

Algebra III Semester Exam Review

Answer each of the following:

1. Solve

a. $|x+3| > 5$

b. $|2x-3| \leq 5$

2. Determine whether the ordered pair is a solution for the given inequalities. $(1, -2)$

a. $y > 2x - 5$ *yes*

b. $y \leq |x| + 6$ *yes*

3. Identify the vertical and horizontal asymptotes

a. $g(x) = \frac{1}{x+5}$

b. $f(x) = \frac{x+5}{x^2-9x+20}$

c. $h(x) = \frac{x+3}{2x-8}$

4. Find the inverse of each

a. $f(x) = \sqrt{x+2}$

b. $f(x) = x^2 - 2$

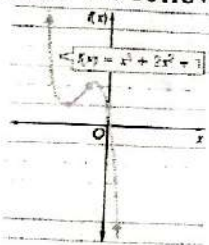
c. $f(x) = \sqrt[3]{3x-5} + 1$

5. Determine whether the function is continuous at $x = 1$

a. $f(x) = \frac{2x}{x^2-1}$ *NO*

b. $f(x) = \frac{3x}{x+2}$ *yes*

6. Answer the following about the graphs.



a. Determine the values of x between in which a real zero is located *b/t 0 and 1*

b. Determine the number of real zeros from the graph above. *1*

7. Use synthetic division

a. $(x^2 - 7x + 12) \div (x - 3)$

b. $(x^3 + 2x^2 - 5x + 7) \div (x - 1)$

8. Given the equation $x^2 - 6x - 16 = 0$, answer the following:

a. Find the leading coefficient and degree

b. Factor and Solve

c. solve by completing the square

d. solve by quadratic formula

e. find the discriminant

f. classify the roots

9. Evaluate

a. $\log_5 \sqrt{5}$

c. $\ln e^{5x}$

e. $\sqrt{5} \cdot \sqrt{10}$

g. $\left(\frac{2}{3}\right)^{-2}$

b. $\log_3 9$

d. $e^{\ln 7}$

f. $\log_5 9$

h. $\left(\sqrt[3]{64}\right)^2$

10. Answer each of the following:

a. Write $5^3 = 125$ in logarithmic form.

b. Write $\log_3 9 = 2$ in exponential form.

11. Solve

a. $\log_3 (2x+5) = \log_3 10$

b. $\log_5 x + \log_5 10 = \log_5 70$

c. $\ln 7x = 5$ *change to $\ln 7x = 5$*

d. $e^{2x} = 7$

12. Find the next three terms for each of the following:

a. 5, 11, 16, ... *change to 5, 11, 17* b. 8, 4, 2, ...

13. Answer each of the following:

- a. Find the seventh term of the arithmetic sequence in which $a_1 = 3$ and $d = 2$.
 b. Find the sum of the arithmetic series $8 + 5 + 2 + (-1) + \dots + (-13)$.
 c. Find the sum of a geometric series for which $a_1 = 7$, $n = 4$, and $r = 2$.

14. Evaluate

a. $\lim_{n \rightarrow \infty} \frac{2x+5}{3x+4}$ $\frac{2}{3}$

c. $\lim_{n \rightarrow \infty} \frac{2x+5}{3x^2+4}$ 0

e. $\sum_{n=1}^3 2 \cdot 3^{n-1}$

b. $\lim_{n \rightarrow \infty} \frac{2x^2+5}{3x+4}$ DNE

d. $\lim_{n \rightarrow \infty} 5$ 5

f. $(x+y)^5$

15. Answer each of the following:

- a. Ed purchased six new shirts and four new pairs of pants. How many new outfits can he make with these items? $6 \cdot 4 = 24$
 b. Evaluate $P(7, 3)$ and $C(7, 3)$ $P(7, 3) = 210$ $C(7, 3) = 35$
 c. A green die and a yellow die are tossed. What is the probability that a 2 will appear on both dice? $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$
 d. A bag contains 3 cherry, 4 strawberry, and 5 grape-flavored candies. What is the probability of selecting a cherry or a grape flavored candy? $\frac{3}{12} + \frac{5}{12} = \frac{8}{12} = \frac{2}{3}$
 e. How many different arrangements of the letters of the word *Nathan* are possible? $\frac{6!}{2!2!} = 180$
 f. If the odds of achieving something is $\frac{3}{2}$, what is the probability? $\frac{3}{5}$
 g. Give an example of independent and dependent events.

16. Find the limit of each

a. $\lim_{x \rightarrow 5} (x+3)$

c. $\lim_{x \rightarrow 2} \sqrt{4x-3}$

e. $\lim_{x \rightarrow 4} \frac{x^2-6x+8}{x-4}$

b. $\lim_{x \rightarrow 2} (-x-1)$

d. $\lim_{x \rightarrow -3} \frac{x^3+27}{x^2-9}$

f. $\lim_{x \rightarrow 1} \frac{x-1}{x^2+2x-3}$

17. Find the derivative of each of the following:

a. $y = 3$

b. $f(x) = 7x^3$

c. $y = (3x^2+1) \cdot 2x^4$ d. $f(x) = 3x + x^{-3}$

18. Given $f(x) = \begin{cases} 8 & , x < 3 \\ 3x-1 & , 3 \leq x < 5 \\ x^2 & , x \geq 5 \end{cases}$

a. $f(4)$

b. $f(0)$

c. $f(3)$

d. $f(5)$

19. Write an equation in slope intercept form for each line described.

a. slope = 2 and y-int = -3

c. passes through $(8, 1)$ and $(-3, 1)$

b. slope = 3 and passes through $(2, 5)$

d. passes through $(1, -1)$ and parallel to

$y = 2x - 3$. Then find out perpendicular.

perpendicular $(4, -1)$

$$1a) |x+3| > 5$$

$$x+3 > 5 \text{ or } x+3 < -5$$

$$\boxed{x > 2 \text{ or } x < -8}$$

$$1b) |2x-3| \leq 5$$

$$-5 \leq 2x-3 \leq 5$$

$$-2 \leq 2x \leq 8$$

$$\boxed{-1 \leq x \leq 4}$$

$$3A) g(x) = \frac{1}{x+5}$$

$$x+5=0$$

$$\boxed{\text{VA } x = -5}$$

$$\boxed{\text{HA } y = 0}$$

$$3B) f(x) = \frac{x+5}{x^2-9x+20}$$

$$x^2-9x+20=0$$

$$(x-4)(x-5)=0$$

$$\boxed{\text{VA } x=4, x=5}$$

$$\boxed{\text{HA } y=0}$$

$$3C) h(x) = \frac{x+3}{2x-8}$$

$$2x-8=0$$

$$\boxed{x=4}$$

$$\boxed{y = \frac{1}{2}}$$

$$4A) f(x) = \sqrt{x+2}$$

$$x = y^2 + 2$$

$$x^2 = y^2 + 2$$

$$x^2 - 2 = y^2$$

$$\boxed{f^{-1}(x) = x^2 - 2}$$

$$4B) f(x) = x^2 - 2$$

$$x = y^2 - 2$$

$$x + 2 = y^2$$

$$\pm \sqrt{x+2} = y$$

$$\boxed{f^{-1}(x) = \pm \sqrt{x+2}}$$

$$4C) f(x) = \sqrt[3]{3x-5} + 1$$

$$x = \sqrt[3]{3y-5} + 1$$

$$x-1 = \sqrt[3]{3y-5}$$

$$(x-1)^3 = 3y-5$$

$$\boxed{f^{-1}(x) = \frac{(x-1)^3 + 5}{3}}$$

$$7A) \begin{array}{r} 3 \overline{) 1 - 7 12} \\ \underline{ 3 - 12} \\ 1 - 4 0 \end{array}$$

$$\boxed{x-4}$$

$$7B) \begin{array}{r} 1 \overline{) 1 2 - 5 7} \\ \underline{ 1 3 - 2} \\ 1 3 - 2 5 \end{array}$$

$$x^2 + 3x - 2 + \frac{5}{x-1}$$

$$\boxed{x^2 + 3x - 2 \text{ R } 5}$$

$$8A) \text{ degree 2}$$

$$\text{leading coeff } 1$$

$$8B) x^2 - 6x - 16 = 0$$

$$(x+2)(x-8) = 0$$

$$\boxed{x = -2} \quad \boxed{x = 8}$$

$$8C) x^2 - 6x - 16 = 0$$

$$-\frac{b}{2} = -3 \quad x^2 - 6x = 16$$

$$(-3)^2 = 9 \quad x^2 - 6x + 9 = 16 + 9$$

$$(x-3)^2 = 25$$

$$x-3 = \pm \sqrt{25}$$

$$x-3 = \pm 5$$

$$x = 3 \pm 5$$

$$\boxed{x = 8}$$

$$\boxed{x = -2}$$

$$x^2 - 6x - 16 = 0$$

$$b^2 - 4ac$$

$$(-6)^2 - 4(1)(-16)$$

$$36 + 64$$

$$8e) \boxed{100}$$

8f) 2 real rational roots

$$8d) x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{6 \pm \sqrt{100}}{2(1)}$$

$$x = \frac{6 \pm 10}{2}$$

$$x = \frac{6+10}{2}$$

$$\boxed{x=8}$$

$$x = \frac{6-10}{2}$$

$$\boxed{x=-2}$$

$$9a) \log_5 \sqrt{5}$$

$$\log_5 5^{\frac{1}{2}}$$

$$\boxed{\frac{1}{2}}$$

$$9b) \log_3 9$$

$$\boxed{2}$$

$$9c) \ln e^{5x} = \boxed{5x}$$

$$9d) e^{\ln 7} = \boxed{7}$$

$$9e) \sqrt{5 \cdot 50}$$

$$\sqrt{250}$$

$$\sqrt{25 \cdot 10}$$

$$\boxed{5\sqrt{10}}$$

$$9f) \log_5 9$$

$$\frac{\log 9}{\log 5}$$

$$\approx 1.3652$$

$$9g) \left(\frac{2}{3}\right)^{-2}$$

$$\left(\frac{3}{2}\right)^2 = \boxed{\frac{9}{4}}$$

$$9h) \left(\sqrt[3]{64}\right)^2$$

$$\boxed{16}$$

$$10A) 5^3 = 125$$

$$\log_5 125 = 3$$

$$10B) \log_3 9 = 2$$

$$3^2 = 9$$

$$11A) \log_3 (2x+5) = \log_3 10$$

$$2x+5 = 10$$

$$\boxed{x = \frac{5}{2}}$$

$$11B) \log_5 x + \log_5 10 = \log_5 70$$

$$10x = 70$$

$$\boxed{x=7}$$

$$11c) \ln 7x = 5$$

$$\frac{7x}{7} = \frac{e^5}{7}$$

$$x \approx 21.2019$$

$$11d) e^{2x} = 7$$

$$\ln e^{2x} = \ln 7$$

$$2x = \ln 7$$

$$x = \frac{\ln 7}{2}$$

$$x \approx 0.9730$$

$$12A) 5, 11, 17, \dots$$

$$23, 29, 35, \dots$$

$$12B) 1, \frac{1}{2}, \frac{1}{4}, \dots$$

$$13A) n=7 \quad a_1=3 \quad d=2$$

$$a_n = 3 + 2(7-1)$$

$$= 15$$

$$13B) -13 = 8 - 3(n-1)$$

$$-21 = -3(n-1)$$

$$7 = n-1$$

$$n=8$$

$$S_n = \frac{8}{2}(8-13)$$

$$= -20$$

$$13C) S_n = \frac{7-7(2)^n}{1-2}$$

$$= 105$$

$$14E) 2 \cdot 3^{1-1} + 2 \cdot 3^{2-1} + 2 \cdot 3^{3-1}$$

$$2(1) + 2(3) + 2(9)$$

$$2 + 6 + 18$$

$$\boxed{26}$$

$$14F) x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

$$16A) \lim_{x \rightarrow 5} (x+3)$$

$$5+3 = \boxed{8}$$

$$16B) \lim_{x \rightarrow 2} (-x-1)$$

$$-2-1 = \boxed{-3}$$

$$16C) \lim_{x \rightarrow 2} \sqrt{4x-3}$$

$$\sqrt{4(2)-3} = \boxed{\sqrt{5}}$$

$$16D) \lim_{x \rightarrow -3} \frac{x^3+27}{x^2-9}$$

$$\lim_{x \rightarrow -3} \frac{(x+3)(x^2-3x+9)}{(x+3)(x-3)}$$

$$\frac{(-3)^2-3(-3)+9}{-3-3} = \frac{27}{-6} = \boxed{-\frac{9}{2}}$$

$$16E) \lim_{x \rightarrow 4} \frac{x^2-6x+8}{x-4}$$

$$\lim_{x \rightarrow 4} \frac{(x+2)(x-4)}{x-4}$$

$$4+2 = \boxed{6}$$

$$16F) \lim_{x \rightarrow 1} \frac{x-1}{x^2+2x-3}$$

$$\lim_{x \rightarrow 1} \frac{x-1}{(x-1)(x+3)}$$

$$\frac{1}{1+3} = \boxed{\frac{1}{4}}$$

$$17a) y = 3$$

$$\frac{dy}{dx} = 0$$

$$17b) f(x) = 7x^3$$

$$f'(x) = 21x^2$$

$$17c) y = 6x^6 + 2x^4$$

$$\frac{dy}{dx} = 36x^5 + 8x^3$$

$$17d) f(x) = 3x + x^{-3}$$

$$f'(x) = 3 - 3x^{-4}$$

$$f'(x) = 3 - \frac{3}{x^4}$$

$$18A) 3(4) - 1$$

$$\boxed{11}$$

$$18B) f(6) = \boxed{8}$$

$$18C) f(3) = 3(3) - 1$$

$$\boxed{8}$$

$$18d) f(5) = 5^2 - \boxed{25}$$

$$19A) y = 2x - 3$$

$$19B) m = 3 \quad (2, 5)$$

$$y - 5 = 3(x - 2)$$

$$y - 5 = 3x - 6$$

$$y = 3x - 1$$

$$19C) (8, 1) \text{ and } (-3, 1)$$

$$m = 0$$

$$y = 1$$

$$19D) (1, -1) \quad y = 2x - 3$$

$$m = 2 \quad \text{or } m = 2$$

$$y + 1 = 2(x - 1)$$

$$y + 1 = 2x - 2$$

$$y = 2x - 3$$

$$(4, -1) \perp y = 2x - 3$$

$$m = 2 \quad \perp m = -\frac{1}{2}$$

$$y + 1 = -\frac{1}{2}(x - 4)$$

$$y + 1 = -\frac{1}{2}x + 2$$

$$y = -\frac{1}{2}x + 1$$