ▪ Using graphing calculators or computers for transformations on more complex functions. ▪ Understanding and comparing the properties of classes of functions, including exponential, polynomial, rational, and trigonometric functions. ▪ Identifying a linear function vs. a non-linear function. ▪ Defining symmetry in a given graph or figure. ▪ Identifying increasing or decreasing on an interval of a graph.

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). • **F-IF.2.** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. • **F-IF.4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. * • **F-IF.5.** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* • **F-IF.6.** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* • **F-IF.7.**		

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. c. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. • **F-IF.8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. • **F-IF.9.** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. • **F-BF.1.** Write a function that describes a relationship between two quantities. b. Determine an explicit expression, a recursive process, or steps for calculation from a context. c. Combine standard function types using		

arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

d. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

- **F-BF.3.**

Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(k x), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Modeling

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

Unit VII – Relationships
Stage 2 - Evidence

Evaluative Criteria	Assessment Evidence
RUBRIC/SCALE **3 - POINT RESPONSE** The response shows complete understanding of the problem's essential mathematical concepts. The student executes procedures completely and	PERFORMANCE TASK (S): Sharifa is offered two jobs: one at Tom's World of Music and one at Rosie's Café. Tom's promises her a salary of $30,000 and a raise of $500 each year. Rosie's offers her a salary of $30,000 and a raise of 4% of her current salary each year.

gives relevant responses to all parts of the task. The response contains few minor errors, if any. The response contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made.

2 - POINT RESPONSE
The response shows nearly complete understanding of the problem's essential mathematical concepts. The student executes nearly all procedures and gives relevant responses to most parts of the task. The response may have minor errors. The explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences.

1 - POINT RESPONSE
The response shows limited understanding of the problem's essential mathematical concepts. The response and procedures may be incomplete and/or may contain major errors. An incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made.

0 - POINT RESPONSE
The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.

- Which job should she accept? Explain your answer. Show all work.
- Research to find two jobs that require equal degrees of education (for example: a teacher and a nurse who both require a bachelor's degree). Discuss the salary and opportunities presented by both occupations and identify pros and cons of each. Of the two occupations that you've researched, which job would you consider and why?

(Adapted from www.state.nj.us by Jaclyn E. Varacallo)

SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

The table below shows a pattern. What type of model represents the function shown? How do you know?

x	y
0	0
1	-1
2	0
3	3
4	8

SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

A local jewelry dealer calculates the price at which he will buy back gold jewelry based on the following formula: $B = .75(G - 15) + .15G$, where B is the "buyback" price and G is the current market value of the gold. You bring in a gold bracelet with a current market value of $105. What will the dealer pay you for the bracelet?

SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

James came home from school with 16 hours of homework. He had three times as much

	English homework as he had math. How much math homework did he have? **SAMPLE CONCEPTUAL UNDERSTANDING ITEM** Judge Esther Odometer developed a formula to determine the fine for speeding on the parkway in her town. The formula she developed is: $F = 12(R - 60) + 55$ In this equation, F represents the total amount of fine, R represents how fast (rate of speed) the car was going in miles per hour. **SAMPLE CONCEPTUAL UNDERSTANDING ITEM** (WWW.STATE.NJ.US) A car has been stopped for traveling 75 mph, 80 mph, 87 mph, 90 mph, and 100 mph over the course of the past year. Show the domain and range for this relation. Judge Esther charges you a fine of $175 for speeding. Is this a reasonable fine? Why or why not **SAMPLE CONCEPTUAL UNDERSTANDING ITEM** (WWW.STATE.NJ.US) Mike dropped a basketball ball from his bedroom window a height of 20 feet. Each time the ball bounced, it reached a maximum height of approximately half that of its previous height. 1. Draw a graph to represent the relationship between the number of times the ball bounces and the height reached by the ball. 2. What is the total of the heights the ball reached after the 4th bounce? 3. What do you think this total would have been if the ball had bounced 20 times?
Student Responses should be: ➢ ***Accurate*** ➢ ***Clear*** ➢ ***Effective*** ➢ ***Organized*** ➢ ***Thorough*** ➢ ***Thoughtful***	OTHER EVIDENCE: ***Students will show they have achieved Stage 1 goals by . . .*** • Providing written or oral response to one of the essential questions. • Developing a journal of Rules for Success. The students will keep an ongoing journal all year of accumulating insight about which rules and properties. Include examples that

	show the rule or property correctly applied, as well as common mistakes. • Passing all quizzes and tests relating to the unit. • Braingenie.com ➢ **Diagnostic/Pre – Assessment:** ✓ Pre-test (5-10 open ended questions) will be given covering multiple concepts. ➢ **Open-Ended (Formative) Assessment:** ✓ Homework is assigned daily, from the textbook, Chapter Resource Practice Workbook, or other sources. *(Synthesis, Analysis, Evaluation)* ✓ Introductory and Closing Activities will be done every day to pre-assess student knowledge and assess understanding of topics.*(Synthesis, Analysis, Evaluation)* ✓ Excerpts from previous HSPA exams including multiple choice and open-ended problems should be given every class to assess student understanding and measure their individual skills. ➢ **Summative Assessment:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Synthesis, Analysis, Evaluation)* ✓ Students will be given quizzes that provide a brief review of the concepts and skills in the previous lessons. ✓ Students will be given a unit test that provides a review of the concepts and skills in the unit. ✓ Students will take the HSPA ✓ Students will work on Performance Assessment process similar to the ASHA process

Unit VII – Relationships
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

➢ *Throughout the unit teacher made open-ended performance assessment tasks (PATs) will be used as instructional materials with a stress on communicating reasoning and justification for solutions through well reasoned and well written explanations.*

Group Activity: Students will work with TI-84 graphing calculators to explore the effects of various transformations on the graphs of:

A. Linear functions
B. Quadratic functions
C. Absolute value functions

D. Other functions

- ***Whole group instruction*** *to introduce new topics*
- ***Small group instruction*** *(based on student needs as identified by ongoing formative assessments)*
- ***Individual instruction*** *(As needed based on student needs and formative assessment results)*

TECHNOLOGY USE *in daily teaching as formative assessment tool and as independent practice tool for practicing and deepening knowledge*

- ***Class Wiki*** *for distributing materials and resources to students*
- ***Using iPads: www.socrative.com*** *(for formative assessments and classroom surveys)*
- ***Using iPads: www.****Braingenie.com*
- ***Using iPads: www.****Interactmath.com*
- ***Using Internet:*** *www.Khanacademy.org*

Discuss multiple strategies to solve a given real life problem:

- ***Concept Application:*** *Frame (Created by Dr. M. K. Sran)*
- ***Concept Application:*** *Fundraiser (Created by Dr. M. K. Sran)*
- ***Released PATs:*** *Prom Expenses (Created by Dr. M. K. Sran)*
- ***Released PATs:*** *Profit Percent (Created by Dr. M. K. Sran)*

Unit VIII - Equivalence
Stage 1 Desired Results

ESTABLISHED GOALS

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- **A-CED.4.**

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

- **A-REI.2.**

Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

- **A-REI.3.**

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- **A-REI.4.**

Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula

Transfer

Students will be able to independently use their learning to…

- Solve problems by simplifying them, using equivalent statements based on the properties of real numbers and the order of operations.
- Analyze when any rule in any system (language, law, math) is an essential principle or merely conventional.

Meaning

UNDERSTANDINGS

Students will understand that…

- The factors and x-intercepts of a polynomial are directly **related**.
- Multiplying polynomials and factoring polynomials are reverse **processes** of each other.

ESSENTIAL QUESTIONS

1. Why can't you solve for the zeros of a polynomial if the polynomial equation is set equal to anything other than zero?
2. How are factoring a polynomial and multiplying a polynomial related?
3. How does factoring a polynomial help to yield information from a real-life model?

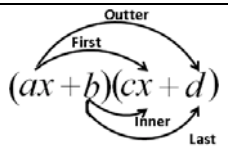
Acquisition

Students will know…

- When adding or subtracting polynomials, combine like terms.
- The **degree** of a polynomial is the highest exponent.
- The **leading coefficient** is the number next to the variable with the highest exponent.
- A **term** is called a monomial, involving multiplication between constants, which can be multiplied by variables.
- **FOIL** is a double distributing method of multiplication for two binomials $(ax + b)(cx + d)$:

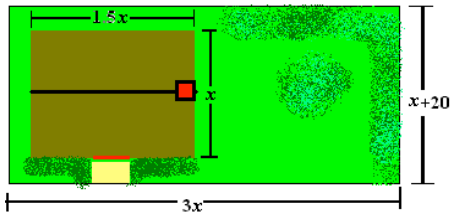
Students will be skilled at…

- Evaluating and simplifying expressions.
- Adding and subtracting polynomials
- Multiplying a polynomial by a monomial or binomial.
- Dividing a polynomial by a monomial
- Selecting and using appropriate methods to solve equations and inequalities.
  - Linear equations - algebraically
  - Quadratic equations - factoring (when the coefficient of x^2 is 1) and using the quadratic formula
  - All types of equations using graphing, computer, and graphing calculator

from this form. **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. • **F-IF.8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **b.** Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay. • **A-APR.1.** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. • **A-APR.7.** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	 ▪ **Zero product property** states that if $ab = 0$ then $a = 0$ or$b = 0$. ▪ **Greatest common factor** is the common factor of all the terms. ▪ A polynomial is prime if it is not the product of polynomials having integer coefficients. ▪ To factor a polynomial completely write as the product of monomial factors or prime factors with at least two terms.	techniques ▪ Judging the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

Modeling Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).		

Unit VIII - Equivalence
Stage 2 - Evidence

Evaluative Criteria	Assessment Evidence
RUBRIC/SCALE **3 - POINT RESPONSE** The response shows complete understanding of the problem's essential mathematical concepts. The student executes procedures completely and gives relevant responses to all parts of the task. The response contains few minor errors, if any. The response contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made. **2 - POINT RESPONSE** The response shows nearly complete understanding of the problem's essential mathematical concepts. The student executes nearly all procedures and gives relevant responses to most parts of the task. The response may have minor errors. The explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences. **1 - POINT RESPONSE** The response shows limited understanding of the problem's essential mathematical concepts. The response and procedures may be incomplete and/or may contain major errors. An incomplete	PERFORMANCE TASK (S): You plan to build a house that is 1 ½ times as long as it is wide. You want the land around the house to be 20 feet wider than the width of the house, and twice as long as the length of the house. You have an acre of land that you can use to build a house that you wish to build from the most economical standpoint. Calculate how much it would cost to build the house based on your materials and the size of the house. Choose a place that you'd wish to live and use estimates of taxes and utilities of that are to calculate the to live in the home for a year.  *(Adapted from McDougal-Littell: Algebra 1, 2004 by Jaclyn E. Varacallo)*

explanation of how the problem was solved may contribute to questions as to how and why decisions were made. **0 - POINT RESPONSE** The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made. 	
Student Responses should be: ➢ ***Accurate*** ➢ ***Clear*** ➢ ***Effective*** ➢ ***Organized*** ➢ ***Thorough*** ➢ ***Thoughtful***	OTHER EVIDENCE: ***Students will show they have achieved Stage 1 goals by . . .*** • Providing written or oral response to one of the essential questions. • Developing a journal of Rules for Success. The students will keep an ongoing journal all year of accumulating insight about which rules and properties. Include examples that show the rule or property correctly applied, as well as common mistakes. • Passing all quizzes and tests relating to the unit. • Braingenie.com ➢ **Diagnostic/Pre – Assessment:** ✓ Pre-test (5-10 open ended questions) will be given covering multiple concepts. ➢ **Open-Ended (Formative) Assessment:** ✓ Homework is assigned daily, from the textbook, Chapter Resource Practice Workbook, or other sources. *(Synthesis, Analysis, Evaluation)* ✓ Introductory and Closing Activities will be done every day to pre-assess student knowledge and assess understanding of topics.*(Synthesis, Analysis, Evaluation)* ✓ Excerpts from previous HSPA exams including multiple choice and open-ended problems should be given every class to assess student understanding and measure their individual skills. ➢ **Summative Assessment:** Assessment questions should be open-ended and should follow

	the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Synthesis, Analysis, Evaluation)* ✓ Students will be given quizzes that provide a brief review of the concepts and skills in the previous lessons. ✓ Students will be given a unit test that provides a review of the concepts and skills in the unit. ✓ Students will take the HSPA ✓ Students will work on Performance Assessment process similar to the ASHA process

Unit VIII - Equivalence
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

- *Throughout the unit teacher made open-ended performance assessment tasks (PATs) will be used as instructional materials with a stress on communicating reasoning and justification for solutions through well reasoned and well written explanations.*
- ***Whole group instruction*** *to introduce new topics*
- ***Small group instruction*** *(based on student needs as identified by ongoing formative assessments)*
- ***Individual instruction*** *(As needed based on student needs and formative assessment results)*

TECHNOLOGY USE *in daily teaching as formative assessment tool and as independent practice tool for practicing and deepening knowledge*

- ***Class Wiki*** *for distributing materials and resources to students*
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- ***Using iPads: www.****Interactmath.com*
- ***Using Internet:*** *www.Khanacademy.org*

- *10.1* ***Application*** *Lesson Opener (McDougal Littell,* Algebra 1*, 2004, Chapter 10 Resource Books*, p.13)
- *10.2* ***Graphing Calculator Activity*** *(McDougal Littell,* Algebra 1*, 2004, Chapter 10 Resource Books*, p.27)
- *10.3* ***Application*** *Lesson Opener (McDougal Littell,* Algebra 1*, 2004, Chapter 10 Resource Books*, p.40)
- *10.****3 Cooperative Learning Activity*** *(McDougal Littell,* Algebra 1*, 2004, Chapter 10 Resource Books*, p.47)
- *10.5 Activity Lesson Opener (McDougal Littell,* Algebra 1*, 2004, Chapter 10 Resource Books*, p.68)
- *10.7 Activity Lesson Opener (McDougal Littell,* Algebra 1*, 2004, Chapter 10 Resource Books*, p.95)

Discuss multiple strategies for solving the following real life applications:

Concept Application: *Quadratic Function (Created by Dr. M. K. Sran)*
Concept Application: *Scientific Notation (Created by Dr. M. K. Sran)*

Unit IX- Mathematical Modeling
Stage 1 Desired Results

ESTABLISHED GOALS

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- **A-CED.1.**

Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

- **A-CED.2.**

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **A-CED.3.**

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Transfer

Students will be able to independently use their learning to…

- Utilize patterns both, graphically and algebraically, to anticipate outcomes of real life events.
- Utilize and manipulate pieces of the "big picture" to alter the outcome of an interdependent relationship (such as costs of labor and the resulting profit of company).

Meaning

UNDERSTANDINGS

Students will understand that…

- Functions model real-life **patterns** to enable us to make **predictions**.
- Equations describe the **relationship** between a dependent and an independent variable.

ESSENTIAL QUESTIONS

1. Why is it necessary to utilize different types of functions in order to model real-world phenomena?
2. How are functions useful in making predictions?

Acquisition

Students will know…

- Linear, quadratic, exponential, periodic (sine and cosine) and step functions can be used to model real – world situations.
- **Direct variation** is when the variable x and y vary directly if for a constant k; $\frac{y}{x} = k$ or $y = kx$; $k \neq 0$
- k is the **constant of variation**.
- **Inverse variation** is when the variable x and y vary inversely if for a constant k; $xy = k$ or $y = \frac{k}{x}$; $k \neq 0$.
- **Absolute value** of a number is the

Students will be skilled at…

- Using functions to model real-world phenomena and solving problems that involve varying quantities.
- Analyzing and describing how a change in an independent variable leads to change in a dependent one.
- Converting recursive formulas to linear or exponential functions.

• **A-CED.4.** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. • **A-REI.1.** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. • **A-REI.2.** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. • **A-REI.3.** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. • **A-REI.4.** Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. • **A-REI.5.**	distance of a value from zero on the number line. ▪ The graph of a **linear inequality** in one variable is the set of points on a number line that represent all solutions of the inequality. ▪ An ordered pair, (x, y) is a solution of a linear inequality if the inequality is true when the values of x and y are substituted into the inequality.	

Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. • **A-REI.6.** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. • **A-REI.7.** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3. • **A-REI.8.** (+) Represent a system of linear equations as a single matrix equation in a vector variable. • **A-REI.10.** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). • **A-REI.11.** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* • **A-REI.12.** Graph the solutions to a linear inequality in two		

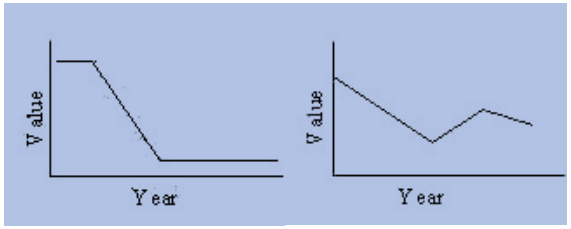
variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. • **F-IF.1.** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). • **F-IF.2.** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. • **F-IF.3.** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. • **F-IF.4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* • **F-IF.5.** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of		

person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* • **F-IF.6.** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* • **F-IF.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. a. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. • **F-IF.8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, y =		

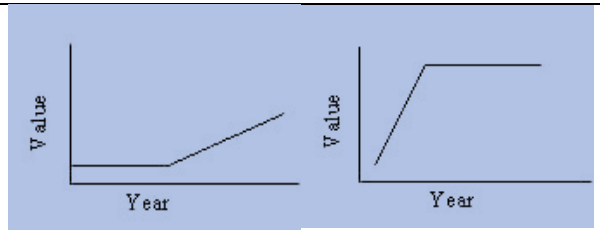
$(1.01)^{12t}$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay. • **F-IF.9.** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. • **F-BF.1.** Write a function that describes a relationship between two quantities.* a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. • **F-BF.2.** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.* • **F-LE.1.** Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal		

differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. • **F-LE.2.** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). • **F-LE.3.** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. • **F-LE.5.** Interpret the parameters in a linear or exponential function in terms of a context. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*). **Modeling** Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).		

Unit IX- Mathematical Modeling
Stage 2 - Evidence

Evaluative Criteria	Assessment Evidence
RUBRIC/SCALE **3 - POINT RESPONSE** The response shows complete understanding of the problem's essential mathematical concepts. The student executes procedures completely and gives relevant responses to all parts of the task. The response contains few minor errors, if any. The response contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made. **2 - POINT RESPONSE** The response shows nearly complete understanding of the problem's essential mathematical concepts. The student executes nearly all procedures and gives relevant responses to most parts of the task. The response may have minor errors. The explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences. **1 - POINT RESPONSE** The response shows limited understanding of the problem's essential mathematical concepts. The response and procedures may be incomplete and/or may contain major errors. An incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made. **0 - POINT RESPONSE** The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be	PERFORMANCE TASK (S): You are the service manager at an auto repair shop. You charge $22 per hour for labor plus the cost of any parts. A car needed $256 of new parts. The final bill for the car was $421. • How long did it take to repair the car? Explain your answer. • Write an algebraic equation to solve this problem. Show all work. • If you could charge anything you wanted, what would you charge? • Research to find out how your cost compares with typical auto repair shops today. How do you think your consumers would react to your new prices from the original problem ($22/hr + parts)? *(Adapted from www.state.nj.us by Jaclyn E. Varacallo)* **SAMPLE CONCEPTUAL UNDERSTANDING ITEM** (WWW.STATE.NJ.US) The weekly pay a worker at a restaurant earns, *P*, varies directly as the number of hours, *h*, which they work. Express this relation as a formula. **SAMPLE CONCEPTUAL UNDERSTANDING ITEM** (WWW.STATE.NJ.US) In 2000, a particular technology stock dropped quickly, and then leveled off. Which of the graphs below best represents that stock's performance? 

able to understand the explanation. The reader may not be able to understand how and why decisions were made.



SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

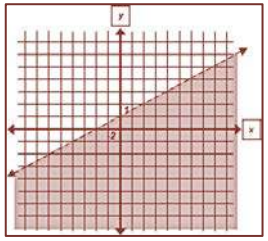
A bag of jelly beans includes only r red and y yellow jellybeans. If Violet removes 1 red jellybean, what fractional part of the jelly beans in the bag is red?

SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

If @ is defined for all positive numbers a and b by $a@b = 2ab - b^2$, then $5@3 =$

SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

What is the inequality for the following graph?



SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US)

A Web designer is building pages that are 600 pixels wide and 480 pixels high. She wishes to place an image on the page that is proportionate to those dimensions. The image is 320 pixels wide. How high will it have to be?

	SAMPLE CONCEPTUAL UNDERSTANDING ITEM (WWW.STATE.NJ.US) Diana had $1200 in her checking account. She withdrew the same amount each month for 5 months to pay for a car loan. At the end of 6 months, she deposited an additional $600 into her account. Her new balance was $800. How much money did she withdraw each month? What was her account balance after the 6 months before her deposit of $600? Explain and show all your work. **SAMPLE CONCEPTUAL UNDERSTANDING ITEM** (WWW.STATE.NJ.US) John and Ellen work in a clothing store after school. John's boss told him to reduce every item in the store for a 2-day sale by 30%. After the sale, John's boss told her to increase every sale item's sale price by 30%. John started marking each item with the original price. Ellen said, "That is wrong! If you increase the sale price by 30% you will not get the original price." Who is right? Show your work. You may wish to include a simple, specific example to support your answer **SAMPLE CONCEPTUAL UNDERSTANDING ITEM** (WWW.STATE.NJ.US) You are the service manager at an auto repair shop. You charge $22 per hour for labor plus the cost of any parts. A car needed $256 of new parts. The final bill for the car was $421. How long did it take to repair the car? Explain your answer. Write an algebraic equation to solve this problem. Show all work.
Student Responses should be: ➢ ***Accurate*** ➢ ***Clear*** ➢ ***Effective*** ➢ ***Organized*** ➢ ***Thorough*** ➢ ***Thoughtful***	OTHER EVIDENCE: ***Students will show they have achieved Stage 1 goals by . . .*** • Providing written or oral response to one of the essential questions. • Developing a journal of Rules for Success. The students will keep an ongoing journal all year of accumulating insight about which rules and properties. Include examples that show the rule or property correctly applied, as well as common mistakes.

	• Passing all quizzes and tests relating to the unit. • Braingenie.com ➢ **Diagnostic/Pre – Assessment:** ✓ Pre-test (5-10 open ended questions) will be given covering multiple concepts. ➢ **Open-Ended (Formative) Assessment:** ✓ Homework is assigned daily, from the textbook, Chapter Resource Practice Workbook, or other sources. *(Synthesis, Analysis, Evaluation)* ✓ Introductory and Closing Activities will be done every day to pre-assess student knowledge and assess understanding of topics.*(Synthesis, Analysis, Evaluation)* ✓ Excerpts from previous HSPA exams including multiple choice and open-ended problems should be given every class to assess student understanding and measure their individual skills. ➢ **Summative Assessment:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Synthesis, Analysis, Evaluation)* ✓ Students will be given quizzes that provide a brief review of the concepts and skills in the previous lessons. ✓ Students will be given a unit test that provides a review of the concepts and skills in the unit. ✓ Students will take the HSPA ✓ Students will work on Performance Assessment process similar to the ASHA process

Unit IX- Mathematical Modeling
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

- *Throughout the unit teacher made open-ended performance assessment tasks (PATs) will be used as instructional materials with a stress on communicating reasoning and justification for solutions through well reasoned and well written explanations.*
- ***Whole group instruction*** *to introduce new topics*
- ***Small group instruction*** *(based on student needs as identified by ongoing formative assessments)*
- ***Individual instruction*** *(As needed based on student needs and formative assessment results)*

TECHNOLOGY USE *in daily teaching as formative assessment tool and as independent practice tool for practicing and deepening knowledge*

- ***Class Wiki*** *for distributing materials and resources to students*
- ***Using iPads: www.socrative.com*** *(for formative assessments and classroom surveys)*

- ***Using iPads: www.Braingenie.com***
- ***Using iPads: www.Interactmath.com***
- ***Using Internet: www.Khanacademy.org***

- **Group Activity:** Students will work on an assignment to decide which product to buy from the given choices of Brands and prices given by using proportional reasoning *(Adapted from http://www.doe.state.la.us/topics/comprehensive_curriculum.html)*
  - ✓ ***Students will identify the items from the given list that they think will be a better buy***
  - ✓ ***They will use proportional reasoning to determine which brand will provide a better buy***
  - ✓ ***They will determine the total amount of money saved by buying the "better" priced item on the shopping list***

- **Group Activity***:* Students will use a ruler and scale factor to find the area and perimeter of each room in a given layout of a house.
  - ✓ **Students will determine the actual measurements form the given scaled drawing and using the scale factor**
  - ✓ **Students will find the area of each room with given conditions**
  - ✓ **Find the cost of painting each room when given the heights of the walls and dimensions of the windows and doors**

- **Individual Project:** You are an entrepreneur and have 10 employees working for you. Since you do not have a person to handle the payroll you have to calculate the weekly wages, commission, and total gross earnings for each one of your employees. You have the following data:
  - ✓ ***Hours worked for each employee***
  - ✓ ***Your daily sales for each individual***
  - ✓ ***Hourly rate of pay for each employee***
  - ✓ ***Commission rate for each employee***
  - ✓ Calculate how much money each employee has earned. Who made the most money? Who made the least money *(Adapted from http://www.doe.state.la.us/topics/comprehensive_curriculum.html)*

- **Lesson Activity:** *to analyze and solve linear equations and pairs of simultaneous linear equations*
- This lesson is intended to help assess how well students are able to:
  - Interpret a situation and represent the variables mathematically
  - Select appropriate mathematical methods to use
  - Systematically vary constraints and study the effects
  - Interpret and evaluate data and find break-even point
  - Clearly communicate their reasoning
- **Class Activity: Water displacement** *(Adapted from http://www.doe.state.la.us/topics/comprehensive_curriculum.html)*

- ***7.4 Cooperative Learning Activity*** (*McDougal Littell,* Algebra 1*, 2004, Chapter 7 Resource Books*, p.60)
- ***7.5 Graphing Calculator Lesson Opener*** (*McDougal Littell,* Algebra 1*, 2004, Chapter 7 Resource Books*, p.66)
- ***9.3 Graphing Calculator Lesson Opener*** (*McDougal Littell,* Algebra 1*, 2004, Chapter 9 Resource Books*, p.37)
- ***9.3 Graphing Calculator Activity*** (*McDougal Littell,* Algebra 1*, 2004, Chapter 9 Resource Books*, p.40)
- ***9.4 Visual Approach Lesson Opener*** (*McDougal Littell,* Algebra 1*, 2004, Chapter 9 Resource Books*, p.55)
- ***9.6 Activity Lesson Opener*** (*McDougal Littell,* Algebra 1*, 2004, Chapter 9 Resource Books*, p.85)

- Discuss multiple strategies to solve the real world applications:

Concept Application: *Sale (Created by Dr. M. K. Sran)*
Concept Application: *Catering (Created by Dr. M. K. Sran)*
Released PATs: *Counting Numbers*

Benchmark Assessment Quarter 1

1. Students will demonstrate how numbers are applied in real-world situations
2. Students will be able to use properties of three dimensional objects using two dimensional representations
3. Students will be able to use geometric models to solve real-life problems
4. Students will successfully explain the impact of change of an object's dimensions on its surface area, volume, and perimeter
5. Students will be able to demonstrate that mathematical ideas are interconnected and build on each other

Benchmark Assessment Quarter 2

1. Students will demonstrate how theoretical and empirical probabilities are related.
2. Students will be able to use probability and expected value to decide whether a game is fair.
3. Students will be able understand that theoretical probability is dependent on the sample space of an event.
4. Students will successfully use systematic listing and counting to organize outcomes.
5. Students will be able to use visual representations of data to perform analysis.

Benchmark Assessment Quarter 3

1. Students will demonstrate how linear models are used to approximate real-life situations.
2. Students will be able to demonstrate that the factors and x-intercepts of polynomials are related.
3. Students will be able to show that a graph and its equation are in an interdependent relationship.
4. Students will successfully find the zeros of a polynomial.
5. Students will be able to use transformations in real life situations.

Benchmark Assessment Quarter 4

1. Students will demonstrate how linear functions are used to solve real world applications problems in business and other areas.
2. Students will be able to use properties of three-dimensional objects using two-dimensional representations to solve applications in real life situations.
3. Students will be able to use geometric models to solve real-life problems.
4. Students will successfully explain the impact of change of an object's dimensions on its surface area, volume, and perimeter and use this to solve problems arising in real life contexts.
5. Students will be able to demonstrate that mathematical ideas are interconnected and build on each other