

G-MG [686730]

Student _____

Class _____

Date _____

Read the following and answer the questions below:

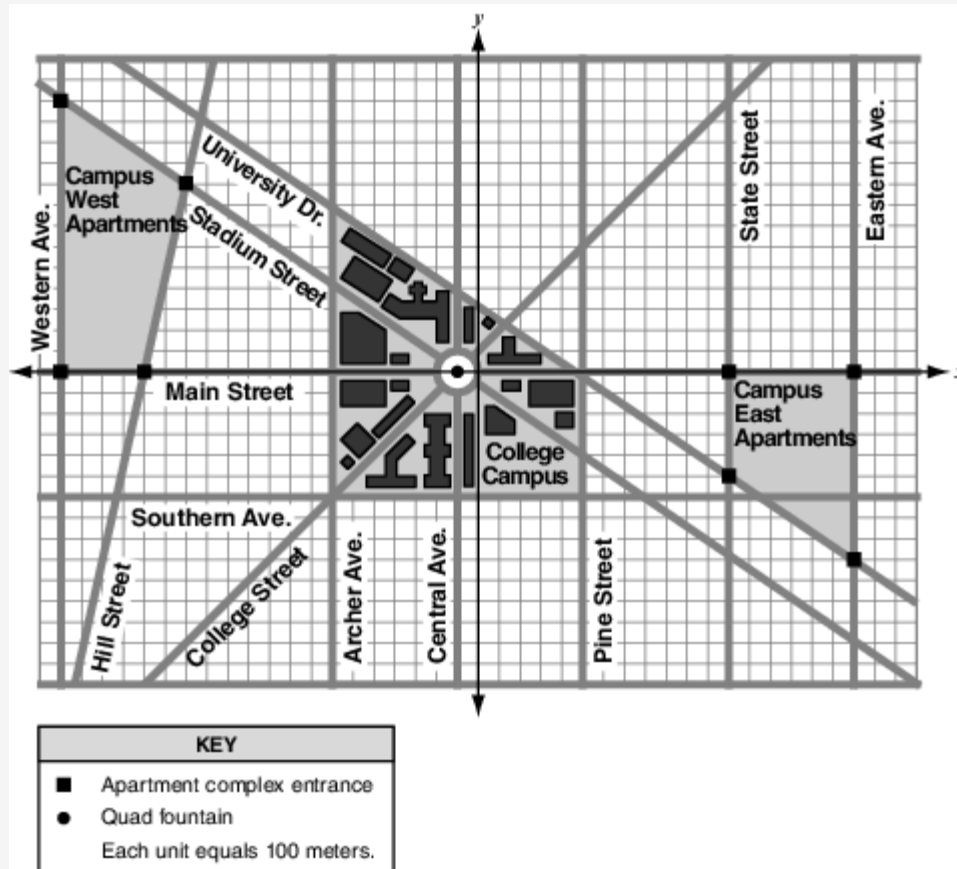
College Apartments

College Apartments

The number of students enrolled in a local college is increasing rapidly, and there is limited space in the high-rise residence halls on the college campus. Because of this, more students are moving to apartments near campus. The college is working with a real estate development company to plan two new apartment complexes to provide housing for the additional students.

The college has requested that each apartment complex have housing for at least 300 students, as well as such amenities as laundry and common study areas, so that it provides a similar environment to that of the residence halls. Each of the residence halls on campus is 11 stories tall; 10 floors each contain 32 one-bedroom dormitories, and 1 floor contains a study lounge, cafeteria, social area, and laundry room.

As shown in the map of the area near the college campus below, one new apartment complex will be to the west of campus and one will be to the east. The new developments are titled Campus West Apartments and Campus East Apartments, respectively.



Michael and Nancy work for the development company, and they are each managing the teams that are designing these two apartment complexes. They both want to create apartments that meet the college's requirements and that students want to live in.

Michael's team is developing a plan for an apartment complex where students will live in 2 large three-story apartment buildings. One building will have 20 two-bedroom apartments on each floor, and the other building will have 16 four-bedroom apartments on each floor. There will also be a separate, smaller building that contains the management office, a common area with a food court, a study center, and a laundry facility. Michael is also planning to put picnic tables outside so students can have an additional area in which to socialize, and students will have the option of getting a parking space in the apartment complex's parking lot.

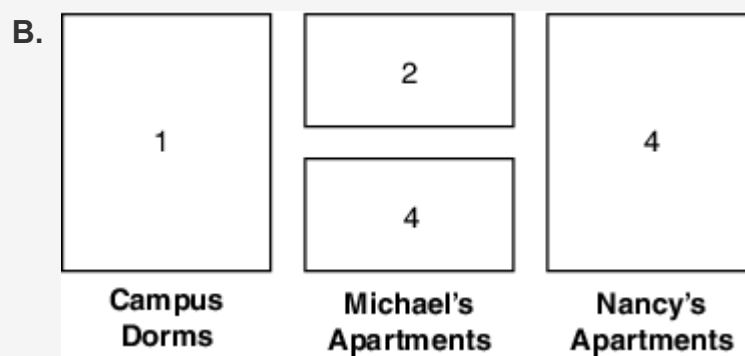
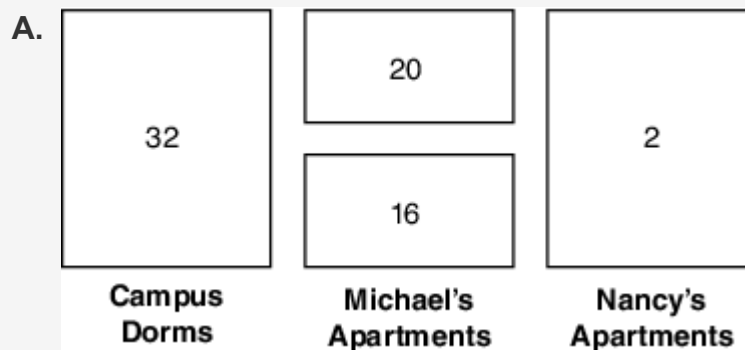
Instead of a few large apartment buildings, Nancy's team has decided that many smaller apartment buildings will make the apartment complex feel

more welcoming. This development will be made up of 15 four-story apartments that each have 2 three-bedroom apartments on each floor. Laundry will be included in each apartment. In this complex, there will also be two smaller buildings that serve as common areas. These buildings will include computer labs, pool tables, and a mini-market. There will not be a parking lot, but there will be an outdoor basketball court and a sand volleyball court.

Students on campus are eager to secure apartments and roommates for next year and are comparing their options to determine which development is best for their needs.

1. Read "College Apartments" and answer the question.

Michael and Nancy are creating mathematical models to illustrate how many students each type of apartment building or dorm can house on **each of its floors**. They assume that each student will have his or her own bedroom. Which model is the **best** comparison?



C.

32×1	20×2 16×4	2×3
Campus Dorms	Michael's Apartments	Nancy's Apartments

D.

$10 \times 32 \times 1$	$3 \times 20 \times 1$ $3 \times 16 \times 4$	$15 \times 3 \times 2$
Campus Dorms	Michael's Apartments	Nancy's Apartments

Read the following and answer the questions below:

The Lionfish Threat

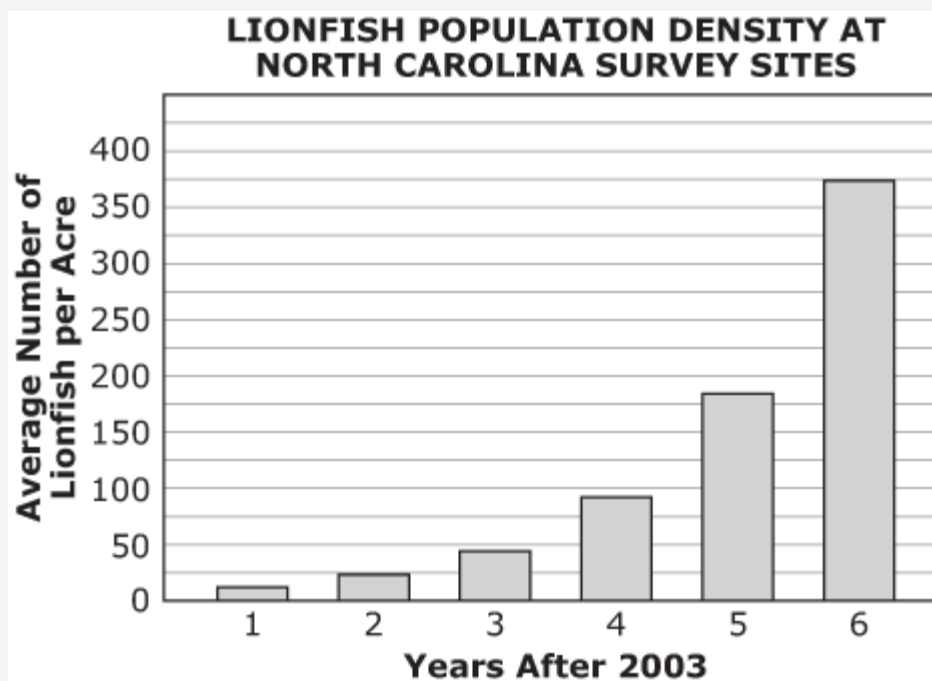
The Lionfish Threat

Fishermen and environmentalists are concerned about the spread of an invasive species known as the lionfish. Originally found in the Pacific and Indian Oceans, this ornamental fish has been highly popular with aquarium owners. It is believed that one such enthusiast may have released several specimens into the waters off the Atlantic coast of Florida or the Carolinas. The impact has had significant consequences on the local reef system.

Lionfish prey upon commercially important species such as snapper and lobster. In some areas, the introduction of lionfish has reduced those populations to one-tenth of their former size. Lionfish also compete against other predators such as the coney grouper. They are able to consume prey

species at two-and-a-half times the rate of their competitors. It is for this reason that lionfish may be contributing to a decline in the number of coney as available food sources become harder to find.

Marine biologists are concerned about the long-term effects of lionfish predation along the Atlantic Coast. To monitor the extent of increase in lionfish numbers, they set up a series of survey sites off the coast of North Carolina. They have been able to count the number of fish observed at each site and calculate the average. This makes it possible to compare the annual population density of lionfish per acre. The bar graph below shows the data collected annually from the survey sites.



2. Read the passage “The Lionfish Threat” and then answer the questions.

Part A

On average, approximately how many more lionfish were there per acre in 2009 than in 2008?

Part B

Approximately how many times greater was the lionfish population per acre in 2009 than in 2004?

Part C

If x represents the total number of lionfish observed in 2004, write an expression, in terms of x , that represents the number of acres of habitat in 2004.

Part D

If 1 square mile = 640 acres, what was the approximate lionfish population density per square mile in 2009?

Part E

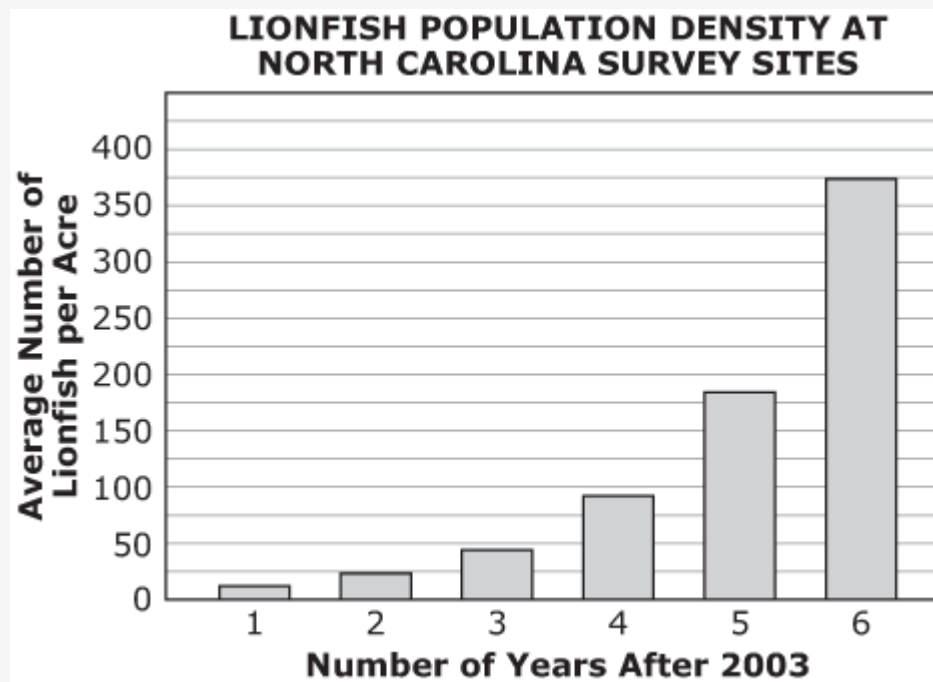
If the lionfish density per acre in 2010 was half the density in 2009, one of the causes might be a decrease in lionfish population. What might another cause be?

Part F

How would it be possible for the combined snapper and lobster population in 2010 to be one tenth the 2009 population, but for the population density of snapper and lobster to remain the same?

Part G

Explain whether the graph below can be defined by a linear function. If so, state the linear function.



Read the following and answer the questions below:

Designing a Hotel

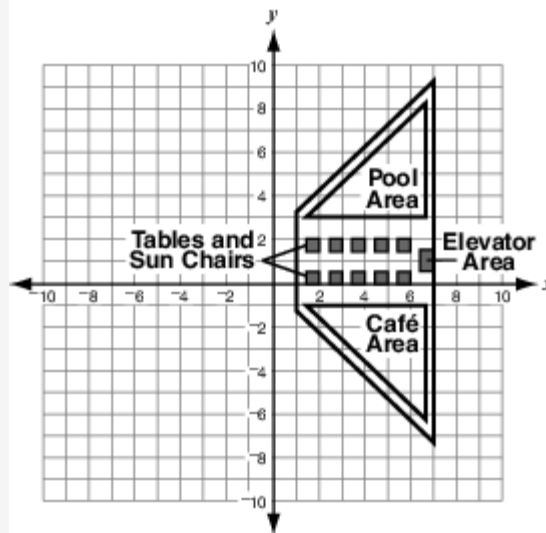
Designing a Hotel

Yasmin is a high school student who wants to be an architect. A local architecture design firm is sponsoring a contest to design a new high-rise hotel building that will be located in the downtown area of the city in which Yasmin lives. The winner of the contest will earn an internship with the architecture firm, and Yasmin has decided to submit a design. She hopes that she will be able to learn more about architecture by creating the design and learn even more if she wins the internship.

Yasmin wants hotel guests to be able to see the city and enjoy the warm weather, so one of the first elements she puts in her design plan is a rooftop deck. She asks her friends at school what they like to do when they visit a hotel, and the most common response is to go swimming. Yasmin decides to combine the two ideas and put the hotel pool on the roof.

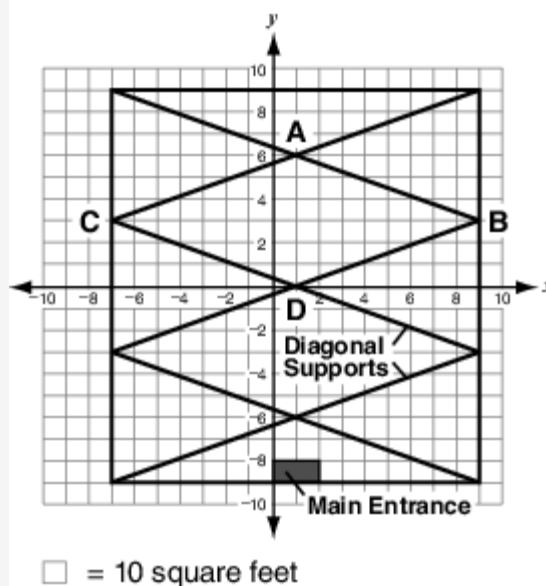
She also incorporates a café area so that people can eat while enjoying the view. Then, she adds an area between the café and the pool where people can sit at tables in sun chairs, admiring the city skyline.

YASMIN'S DESIGN OF THE HOTEL'S ROOFTOP DECK



Another thing Yasmin wants is for her building to stand out among the other tall buildings downtown. She uses nonvertical supports to create an interesting pattern on the front of the building. The main structural supports will be inside and will not be visible from the outside.

YASMIN'S DESIGN OF THE FRONT OF THE HOTEL



Lastly, Yasmin plans to cover most of the outside of the building with dark glass. This way, she thinks, the building will look shiny and modern, and all of the hotel's guests will be able to have a great view of the city from their rooms.

Yasmin submits her design plans to the contest before the deadline and waits anxiously to find out whether she has won. She knows that winning this contest could be the start of the career she's hoping for.

3. Read "Designing a Hotel" and answer the question.
Which geometric solid **best** models the shape of the hotel?

- A. a trapezoidal pyramid
- B. a right trapezoidal prism
- C. an oblique rectangular prism
- D. a truncated rectangular pyramid

-
4. A farmer planted tomato plants and green pepper plants in a rectangular field with a width of 40 meters and a length of 50 meters.

- In 40% of the field, he planted 70 rows of tomato plants with 80 plants in each row.
- In 60% of the field, he planted 100 rows of green pepper plants with 120 plants in each row.

Which is closest to the overall density of the field, in plants per square meter?

- A. 4.7
- B. 5.3

C. 8.5

D. 8.8

5. **Pet Fence**

Dana is planning to build an enclosure in her yard so that her dogs can play in a secure area. She is planning to use fencing that comes in rigid 6-foot-long sections. She cannot bend the individual sections, but she can join them at any angle to form different polygons. Dana has enough money to buy 24 sections of fencing, including one with a gate. Dana plans to use all 24 sections of fencing when building the enclosure for her dogs.

Part A. Dana first considers making a rectangular enclosure. In the table below, list all possible ways Dana could use the fencing to make an enclosure that has an area of at least 900 square feet. What is the greatest rectangular area Dana could enclose with the 24 sections of fencing? Explain your answer.

Length (in number of fence sections)	Width (in number of fence sections)	Area (in fence sections by fence sections)	Length (in feet)	Width (in feet)	Area (in square feet)

Part B. Dana decides to sketch models of the rectangular enclosures. She uses tick-marks to show each section of fencing on the models, and she labels what will be the actual length and width of the enclosures. If ---|---| represents two pieces of fencing placed next to each other, use a ruler or graph paper to sketch models of all of the possible enclosures that have an area of at least 1,000 square feet. Label the models with what will be the actual lengths and widths of the enclosures. How does the area of each enclosure, in square feet, relate to the area of each enclosure in fence section by fence section? Use the models you drew to help explain your answer.

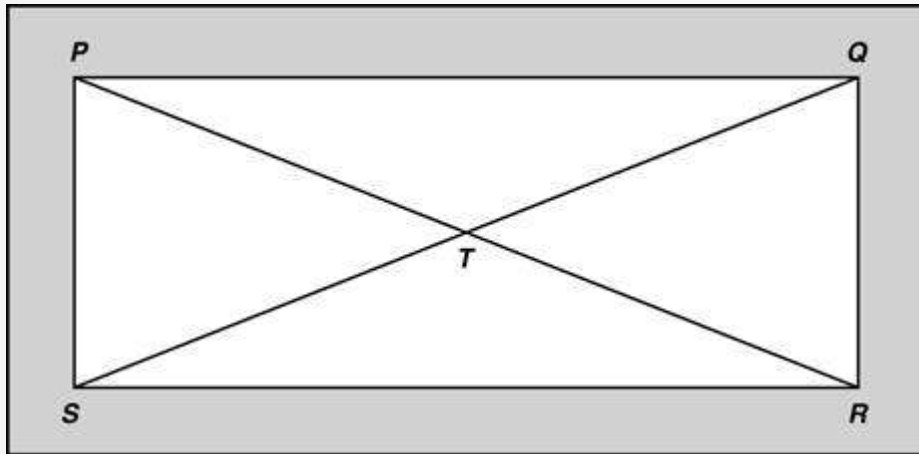
Part C. Dana is also considering making the enclosure in the shape of a regular hexagon. Use a ruler or graph paper to sketch a model of a regular hexagon with tick-marks to show how many fence sections would be needed for each side. Include the length of each side, in feet. Then, divide the hexagon into sections so that you can compute its area in square feet. Show how you chose to divide the hexagon and show your work for computing the area. When appropriate, leave side lengths in radical form. For your final answer, round the area to the nearest square foot.

Part D. Dana's sister suggested she make the enclosure in the shape of a regular octagon. Use a ruler or graph paper to sketch a model of a regular octagon with tick-marks to show how many fence sections would be needed for each side. Include the length of each side, in feet. Then, divide the octagon into sections so that you can compute its area in square feet, and sketch your divisions on your model. Show your work and label the lengths you used in your calculations. When appropriate, leave side lengths in radical form. For your final answer, round the area to the nearest square foot.

Part E. If Dana uses all 24 pieces of fencing as the sides of the enclosure, how could Dana construct the enclosure in order to maximize the area? Describe the configuration and explain your answer.

6. Brianna made a frame. She connected the opposite corners of the frame with wire, as shown in the

diagram below.



Brianna wants to make sure that the interior angles on the frame are right angles. Which method could Brianna use to verify that $\angle SPQ$, $\angle PQR$, $\angle QRS$, and $\angle RSP$ are all right angles?

- A. Measure $\overline{PR}$ and $\overline{QS}$ to make sure they are equal, measure $\overline{PS}$ and $\overline{QR}$ to make sure they are equal, and measure $\overline{PQ}$ and $\overline{RS}$ to make sure they are equal.
- B. Measure $\overline{ST}$ and $\overline{QT}$ to make sure they are equal, measure $\overline{PT}$ and $\overline{RT}$ to make sure they are equal, and measure $\overline{PQ}$ and $\overline{RS}$ to make sure they are equal.
- C. Measure $\overline{PS}$ and $\overline{QR}$ to make sure they are equal, and measure $\overline{PQ}$ and $\overline{RS}$ to make sure they are equal.
- D. Measure $\overline{PT}$ and $\overline{QT}$ to make sure they are equal, and measure $\overline{ST}$ and $\overline{RT}$ to make sure they are equal.

7. The table below lists the population and area of four different states from the same year.

Population and Area

State	Population	Area (square miles)
Alaska	626,932	570,374
Georgia	8,186,453	57,919
Kansas	2,688,418	81,823
Texas	20,851,820	261,914

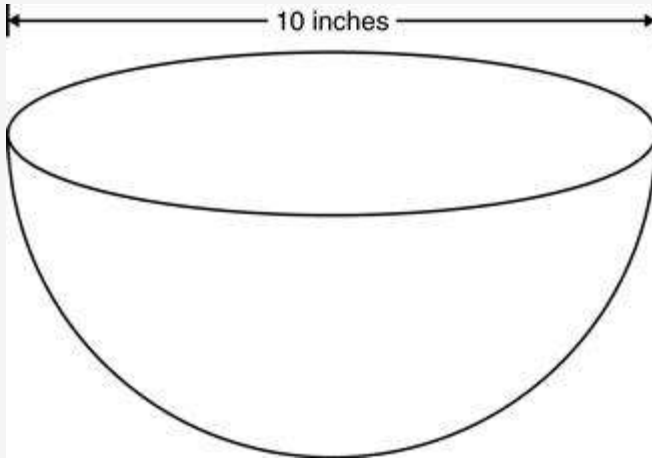
Based on the information in the table, which state had the greatest population density in that year?

- A.** Alaska
- B.** Georgia
- C.** Kansas
- D.** Texas

- 8.** Mr. Parker purchased gold in the shape of a rectangular prism. The gold has a length of 7 centimeters, a width of 5 centimeters and a mass of 202 grams. Since gold has a density of approximately 19.3 g/cm^3 , which value is closest to the thickness of his gold bar?

- A.** 0.3 cm
- B.** 0.9 cm
- C.** 3.3 cm
- D.** 6.5 cm

- 9.** Gerald has a punch bowl in the shape of a hemisphere as shown below.



One cup is equivalent to approximately 14.44 cubic inches. Which of the following is closest to the maximum number of cups the punch bowl can hold?

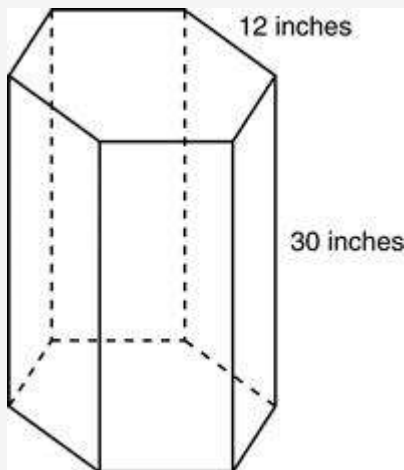
- A.** 290 cups
- B.** 145 cups
- C.** 36 cups
- D.** 18 cups

10. The density of water is 1000 kilograms per cubic meter, and the density of ice is about 916 kilograms per cubic meter. If 575 kilograms of water and 275 kilograms of ice is combined in a container, about how much volume would the mixture take up?

- A.** 5.0 m^3
- B.** 2.3 m^3
- C.** 0.9 m^3

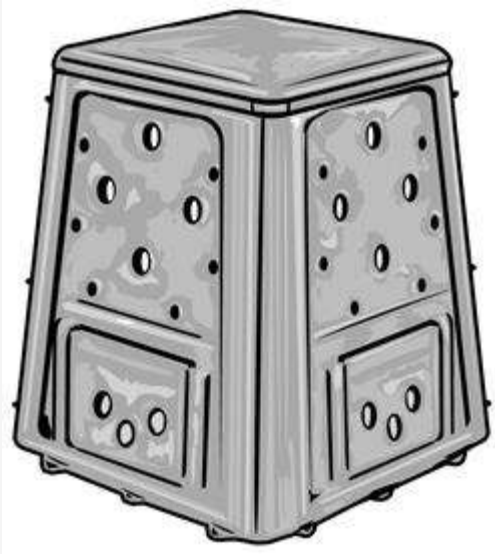
D. 0.4 m^3

- 11.** Charles bought a fish tank in the shape of a prism with a base shaped like a regular hexagon. The dimensions of the fish tank are shown below.



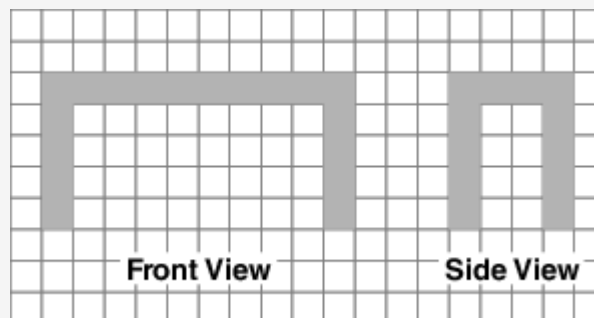
Charles wants to completely fill the fish tank with water. Which of the following is closest to the number of gallons of water Charles needs to fill the fish tank? (1 gallon $\approx$ 231 cubic inches)

- A.** 18.7 gallons
- B.** 48.6 gallons
- C.** 56.1 gallons
- D.** 97.2 gallons
- 12.** The figure below shows a compost bin.



Which process is most likely to be used to estimate the total surface area of the compost bin?

- A. computing the area of 6 rectangles
 - B. computing the area of 4 trapezoids and 2 rectangles
 - C. finding the perimeter of the base and multiplying by the height
 - D. finding the mean of the perimeters of the base and the top and multiplying by the height
13. Katie is using graph paper to sketch a design for a flat wooden bench that will have a rectangular seat. The front and side views that Katie sketched are shown below.



If Katie uses a scale on the graph paper in which each box represents a 6-inch by 6-inch square, what are the dimensions of the seat of the bench Katie is designing?

- A. 2 feet long by 2.5 feet deep
 - B. 5 feet long by 2 feet deep
 - C. 5.5 feet long by 2.5 feet deep
 - D. 10 feet long by 5 feet deep
14. Mr. Smith built a rectangular-shaped sandbox for his children, measuring 4 feet by 4 feet by 6 inches. Sand has a density of approximately 100 pounds per cubic foot and is sold in 50-pound bags for \$3.50 each. How much will it cost Mr. Smith to completely fill the sandbox?
- A. \$56
 - B. \$98
 - C. \$224
 - D. \$672
15. Julius is in charge of planting a peach orchard that covers 15 acres. He uses the same amount of ground space for each tree. Julius plants 25 trees in the section of the orchard called the South Hill region that measures 450 feet by 24 feet. Approximately how many trees does Julius plant in the orchard in all? (1 acre = 43,560 square feet).
- A. 909

B. 1,151

C. 1,513

D. 1,746

16. The table below shows the area and population for four countries in 2012.

2012 Area and Population		
Country	Area (Square Miles)	2012 Estimated Population
France	211,209.38	65,447,374
Indonesia	741,099.62	234,181,400
Qatar	4,415.85	1,696,563
Thailand	198,456.43	63,525,062

Which country is most densely populated?

A. Qatar

B. France

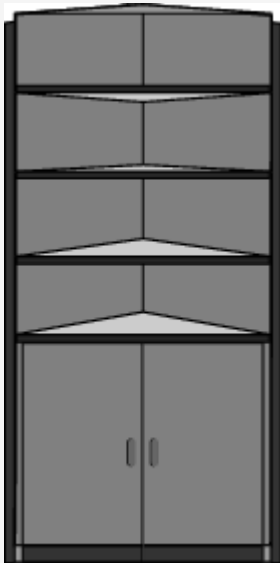
C. Thailand

D. Indonesia

17. Build a Corner Cupboard

You are taking an interest in carpentry and want to design a corner

cupboard for books and knickknacks. A corner cupboard, or cabinet, is shaped like a triangular prism and fits into the corner of a room. You use geometric methods to work out the specifications for the cabinet.



Part A. Given the space you have available, you are thinking that the cabinet should come out 18 inches (in.) from the corner along each wall. How wide will it be across the front? Show your work, and give your answer to the nearest inch.

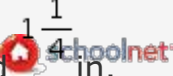
Part B. The back corner of the cabinet measures 90° . What is the measure of the two other angles of the cabinet?

How deep will the cabinet be, from the back corner to the center front of the cabinet? Show your work, and give your answer to the nearest inch.

Part C. You would like the cabinet to be 6 feet tall. At the bottom, there will be 1 shelf enclosed by doors that are 2 feet high. There will be an open shelf at the level of the top of the doors, 3 other evenly spaced shelves, and then the top of the cabinet. You will have to buy extra wood to allow for waste when you cut it, but what is the minimum amount of wood you need for the sides, shelves, and top excluding the doors? Show your work, and give your answer to the nearest square foot.

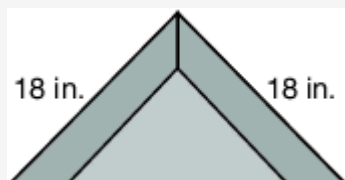
Part D. What is the area of each shelf in square inches? What is the total volume of the cabinet in cubic feet? Give your answer to the nearest cubic foot. Show your work.

Part E. With triangular shelves, not all of the area is always usable space. You have a set of books you want to put on one of the shelves, with

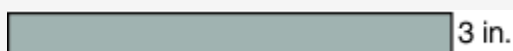
bookends at either end. Each book is 9 in. high, 6 in. wide, and $1\frac{1}{4}$ in. 

thick. If you put the books in a row across the shelf, with the spine of each book at the edge of the shelf, what is the maximum number of books you can put in the row? Draw a sketch, show your work, and explain your answer.

Part F. You are thinking about making a small 3-in. rail for the top shelf, as shown in darker gray below.



Show how you could cut a piece of wood like the one below to make the rail with the least waste possible. Label lengths and angles. What is the minimum length of wood that you would need?



18. The wall of a kitchen has a length of 12 feet (ft) and a height of 10 ft. The wall is to be tiled to cover half of its height and its entire length. How many tiles of dimensions $0.75 \text{ ft} \times 0.50 \text{ ft}$ are required if there is no space between each tile?

- A. 320
- B. 160
- C. 120
- D. 45

19. The cylindrical tank inside a water heater has a 20-inch diameter and a 57-inch height.

- One British thermal unit is needed to raise the temperature of 1 pound of water 1°F when under 1 atmosphere of pressure.
- One cubic foot of water weighs approximately 62.4 pounds.

If the tank is filled with water, which of the following is closest to the number of British thermal units needed to raise the temperature of the water 10°F?

- A.** 6466
- B.** 15,519
- C.** 25,866
- D.** 931,168

20. The following table shows the weight and volume of different objects.

Weight and Volume

Object	Weight (pounds)	Volume (gallons)
A	35.4	4.4
B	63.8	7.5
C	104.2	12.6
D	239.4	31.0

The density of water is approximately 8.34 pounds per gallon. If the density of an object is less than the density of water, the object will float in water. Which object will not float in water?

- A.** Object A
- B.** Object B

C. Object C

D. Object D

21. Platinum has a density of 21.5 grams per cubic centimeters. If a ring made of pure platinum has a mass of 6.3 grams, what is its approximate volume in cubic centimeters?

A. 0.3

B. 3.4

C. 15.2

D. 27.8

22. T and I Construction is paving a rectangular-shaped parking lot with Hot Asphalt Mix. The parking lot is 100 yards by 44 yards and will have a 9-inch base. If the Hot Asphalt Mix has a density of 140 pounds per cubic foot, about how many tons of mix will be needed to pave the parking lot?

A. 77

B. 91

C. 2079

D. 2772

23. A gas at standard temperature and pressure was measured as having a mass of 1485 grams (g) in a volume of 750 liters (L). Which of the following gases could it be?

Density of Gases	
Gas	Density (g/L)
Carbon dioxide	1.98
Carbon monoxide	1.25
Helium	0.18
Oxygen	1.43

- A. carbon dioxide
- B. carbon monoxide
- C. helium
- D. oxygen

24. The owner of an ice cream store wants to make a waffle cone that has the same volume as 3 scoops of ice cream.

- Each scoop of ice cream is in the shape of a sphere.
- The waffle cone will be in the shape of a cone with a diameter that is 1.75 times the diameter of a scoop of ice cream.

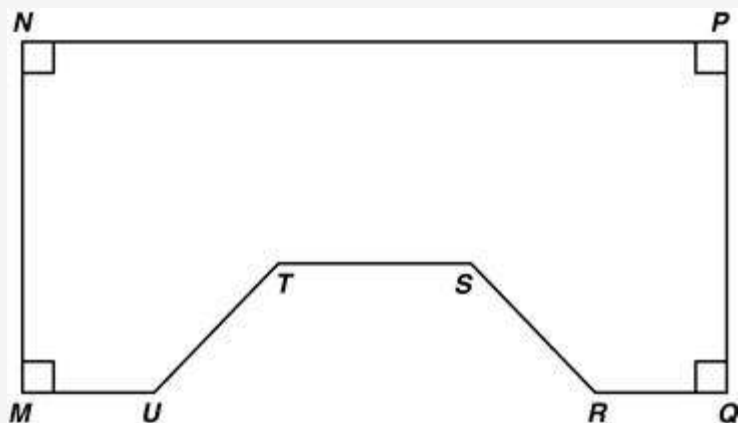
Based on this information, which statement is true about the height of the waffle cone?

- A. The height of the cone should be about 3.4 times the radius of the sphere.
- B. The height of the cone should be about 3.9 times the radius of the sphere.
- C. The height of the cone should be about 6.9 times the radius of the sphere.
- D. The height of the cone should be about 7.1 times the radius of the sphere.

25. Sophie needs to determine the amount of water that will fit into her new vase, shown below.

Which combination of geometric shapes can she use to **best** estimate the vase's volume?

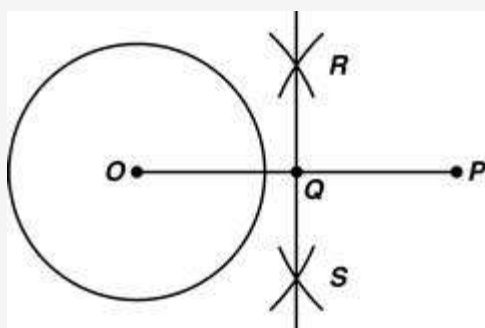
- A. A cone and a hemisphere
 - B. A cylinder and a sphere
 - C. A truncated cone and a sphere
 - D. A truncated pyramid and a hemisphere
26. The top view of a desk is shown below, where $\overline{MN} \cong \overline{QP}$, $\overline{MU} \cong \overline{QR}$, $\overline{TU} \cong \overline{SR}$, and $\angle MUT \cong \angle QRS$.



Natalie wants to construct this desk. Which of the following could not be the shapes used to construct the desk top?

- A. 1 pentagon and 2 congruent rectangles
- B. 1 rectangle and 2 congruent pentagons
- C. 1 rectangle and 2 congruent trapezoids
- D. 1 trapezoid and 2 congruent quadrilaterals

27. Two steps in constructing a tangent line from Point P to Circle O are shown.



Which method will result in finding a point of tangency in the next step?

- A. construct a line through Point R perpendicular to $\overline{RS}$
- B. construct a circle with Center Q and radius of QP

C. construct a line through Point O parallel to $\overline{RS}$

D. construct Triangle ROS

28. A model of an object is made of two geometric figures. Which object can be **best** modeled by a cylinder and a cone?

A. a human torso

B. an ice cream cone

C. a camera with lens

D. a sharpened pencil

29. The population density of Iceland in 2012 was 3.09 persons per square kilometer. If Iceland has an area of 103,000 square kilometers, how much would the population need to increase so that the density increases to 3.25 persons per square kilometer?

A. 10,260

B. 16,480

C. 24,720

D. 31,692

30. The density of pure water at 4°C is 1 g/cm^3 . Three of the substances in the chart will float in water.

Substances with densities less than that of water will float in water.

Mass and Volume of Four Substances		
Substance	Mass	Volume
A	1.9 kg	2.0 m ³
B	2.7 g	3.0 cm ³
C	1000 kg	2400 m ³
D	0.189 g	0.126 cm ³

Which substance will not float in water?

A. Substance A

B. Substance B

C. Substance C

D. Substance D

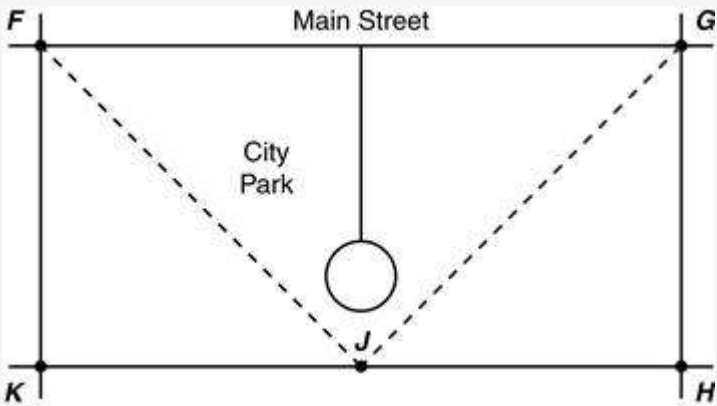
- 31.** The table shows the air conditioning square footage range for a house located in the northern part of the United States. A one-floor rectangular-shaped house is approximately 60 feet long and 40 feet wide.

Square Footage Range	
Air Conditioner - Tons	Square Footage
1.5	700–1100
2.0	1101–1400
2.5	1401–1650
3.0	1651–2100
3.5	2101–2300
4.0	2301–2700
4.5	2701–3300

How many BTUs are needed to cool this house if a 1-ton air conditioner has a cooling capacity of 12,000 BTUs per hour?

- A. 12,000
- B. 18,000
- C. 36,000
- D. 48,000

32. A city council wants to hire a construction company to install pipes underground connecting two locations on Main Street, represented by points F and G on the map shown below.



The pipes can be installed either along Main Street or in City Park along $\overline{FJ}$ and $\overline{GJ}$. The council will use the information in the table below to determine the least expensive construction method.

Construction Costs		
Construction Company	Cost to Install Pipes Along Main Street	Cost to Install Pipes in City Park
1	\$10,000 for every 50 feet	\$7,000 for every 50 feet
2	\$7,500 for every 40 feet	\$5,500 for every 40 feet

On the map, Quadrilateral $FGHK$ is a rectangle with $GH = HJ = JK = KF = 400$ feet. Based on this information, which decision will result in the least expensive construction method?

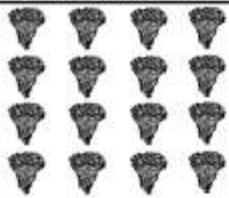
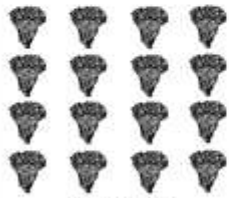
- A. hire Company 1 to install the pipes along Main Street

B. hire Company 1 to install the pipes through City Park

C. hire Company 2 to install the pipes along Main Street

D. hire Company 2 to install the pipes through City Park

- 33.** The spacing of plants in a vegetable garden is important for plants to grow properly. The diagram shows the number of plants that can safely be planted in each square foot of a vegetable garden.

Leaf Lettuce - 16	Beans - 4	Beans - 4
		
	Cauliflower - 1	Cauliflower - 1
		
Leaf Lettuce	Beans	Onions
		
Eggplants	Cauliflower	Corn
		

If the vegetable garden is to have an overall density of 5 plants per square foot, which vegetable should be planted in the empty square foot?

A. Corn

B. Beans

C. Onions

D. Eggplant

- 34.** Sandy is designing cartons to hold cans of beans. The cartons are in the shape of a rectangular prism, and the cans are cylinders measuring 6.858 centimeters (cm) in diameter and 10.16 cm tall. The carton must be able to hold 30 cans on one layer and have a length no greater than 50 cm.

Due to restrictions on the machine that creates the cartons, Sandy first completes her calculations and then rounds her results up to the next 0.25 cm.

Part A

Determine the dimensions for the base of the carton that would best accommodate 30 cans with the least amount of space left over. Show and explain your work.

Part B

Using the dimensions found for part A, determine the amount of area NOT covered by the cans on the base level. Show your work. Round your answer to the nearest hundredth.

Part C

The material for the cans weighs 0.0055 ounces per square centimeter. How many ounces does each can weigh? Round your answer to the nearest hundredth.

- 35.** In the state of Washington, a county is considered rural if it has a population density less than 100 persons per square mile.

2010 Area and Populations of Washington Counties

County	Area (square miles)	Population
Benton	1700	175,177
King	722	252,264
Spokane	1764	471,221
Whatcom	2107	201,140

Based on the information in the table, which county can be considered rural?

- A.** King
 - B.** Benton
 - C.** Spokane
 - D.** Whatcom
- 36.** A container of juice is in the shape of a cylinder with a height of 12 centimeters. If the container holds 355 cubic centimeters of juice, which is closest to the diameter of the container?
- A.** 3.07 centimeters
 - B.** 5.44 centimeters
 - C.** 6.14 centimeters
 - D.** 10.88 centimeters
- 37. TASK:** Show work for all parts using information from the table below, when needed.

	Florida	Minnesota	Vermont	Wisconsin
Land Area (sq. mi.)	53,474	79,610	9,217	

Population	19,318,000		626,000	5,726,000
Population Density		67.6		105.7

Part A: Population density is the number of people living in a state, per square mile. Complete the table above. Round populations to the nearest thousand.

Part B: Which pair of states in the table has the most similar land areas? Explain why their population densities differ.

Part C: Suppose that 500,000 people move from Wisconsin to Minnesota over the next five years. Explain by how much Wisconsin's population density would decrease and Minnesota's would rise. Then, explain why the population density in one state changes more than in the other.

Part D: How many people would have to move from Florida to Minnesota to make their population densities equal?

Part E: The population density of Vermont is very close to that of Minnesota. How many people would have to move from Florida to Vermont to make their population densities equal?

Part F: Explain why the difference in sizes between Minnesota and Vermont contributes to the difference between the answers to Parts D and E.

- 38.** A small cube made of solid copper has a mass of 53.246 grams. Which is closest to the measure of the cube's edge in centimeters, given that the density of copper is 8.96 g/cm^3 ?

A. 1.8

B. 2.4

C. 3.0

D. 5.9

- 39.** A trash container is shown.



Which sum of measures would be best to estimate the maximum amount of trash that would fit in the container?

- A.** the sum of the volume of a rectangular prism and the volume of a triangular prism
 - B.** the sum of the volume of a rectangular prism and the volume of a trapezoidal prism
 - C.** the sum of the surface area of a rectangular prism and the surface area of a triangular prism
 - D.** the sum of the surface area of a rectangular prism and the surface area of a trapezoidal prism
- 40.** A building in the shape of a rectangular prism needs to be painted. Some facts about the building are shown below.
- The building has a width of 50 feet, a length of 80 feet, and a height of 120 feet.
 - There are 60 windows on each lateral face of the building.
 - Each window is a square measuring 5 feet on a side.

Using this information, which expression represents the total lateral surface area, in square feet, that needs to be painted?

- A.**
- B.** $(2)(120)(80) + (2)(120)(50) - (4)(60)(5^2)$
- C.** $(2)(50)(80) + (2)(120)(80) + (2)(120)(50) - (60)(5^2)$

D. $(2)(50)(80) + (2)(120)(80) + (2)(120)(50) - (4)(60)(5^2)$

41. Hector was inspecting air quality at a laboratory researching carbon monoxide. During his inspection, Hector discovered that 2.65 grams of carbon monoxide leaked into the laboratory. The Occupational Safety and Health Administration (OSHA) has set a permissible exposure limit for carbon monoxide at 55 milligrams per cubic meter. The lab measured 12 feet by 14 feet by 10 feet. Approximately how much was the leak, in milligrams per cubic meter, and did the leak exceed OSHA's permissible exposure limit? (1 foot = 0.3048 meters)

- A. The leak was approximately 5.2 milligrams per cubic meter and did not exceed the permissible exposure limit.
- B. The leak was approximately 17.0 milligrams per cubic meter and did not exceed the permissible exposure limit.
- C. The leak was approximately 55.7 milligrams per cubic meter and did exceed the permissible exposure limit.
- D. The leak was approximately 135.7 milligrams per cubic meter and did exceed the permissible exposure limit.

42. The trunk of a tree in Kiana's backyard measures 12 ft high and has a diameter of 2 ft. Which statement about the trunk of Kiana's tree is **true**?

- A. It can be modeled using a cone with a volume of 4π cubic feet.

- B. It can be modeled using a cone with a volume of 16π cubic feet.
- C. It can be modeled using a cylinder with a volume of 12π cubic feet.
- D. It can be modeled using a cylinder with a volume of 48π cubic feet.

43. From Stone Tablets to Atomic Tablets

Use the concept of density to quantify the amount of information that can be stored on different types of media, such as CDs, DVDs, and computer hard drives. Data density is often expressed as the amount of information per unit of area of the storage system.

Data are stored using the binary (base-2) system, which represents numbers and other symbols using only the digits 0 and 1. The smallest piece of information is a bit, which is a single digit, either 0 or 1. One byte is equal to 8 bits. You will use this information and the following equivalencies to answer the questions in this task.

1 KB(kilobyte) = 1,024 bytes ($\approx$ 1 thousand bytes)

1 MB(megabyte) = 1,024 kilobytes = 1,048,576 bytes ($\approx$ 1 thousand kilobytes =1 million bytes)

1 GB(gigabyte) = 1,024 megabytes =1,073,741,824 bytes ($\approx$ 1 thousand megabytes = 1 billion bytes)

1 TB(terabyte) = 1,024 gigabytes = 1,099,511,627,776 bytes ($\approx$ 1 thousand gigabytes = 1 trillion bytes)

An ancient stone carving could hold a few bits of information per square inch. Compare that to the storage density of modern media.

Part 1. A CD-ROM can store about 650 megabytes of data on one side (represented by the darker gray outer circle in the diagram below). A CD is about 4.7 inches in diameter. The radius of the inner track of the CD is 0.9 inch. Describe how to calculate the storage density of the CD in megabytes per square inch. You should show how to set up the equations using the given numbers, but you do not need to solve them.



Part 2. There are approximately 6.45 square centimeters in 1 square inch. Explain how to calculate the storage density of the CD in megabytes per square centimeter if you know the density in megabytes per square inch.

Part 3. A DVD optical disk has the same recording area as a CD-ROM but can store about 7 gigabytes. Find the storage density in megabytes per square centimeter. Use the approximate numbers of bytes given above and round the area to the nearest square centimeter. Then write a general statement comparing the storage density of a CD with that of a DVD.

Part 4. New research into storing data on atoms has opened the possibility of desktop computers with 4 terabytes of memory. If one of these new computers had the same amount of storage area as an old computer with 64 kilobytes of memory, about how many times greater would the data density be? Use the approximate numbers given above.

44. Erica's family has a concrete patio outside the home. The density of the concrete is approximately 2000 kg/m^3 . If the patio has a volume of 6.3 m^3 of concrete, what is the mass of the concrete in this patio?

A. 7.99 kg

B. 317.46 kg

C. 12,600 kg

D. 500,094 kg

45. Which measure would be best to estimate the amount of fabric necessary to make an umbrella?

A. volume of a sphere

B. surface area of a sphere

C. volume of a hemisphere

D. surface area of a hemisphere

46. The density of gold is 19.3 grams per cubic centimeter. A weight of 1 ounce is equivalent to a mass of about 28.35 grams. Which of these is closest to the side length of a cube-shaped block of gold that has a weight of 8 ounces?

A. 2.27 centimeters

B. 3.43 centimeters

C. 9.09 centimeters

D. 11.75 centimeters

47. Which field has a density of approximately 17,000 plants per acre?

A. 85 acres with 1.02×10^6 plants

- B.** 100 acres with 1.7×10^7 plants
- C.** 110 acres with 1.9×10^6 plants
- D.** 205 acres with 3.4×10^5 plants
- 48.** Margaret is using a cardboard model to design a flowerpot. To create this flowerpot model, she cut the tops off of two congruent square pyramids that each had a height of 40 inches and a base length of 15 inches. She used the bottom half of one pyramid and the bottom one-fifth of the other pyramid and then attached the bases as shown.

Margaret would like to use this model to determine how much potting soil a flowerpot this size and shape would hold. Which expression represents the volume of the flowerpot?

A. $\frac{15^2 \cdot 40}{2} + \frac{15^2 \cdot 40}{5}$

B. $\frac{1}{3} \left(\frac{15^2 \cdot 40}{2} + \frac{15^2 \cdot 40}{5} \right)$

C. $(15^2 \cdot 40 - 7.5^2 \cdot 20) + (15^2 \cdot 40 - 12^2 \cdot 32)$

D. $\frac{1}{3} [(15^2 \cdot 40 - 7.5^2 \cdot 20) + (15^2 \cdot 40 - 12^2 \cdot 32)]$

49. Company A sells chocolate candies with a square base of side length 2 centimeters and a height of $\frac{1}{2}$ centimeter. Company B sells chocolate discs with a radius of 1 centimeter and a height of $\frac{1}{2}$ centimeter. Both companies package their chocolates in identical tin containers that each hold one layer of 28 chocolates. Julio buys a tin of chocolates from Company A, and Carmen buys a tin of chocolates from Company B. Which conclusion can be supported based on the information provided?

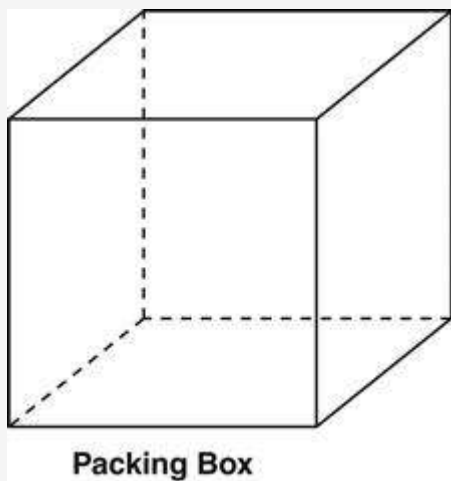
A. Carmen and Julio purchased about the same volume of chocolate.

B. Julio purchased about 12 cm^3 more chocolate than Carmen.

C. Carmen purchased about more chocolate than Julio.

D. Julio purchased about 24 cm^3 more chocolate than Carmen.

50. A cube-shaped packing box like the one shown below is used to pack a stack of dinner plates.



If the volume of the box is 1,000 cubic inches, what is the maximum circumference of the dinner plates

that will fit in the box, to the nearest tenth of an inch?

- A.** 15.7 inches
 - B.** 31.4 inches
 - C.** 78.5 inches
 - D.** 99.3 inches
- 51.** Ice has a density of 0.92 gram per cubic centimeter. What is the total volume of a block of ice that has a mass of 2.3 kilograms?
- A.** 2.1 cubic centimeters
 - B.** 2.5 cubic centimeters
 - C.** 2116 cubic centimeters
 - D.** 2500 cubic centimeters
- 52.** A mixture of 70% isopropyl alcohol and 30% water is sold as rubbing alcohol. The density of isopropyl alcohol is approximately 0.79 g/mL and water has a density of 1.0 g/mL. Excluding the mass of the bottle, what is the approximate mass of a 250-mL bottle of rubbing alcohol?
- A.** 75.0 g
 - B.** 138.25 g
 - C.** 197.5 g
 - D.** 213.25 g

53. In 1999, Hong Kong had a population of approximately 7,000,000 people. The area of Hong Kong is approximately 1042 square kilometers. What was the approximate population density of Hong Kong in persons per square kilometer?

A. 67

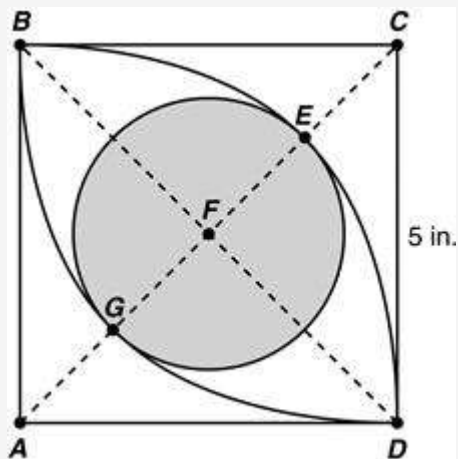
B. 670

C. 6700

D. 67,000

54. A company logo of an eye consists of two arcs and a circle drawn within a square. The company prints its logo on the side of a square box of side length 5 inches as shown.

- Arc BED is drawn from Point A .
- Arc BGD is drawn from Point C .
- Circle F is tangent to Arc BED and Arc BGD at Points E and G , respectively.



What is the area of the circle of the eye?

A. 13.48 in^2

B. 9.20 in^2

C. 6.74 in^2

D. 3.14 in^2

- 55.** During cold months, when the difference between the inside temperature and outside temperature is 30°F , it takes about 3.9 British thermal units (BTU) per cubic foot to heat a room. Which of the following is closest to the number of British thermal units needed to heat a room with a 20-foot by 16-foot rectangular floor and an 8-foot ceiling?

A. 18,982 BTU

B. 9984 BTU

C. 1248 BTU

D. 656 BTU

- 56.** Brian purchased a rectangular-shaped granite countertop for the island in his kitchen. The granite slab measures 72 inches by 42 inches by 1.25 inches. If the density of granite is 170 pounds per cubic foot, how much does the countertop weigh in pounds?

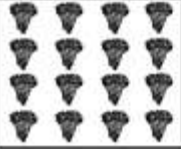
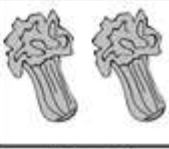
A. 157.5

B. 297.5

C. 371.9

D. 425.0

- 57.** The spacing of plants in a vegetable garden is important so that plants can grow properly. The diagram below shows a vegetable garden that measures 3 feet by 3 feet.

Leaf Lettuce - 16 	Beans - 4 	Beans - 4 
Leaf Lettuce - 16 	Cabbage - 1 	Celery - 2 
Corn - 2 	Corn - 2 	Corn - 2 

What is the approximate average density of plants per square foot in this garden?

- A. 5.4
- B. 8.2
- C. 16.3
- D. 49.0

58. The population of the United States in two different years is shown in the table below.

United States Population	
Year	Population
2000	281,421,906
2010	308,745,538

The total land area of the United States is 3,541,447 square miles. Based on this information, which is closest to the overall change in population density, in people per square mile, for the United States between 2000 and 2010?

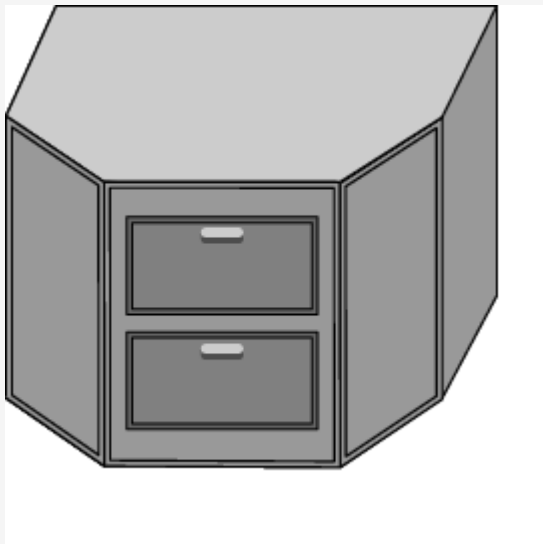
- A. 0.13 people per square mile

B. 0.77 people per square mile

C. 7.72 people per square mile

D. 16.67 people per square mile

59. Which combination of three-dimensional shapes could be used to **best** model the table shown below?



A. 6 rectangular prisms and 1 cube

B. 4 right triangular prisms and 1 cube

C. 2 rectangular prisms and 2 right triangular prisms

D. 3 right triangular prisms and 2 rectangular prisms

Martin's Swimming Pool

Martin is planning to construct a swimming pool behind his house. The architect shows him a plan for the swimming pool. The pool, if viewed from the top looks like a 40 ft long and 20 ft wide rectangle. The pool is divided into two equal sections along its length—the shallow section and the deep section. The shallow section has a constant depth of 5 ft. Once the shallow section ends, the floor of the pool starts sloping until it reaches a **maximum** depth of 20 ft at the other end of the pool.

Part A. What is the length of the slope of the deep section of the pool? Draw a two-dimensional side view of the swimming pool to show the shallow and deep sections. Remember to write the measures of all sides.

Part B. Martin contracts a construction company to dig up earth for the swimming pool. The company charges \$15 per hour and estimates that it will be able to complete the job within 8 hours. How much earth will be dug up? How much earth is the company digging out for every dollar charged? Round your answer to the hundredths place.

Part C. Once the earth has been excavated, the next step is to paint all four lateral sides and to tile the floor of the swimming pool. What area needs to be painted? What area needs to be tiled? Martin has the option to purchase square tiles of length 1 ft or 2 ft or 3 ft, and so on. What is the largest tile he can purchase? The cost of each type of tile, in dollars, is twice the length of its diagonal. If Martin purchases the biggest possible tile, how much would they cost? Round your answers to the hundredths place.

Part D. The pool is filled with water one foot below the top. The water is pumped into the pool through an 18-inch wide pipe with velocity of 1.5 ft/second. Find the area of the cross section of the pump and multiply it by the velocity to calculate the rate, in cubic feet per second, at which the water will be pumped into the pool? Use 3.14 as the value of pi and round your answers to the hundredths place.

Part E. How much water does the shallow section hold? How much more water does the deep section hold as compared to the shallow section?

Part F. Another pipe is attached at the bottom of the pool, which is used to drain water from the pool. The pipe has a diameter of 36 inches and the pump at its end sucks water out with velocity of 0.25 ft/s. One day, when the pool was empty, both the inlet and outlet pipes were opened simultaneously. How many hours will it now take to fill the pool? Use

3.14 as the value of pi and round your answers to the hundredths place.

61. Which geometric shape would be the **best** mathematical model for the trunk of a tree?

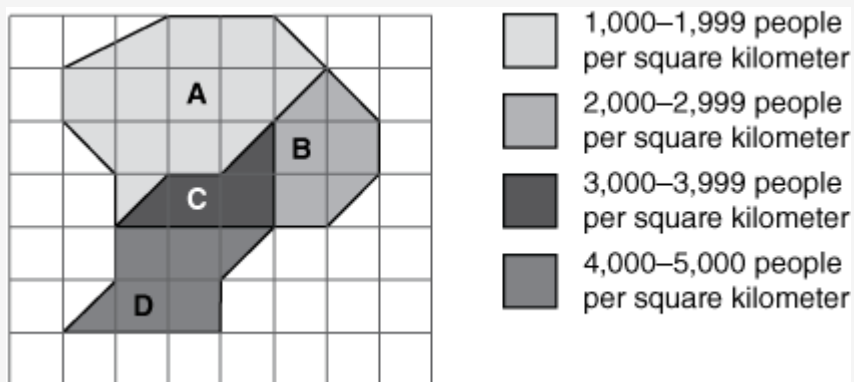
A. a cone

B. a cylinder

C. a triangular prism

D. a rectangular prism

62. A population density map of a county is given below. One square unit represents one square kilometer.

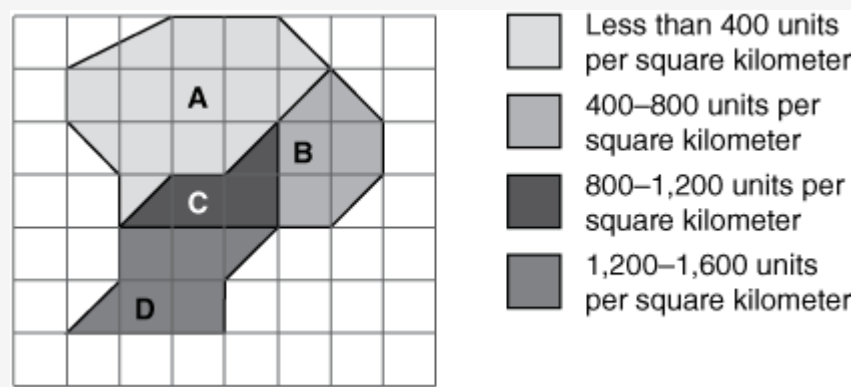


Part A. How many square kilometers is the county?

Part B. Based on the map, what is the smallest possible number of people who live in the county? What is the largest possible number?

Part C. To keep housing affordable and safe, it is recommended that

counties have a ratio of 2 units of housing for every 5 residents. Below is a housing density map.



The county gives out grants for housing developments. Which area should be the top priority for the county officials to encourage the building of additional housing units? Explain. Create a table to show how many units they should build in each area. Be sure to take the range of population into account and support your answer with numbers obtained from the information above.

Use words, numbers, and/or pictures to show your work.

63.

Sarah has 2 spherical balls with radii of 6 mm. One ball is pure gold and the other is pure silver. If the density of gold is 19.3 g/cm³ and the density of silver is 10.5 g/cm³, about how much greater is the gold ball’s mass than the silver ball’s mass?

A.

8 grams

B.

9 grams

C.

8000 grams

D.

4000 grams
64.

A candy company sells taffy in four containers shaped like right prisms. The table below shows the

shapes of the bases and dimensions of each container.

Taffy Containers		
Container	Shape of Base	Dimensions
1	Equilateral Triangle	Each side of base = 8 in. Height of container = 2 in.
2	Rectangle	Length of base = 5 in. Width of base = 6 in. Height of container = 7 in.
3	Square	Each side of base = 4 in. Height of container = 8 in.
4	Regular Hexagon	Each side of base = 5 in. Height of container = 4 in.

Which container has the least ratio of surface area to volume?

- A. Container 1
 - B. Container 2
 - C. Container 3
 - D. Container 4
65. In 1999, the U.S. territory of Guam had an approximate population density of 280 persons per square kilometer and a total population of approximately 150,000 people. What was the approximate area of Guam, in square kilometers?
- A. 5.4
 - B. 54
 - C. 540
 - D. 5400

66. The table shows the number of vegetable plants and the number of planted acres for 4 farmers.

Number of Plants in Planted Acres			
	Type of Plants	Number of Plants	Planted Acres
Farmer A	Corn	1.36×10^6	80
Farmer B	Soybean Plants	1.5×10^7	150
Farmer C	Green Beans	5.2×10^6	50
Farmer D	Pepper Plants	5.8×10^5	20

Which farmer has the most densely planted field?

- A. Farmer A
- B. Farmer B
- C. Farmer C
- D. Farmer D
67. In 1999, the country of Monaco had a population of approximately 32,000 people. The area of the country is approximately $\frac{3}{4}$ of a square mile. What was the approximate population density of Monaco in persons per square mile?
- A. 24,000
- B. 32,000
- C. 34,000
- D. 43,000

68. A container shaped like a rectangular prism holds 4.0 kilograms of an unknown gas. The dimensions of the container are 2 meters by 2 meters by 0.5 meter.

Density of Gases	
Gas	Density (g/cm ³)
Helium	0.000178
Hydrogen	0.00009
Carbon Dioxide	0.00125
Carbon Monoxide	0.001977

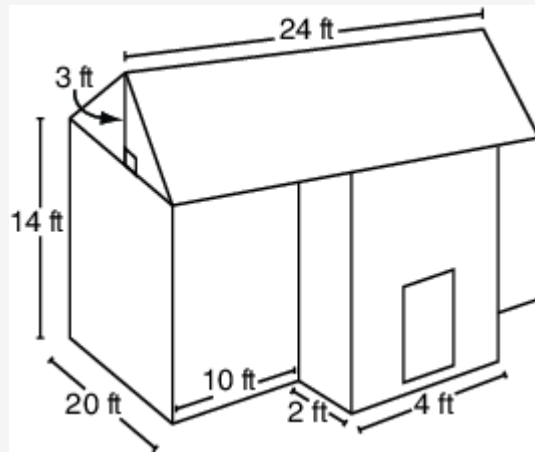
Based on the densities given in the table, what is the unknown gas in the container?

- A. hydrogen
- B. helium
- C. carbon dioxide
- D. carbon monoxide
69. The vertices of Quadrilateral $PQRS$ are $P(2, 4)$, $Q(7, 4)$, $R(11, 7)$, and $S(6, 7)$. The diagonals intersect at Point T . Which conclusion is not true?

- A. $\overline{PT} \cong \overline{TQ}$
- B. $\overline{PQ} \cong \overline{QR}$
- C. $\overline{SQ} \perp \overline{PR}$
- D. $\angle PSQ \cong \angle QSR$

70. Mr. Evans built the model below to show the dimensions of the new barn

he is planning to build on his farm.



Part A. What 3-dimensional shapes did Mr. Evans use to build his model of the barn?

Part B. He plans to paint the exterior walls of the barn, including the door and the triangular parts of the roof, red and the rectangular parts of the roof white. If 1 gallon of paint can cover 10 sq ft, how many gallons of red paint and how many gallons of white paint does Mr. Evans need?

Part C. Inside the barn, Mr. Evans will be stacking and storing extra bales of hay in the rafters. The rafters will be above the ceiling line created by the bottom edge of the roof. About how many cubic feet of hay can he store in the rafters?

Part D. Mr. Evans is also building a model of a grain silo that will be built next to his barn, as show by the diagram below.



Which geometric solids **best** represent the shape of the silo? If the base shape of the silo is 8 ft taller than the barn and has a diameter of 20 ft, how much grain will his silo hold, to the nearest cubic foot?

Use words, numbers, and/or pictures to show your work.

71. In 2012, an estimated 81,757,600 people lived in Germany. Since Germany's area is 357,021 square kilometers, which is closest to Germany's population density per square kilometer in 2012?
- A. 177
- B. 229
- C. 276
- D. 437
72. A standard soccer ball is shown below. The soccer ball is a model of a polyhedron where every vertex connects a regular pentagon and two regular hexagons.



The *deficit* of a vertex of a polyhedron is defined to be 360 minus the sum of the degrees of the angles that meet at the vertex. For a convex polyhedron, the total deficit of all vertices always adds up to 720. How many vertices does the polyhedron modeled by a soccer ball have?

- A. 15
- B. 30
- C. 40
- D. 60

73. The table below shows the type, size, and number of residents for each apartment in an apartment building.

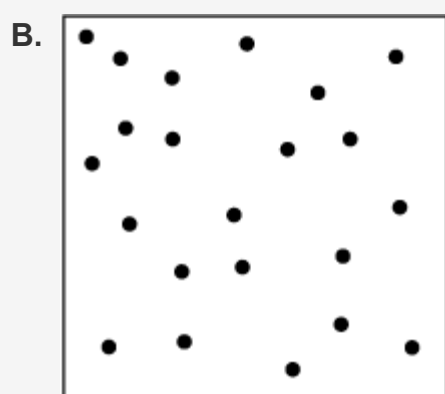
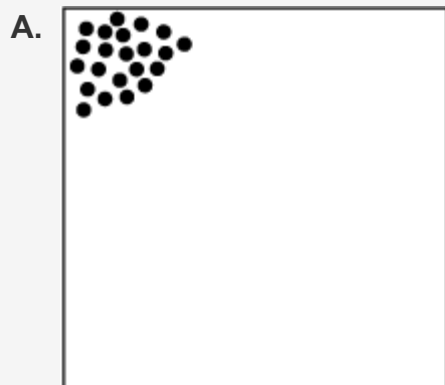
Apartment Building Information

Unit Number	Type of Apartment	Size of Apartment	Number of Residents
1A	Studio	50 square yards	1
1B	One-Bedroom	75 square yards	2
1C	One-Bedroom	75 square yards	1
1D	Two-Bedroom	100 square yards	3
2A	Studio	50 square yards	1
2B	One-Bedroom	75 square yards	1
2C	One-Bedroom	75 square yards	2
2D	Two-Bedroom	100 square yards	4

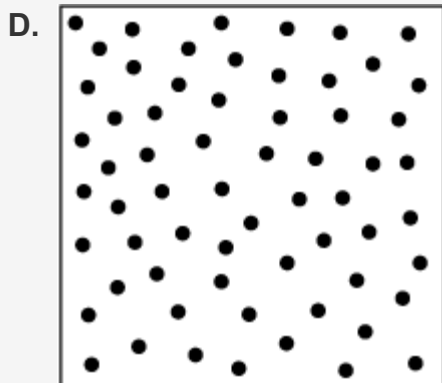
What is the overall density, in residents per square yard, for the entire apartment building?

- A. 0.013 people per square yard
- B. 0.025 people per square yard
- C. 0.028 people per square yard
- D. 0.035 people per square yard

74. A certain town has an area of 63 square kilometers and a population of 1380 people. If a box represents one square kilometer and each dot represents one person, which model is the BEST representation of the population density of this town?



C.



75. As part of a home-improvement project, Andy painted 10 walls in his house. The area of each wall is 142 ft^2 . The paint he bought comes in 2 gallon cans.

About how many cans did Andy need if 1 gallon of paint covers 150 ft^2 ?

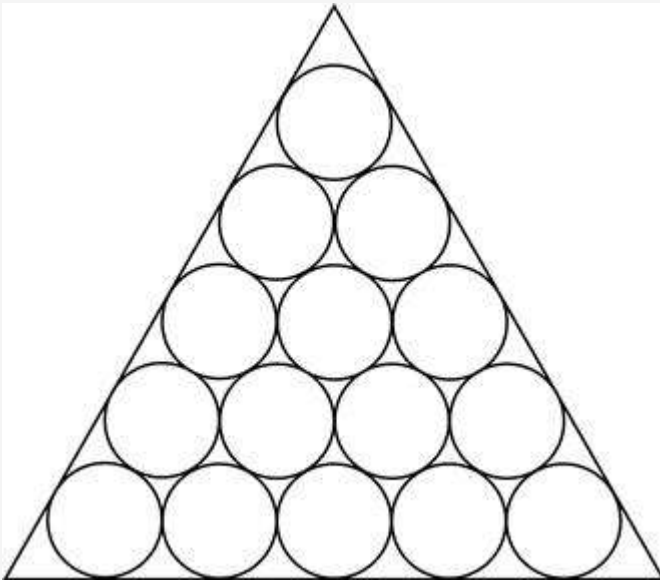
- A. 5
- B. 10
- C. 11
- D. 22
76. Crista has a bag of 10 marbles, where each marble has a diameter of $\frac{5}{8}$ inch. The marbles are made of agate, which has a density of 1.5 ounces per cubic inch. Not including the weight of the bag, about how much do the 10 marbles weigh in ounces? (Use $\pi \approx 3.14$)

- A. 0.2
- B. 1.9

C. 9.4

D. 15.3

77. Amy drew a design consisting of 15 congruent circles surrounded by an equilateral triangle as shown below.



The sides of the triangle are tangent to each of the 12 outer circles. Each circle is tangent to every adjacent circle. Which expression is equal to the perimeter of the triangle in terms of r , the radius of each circle?

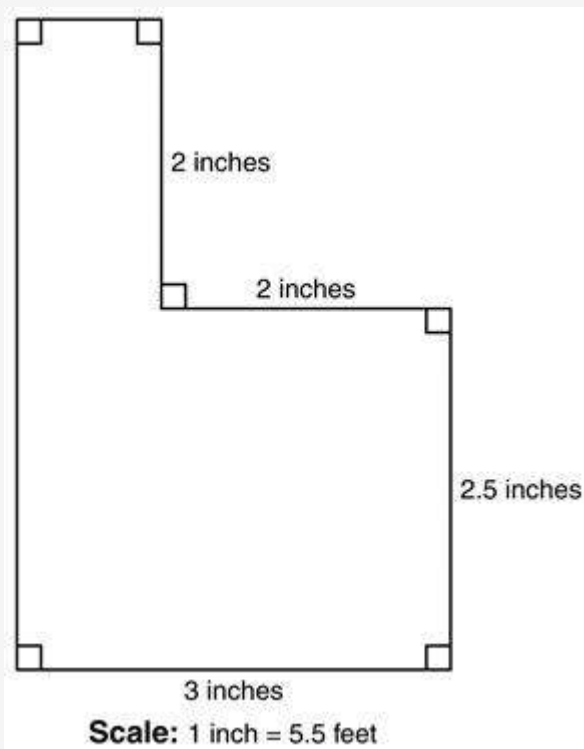
A. $(12 + 2\sqrt{3})r$

B. $(12 + 6\sqrt{3})r$

C. $(24 + 2\sqrt{3})r$

D. $(24 + 6\sqrt{3})r$

78. Roberto put new carpeting in one room of his house. The area of this room is represented in the scale drawing below.



If carpeting costs \$5.40 per square foot, what is the total cost of the new carpeting for this room?

- A. \$256.50
- B. \$282.15
- C. \$1,282.50
- D. \$1,551.83

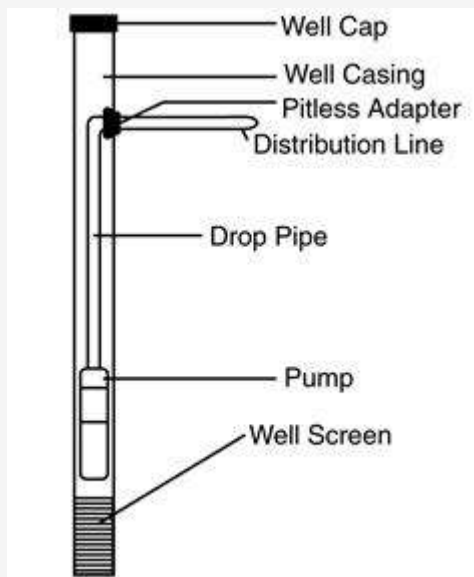
79. Veronica's science class is studying density, and each student is asked to bring in an object to determine its density. Veronica brings in a brick that weighs two pounds and is 6 inches long, 3 inches wide, and 2 inches tall. Her friend Becky brings in a block of wood that weighs 3 pounds and is 1 foot long, 2 inches wide, and 3 inches tall. Which is a true statement about the objects?

- A. The brick has a greater volume than the block and a smaller mass, so

the brick is less dense than the block.

- B.** The block has a smaller volume than the brick and a greater mass, so the brick is more dense than the block.
- C.** The brick is half the volume of the block but more than half the mass, so the brick is more dense than the block.
- D.** The block is twice the volume of the brick and more than double the mass, so the brick is less dense than the block.

80. The figure below shows the components of a cylindrical well.



Which measurement is most likely to be used to determine the diameter of the well cap?

- A.** the capacity of the cylindrical well casing
- B.** the circumference of the cylindrical well casing
- C.** the depth of the cylindrical well casing below the ground

D. the height of the cylindrical well casing above the ground

81. A block of wood is 2 feet 6 inches long, 6 inches wide, and 3 inches tall.

It weighs $16\frac{1}{2}$ pounds. To the nearest pound, what is its density in pounds per cubic foot?

A. 53

B. 51

C. 35

D. 33