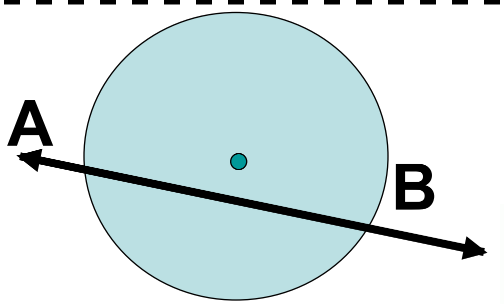
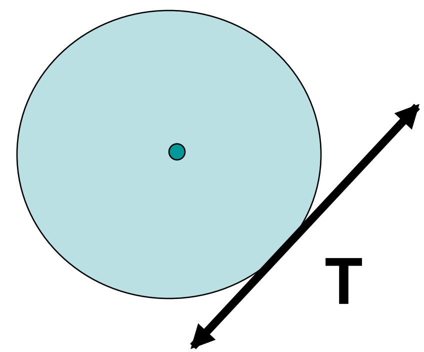
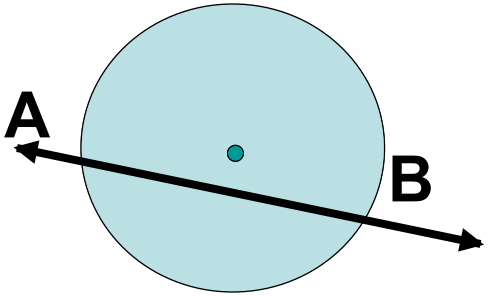
# Secants and Tangents

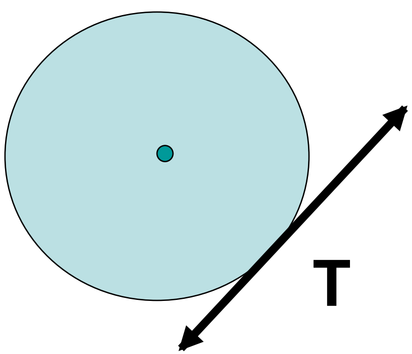


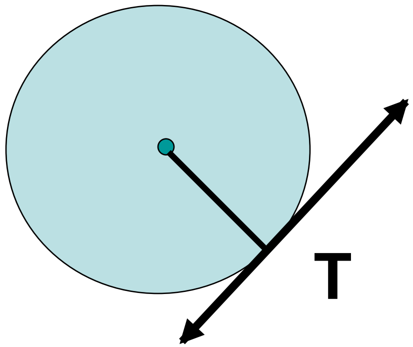



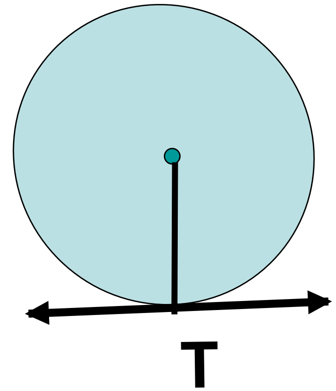



A <u>secant</u> is a line that intersects a circle at exactly two points. (Every secant contains a chord of the circle.)

A <u>tangent</u> is a line that intersects a circle at exactly one point. This point is called the <u>point of tangency</u> or <u>point of contact</u>.



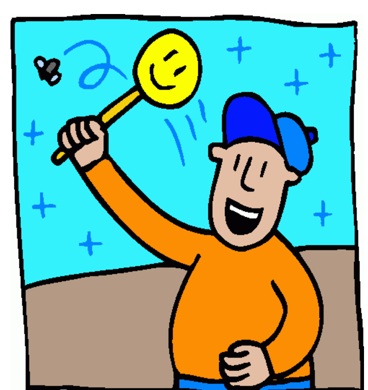




## Tangent lines.

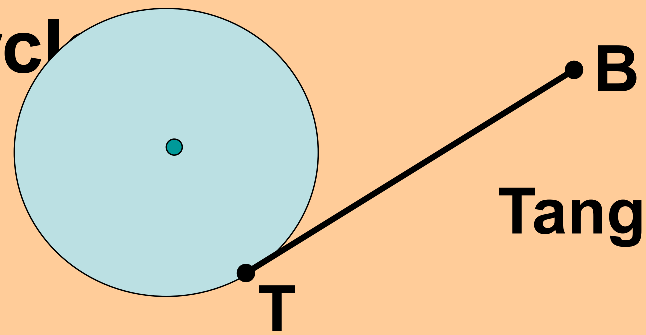
**<u>Postulate</u>: A tangent line is perpendicular to the radius drawn to the point of contact.**

**<u>Postulate:</u> If a line is perpendicular to a radius at its outer endpoint, then it is tangent to the circle.**



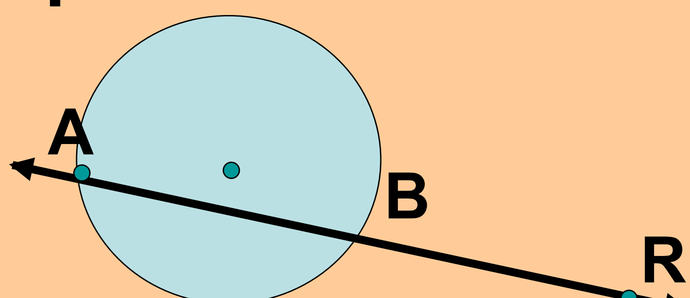
# Secant and Tangent Segments

A <u>tangent segment</u> is a point of a tangent line between the point of contact and a point outside the circl



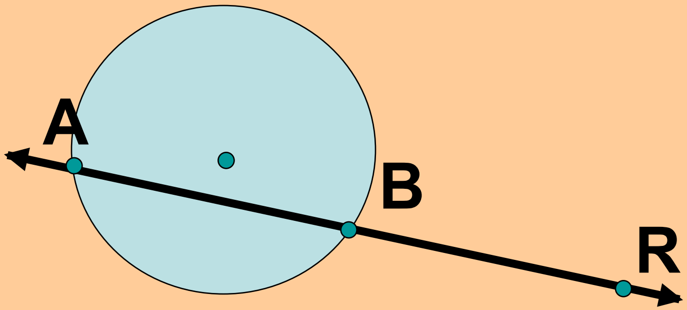

Tangent segment

A <u>secant segment</u> is the part of a secant line that joins a point outside the circle to the farther intersection point of the secant and the circle.



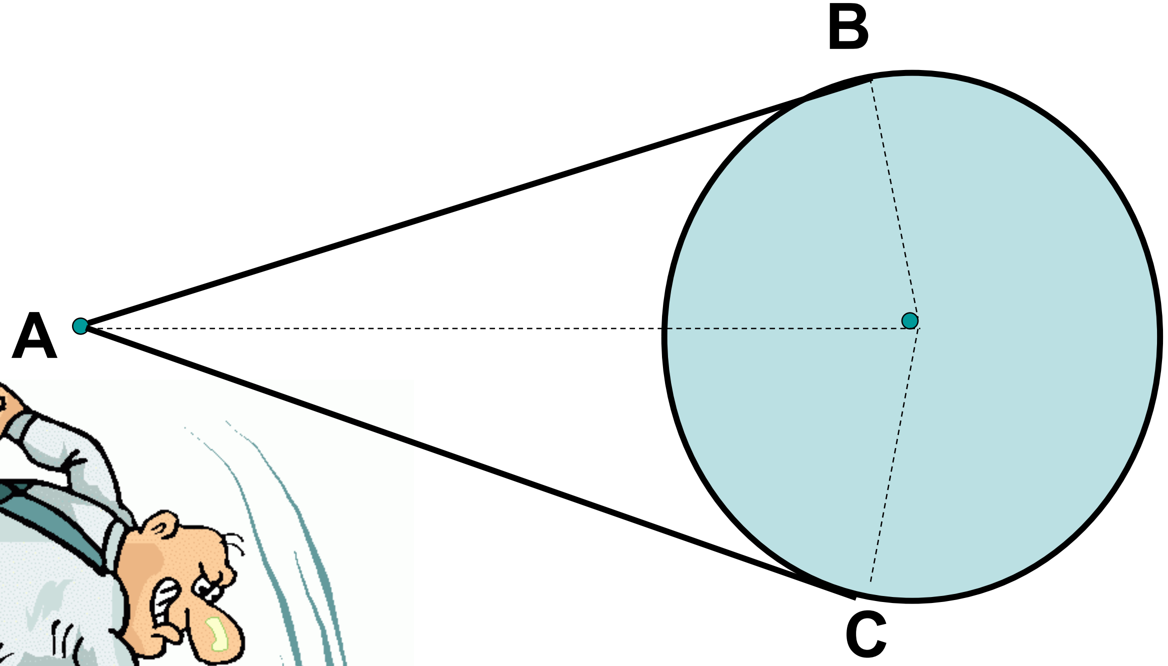

$\overline{AR}$ is the secant segment.

The <u>external part</u> of a secant segment is the part of a secant line that joins the outside point to the nearer intersection point.



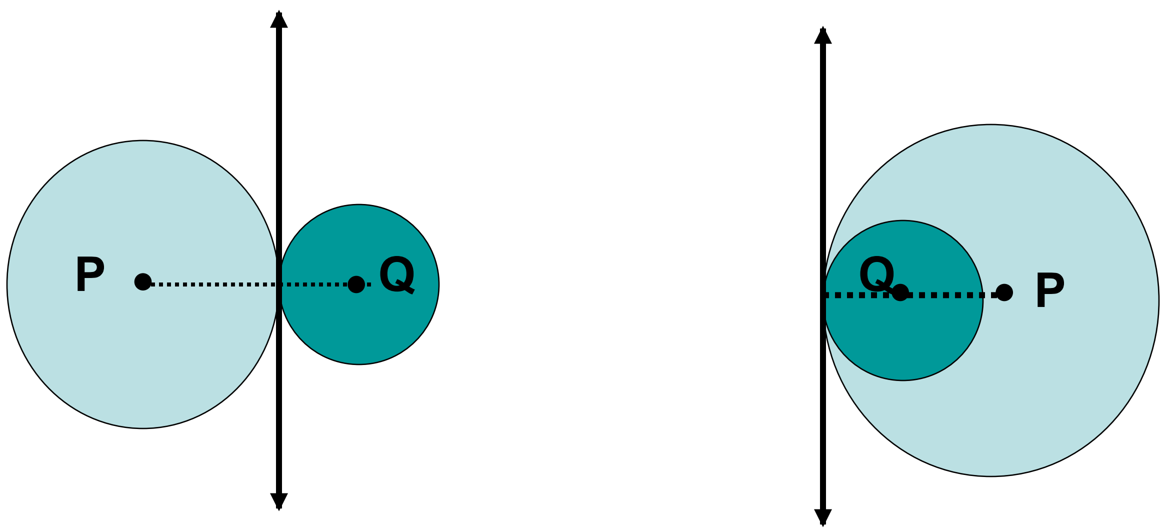

$\overline{BR}$ is the external part.

**Theorem 85**: If two tangent segments are drawn to a circle from an external point, then those segments are congruent. (Two-Tangent Theorem)




# Tangent Circles

**Tangent circles** are circles that intersect each other at exactly one point.




Two circles are **externally tangent** if each of the tangent circles lies outside the other.

Two circles are **internally tangent** if one of the tangent circles lies inside the other.

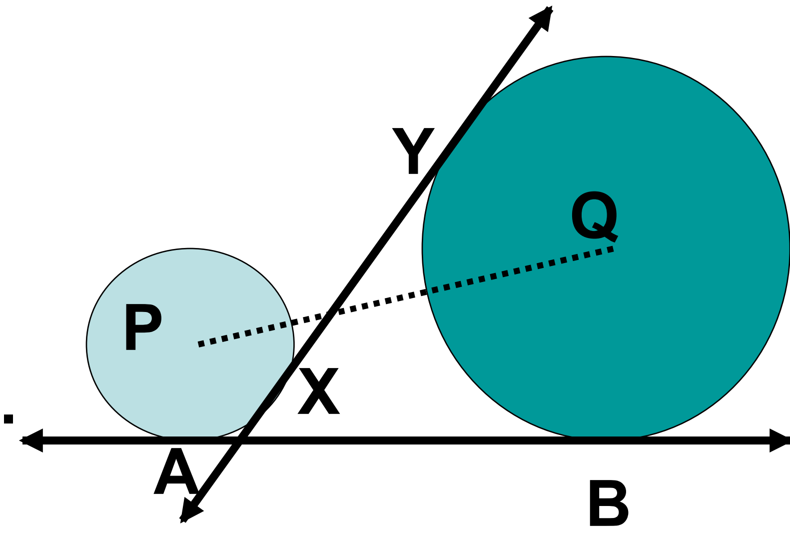
The point of contact lies on the **line of centers**. $\overline{PQ}$

# Common Tangents:

$\overleftrightarrow{PQ}$ is the line of center

$\overleftrightarrow{XY}$ is a common internal tangent.

$\overleftrightarrow{AB}$ is a common external tangent.




**Definition:**

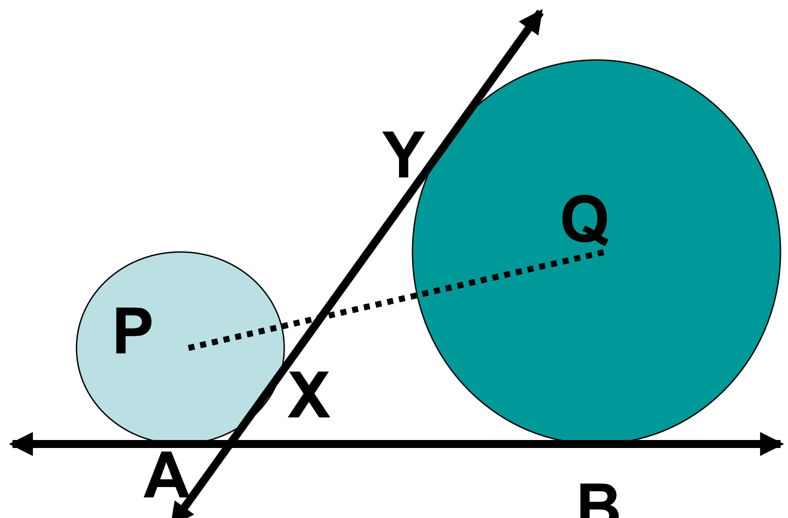
A <u>common tangent</u> is a line tangent to two circles (not necessarily at the same point.)

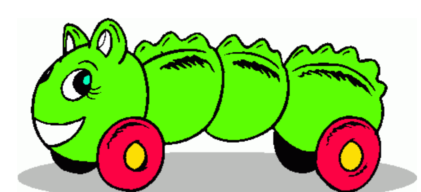
Such a tangent is a <u>common internal tangent</u> if it lies between the circles(intersects the segment joining the centers) or a

<u>common external tangent</u> if it is not between the circles (does not intersect the segment joining the centers.)

In practice, we will frequently refer to a segment as a common tangent if it lies on a common tangent and its endpoints are the tangent's points of contact.

In the diagram for example, $\overline{XY}$ can be called a common internal tangent and $\overline{AB}$ can be called a common external tangent.



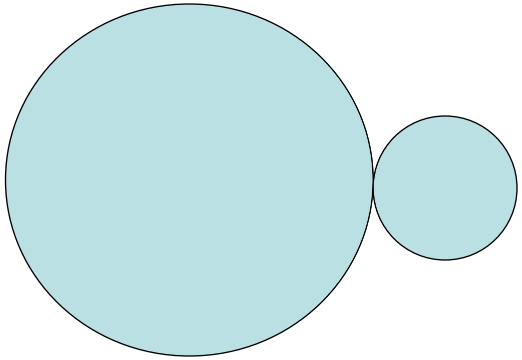



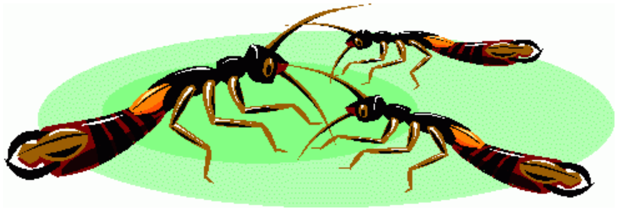
# Practice

- Draw five circles anywhere on your paper, label them 1-5.
  - Draw a tangent line to circle 1.
  - Draw a secant line to circle 2.
  - Draw a common tangent line between circles 3 and 4.
  - Draw an external tangent line between 4 and 5.
  - Draw an internal tangent line between 1 and 2.

# Common Tangent Procedure

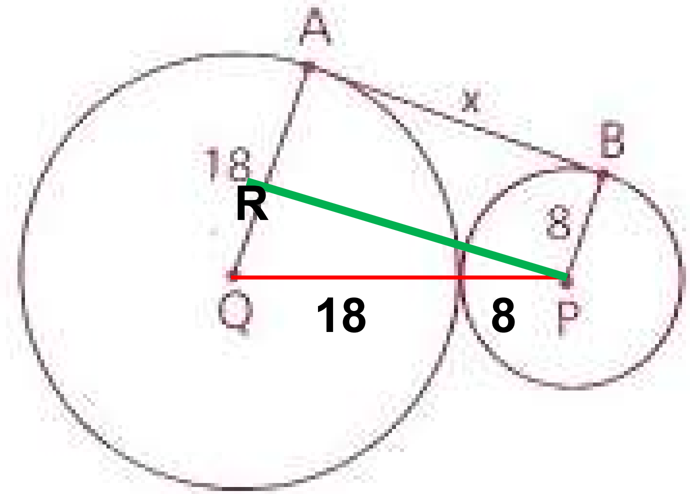
1. Draw the segment joining the centers.
2. Draw the radii to the points of contact.
3. Through the center of the smaller circle, draw a line parallel to the common tangent.
4. Observe that this line will intersect the radius of the larger circle (extended if necessary) to form a rectangle and a right triangle.
5. Use the Pythagorean Theorem and properties of a rectangle.





**Problem #1**

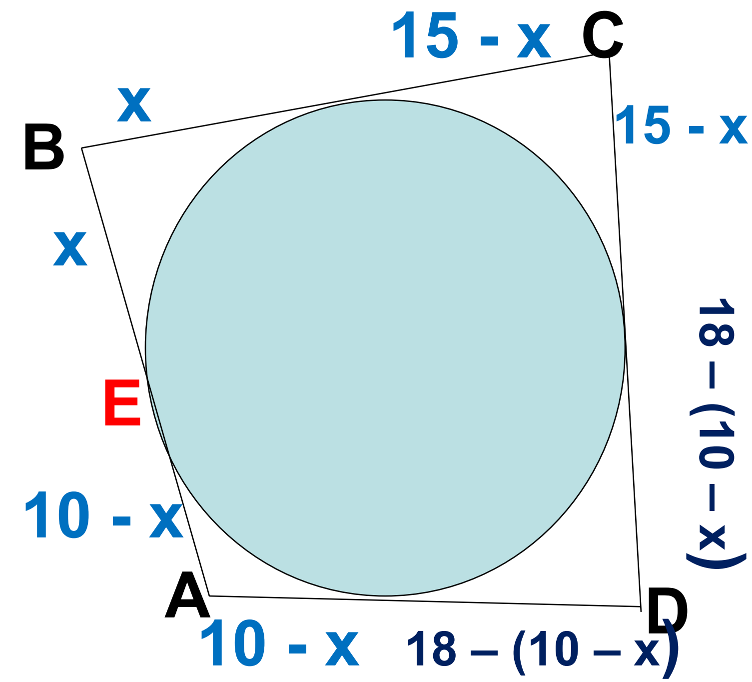
**A circle with a radius of 8 cm is externally tangent to a circle with a radius of 18 cm. Find the length of a common external tangent.**




1. Draw the segment joining the centers.
2. Draw the radii to the points of contact.
3. Through the center of the smaller circle, draw a line parallel to the common tangent.
4. Use the Pythagorean Theorem and properties of a rectangle to solve.
5. In $\Delta RPQ$, $(QR)^2 + (RP)^2 = (PQ)^2$
    1. $10^2 + (RP)^2 = 26^2$
    2. $RP = 24$
6. $AB = 24$ cm

**Problem #2**

Given: Each side of quadrilateral ABCD is a tangent to the circle.
AB = 10, BC = 15, AD = 18.
Find CD.




Let BE = x and "walk around" the figure using the given information and the Two-Tangent Theorem.

$$\begin{aligned} CD &= 15 - x + 18 - (10 - x) \\ &= 15 - x + 18 - 10 + x \\ &= 23 \end{aligned}$$