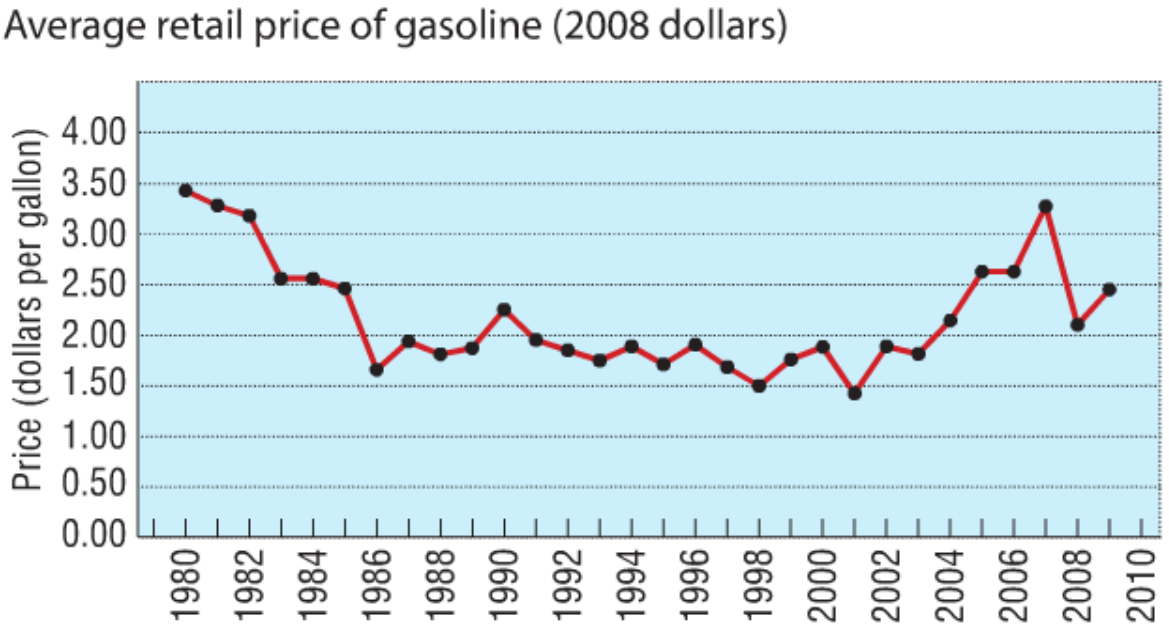
Section 3.2

The Graph of a Function

Average Price of Gasoline at a particular station in Texas.

Year	Price	Year	Price	Year	Price
1980	3.41	1990	2.25	2000	1.85
1981	3.26	1991	1.90	2001	1.40
1982	3.15	1992	1.82	2002	1.86
1983	2.51	1993	1.70	2003	1.79
1984	2.51	1994	1.85	2004	2.13
1985	2.46	1995	1.68	2005	2.60
1986	1.63	1996	1.87	2006	2.62
1987	1.90	1997	1.65	2007	3.29
1988	1.77	1998	1.50	2008	2.10
1989	1.83	1999	1.73	2009	2.45



1 Identify the Graph of a Function

Theorem

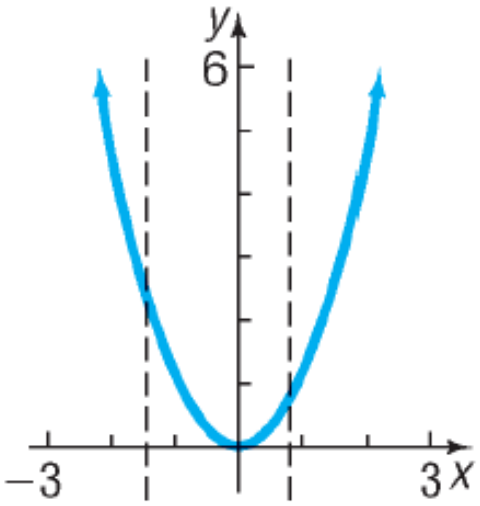
Vertical-line Test

A set of points in the *xy*-plane is the graph of a function if and only if every vertical line intersects the graph in at most one point.

EXAMPLE

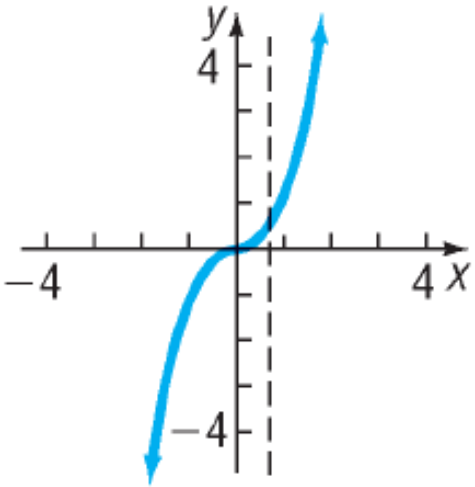
Identifying the Graph of a Function

Which of the following are graphs of functions?



(a) $y = x^2$

A Function



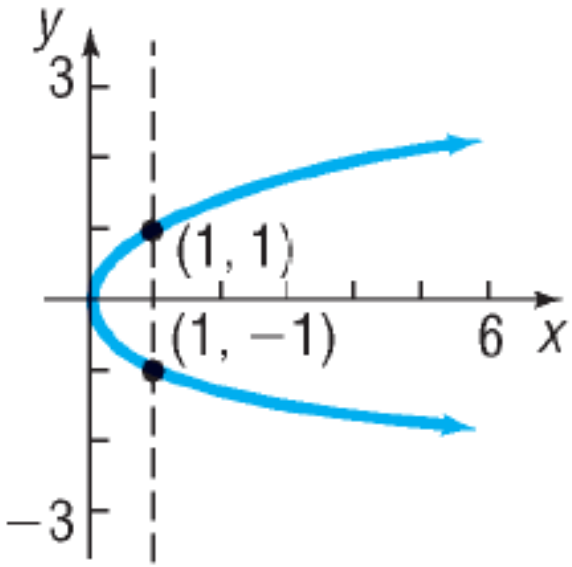
(b) $y = x^3$

A Function

EXAMPLE

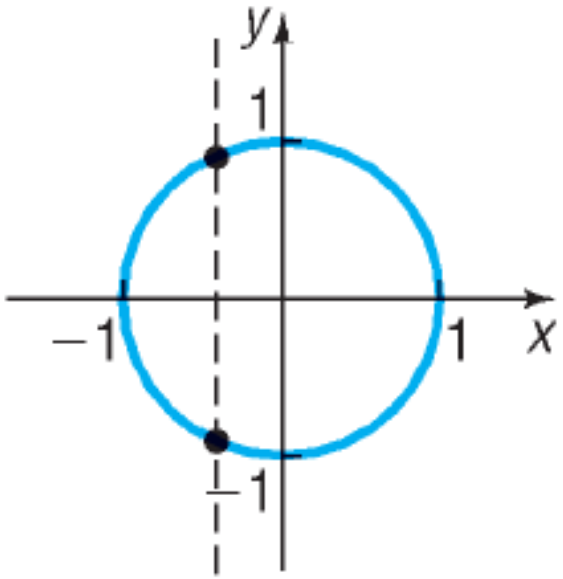
Identifying the Graph of a Function

Which of the following are graphs of functions?



(c) $x = y^2$

Not A Function



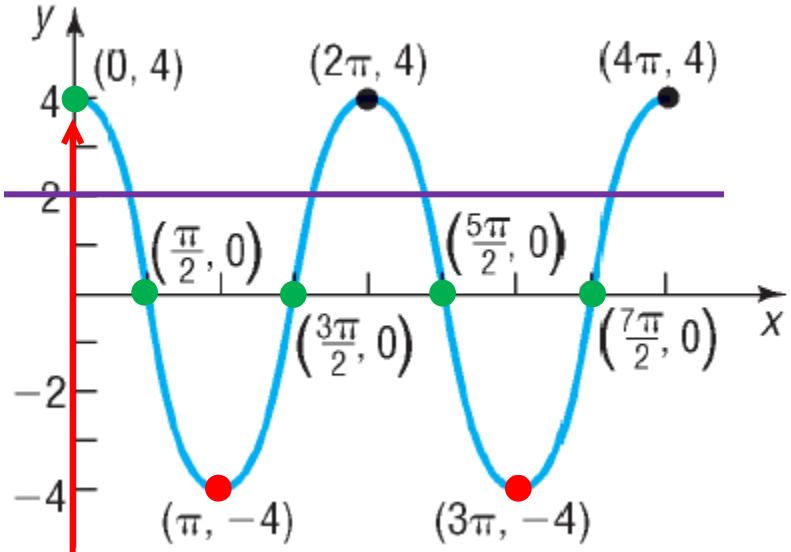
(d) $x^2 + y^2 = 1$

Not A Function

2 Obtain Information from or about the Graph of a Function

EXAMPLE

Obtaining Information from the Graph of a Function



(a) What are $f(0)$, $f\left(\frac{3\pi}{2}\right)$, and $f(3\pi)$?

(b) What is the domain of f?

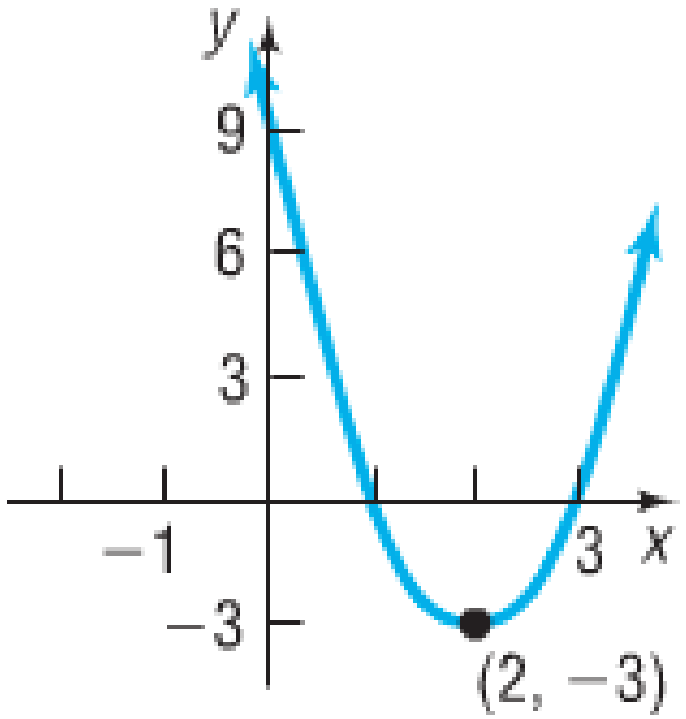
(c) What is the range of f?

(d) List the intercepts.

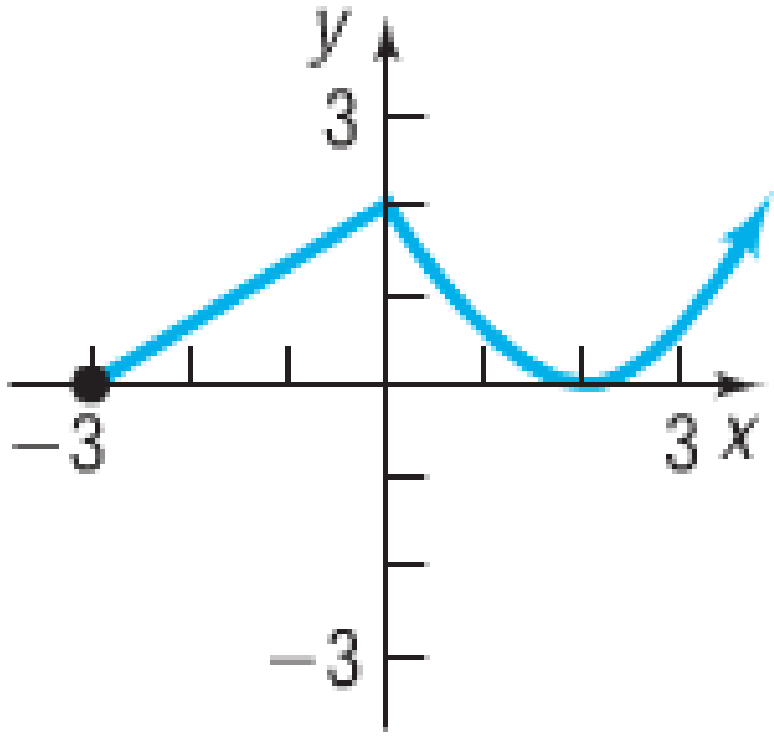
(e) How often does the line $y = 2$ intersect the graph?

(f) For what values of x does $f(x) = -4$?

(g) For what values of x is $f(x) > 0$?



(a) Find $f(0)$ and $f(-6)$.
(b) Find $f(6)$ and $f(11)$.
(c) Is $f(3)$ positive or negative?
(d) Is $f(-4)$ positive or negative?
(e) For what values of x is $f(x) = 0$?
(f) For what values of x is $f(x) > 0$?
(g) What is the domain of f?
(h) What is the range of f?
(i) What are the x-intercepts?
(j) What is the y-intercept?
(k) How often does the line $y = \frac{1}{2}$ intersect the graph?
(l) How often does the line $x = 5$ intersect the graph?
(m) For what values of x does $f(x) = 3$?
(n) For what values of x does $f(x) = -2$?



(a) Find $f(0)$ and $f(-6)$.
(b) Find $f(6)$ and $f(11)$.
(c) Is $f(3)$ positive or negative?
(d) Is $f(-4)$ positive or negative?
(e) For what values of x is $f(x) = 0$?
(f) For what values of x is $f(x) > 0$?
(g) What is the domain of f?
(h) What is the range of f?
(i) What are the x-intercepts?
(j) What is the y-intercept?
(k) How often does the line $y = \frac{1}{2}$ intersect the graph?
(l) How often does the line $x = 5$ intersect the graph?
(m) For what values of x does $f(x) = 3$?
(n) For what values of x does $f(x) = -2$?

EXAMPLE

Obtaining Information from the Graph of a Function

Consider the function $f(x) = \dfrac{x}{x+1}$

(a) Is the point $\left(1, \dfrac{1}{2}\right)$ on the graph of f? Yes $\quad f(1) = \dfrac{1}{1+1} = \dfrac{1}{2}$

(b) If $x = 2$, what is $f(x)$? What point is on the graph of f?

$$f(2) = \frac{2}{2+1} = \frac{2}{3} \qquad \left(2, \frac{2}{3}\right)$$

(c) If $f(x) = 2$, what is x? What point is on the graph of f?

$$f(x) = 2 = \frac{x}{x+1} \qquad 2(x+1) = x \qquad 2x + 2 = x \qquad x = -2 \qquad (-2, 2)$$

$f(x) = 2x^2 - x - 1$

(a) Is the point $(-1, 2)$ on the graph of f?

(b) If $x = -2$, what is $f(x)$? What point is on the graph of f?

(c) If $f(x) = -1$, what is x? What point(s) are on the graph of f?

(d) What is the domain of f?

(e) List the x-intercepts, if any, of the graph of f.

(f) List the y-intercept, if there is one, of the graph of f.

$$f(x) = \frac{x + 2}{x - 6}$$

(a) Is the point $(3, 14)$ on the graph of f?

(b) If $x = 4$, what is $f(x)$? What point is on the graph of f?

(c) If $f(x) = 2$, what is x? What point(s) are on the graph of f?

(d) What is the domain of f?

(e) List the x-intercepts, if any, of the graph of f.

(f) List the y-intercept, if there is one, of the graph of f.

EXAMPLE Average Cost Function

The average cost $\overline{C}$ of manufacturing x computers per day is given by the function

$$\overline{C}(x) = 0.56x^2 - 34.39x + 1212.57 + \frac{20{,}000}{x}$$

Determine the average cost of manufacturing:

(a) 30 computers in a day

(b) 40 computers in a day

(c) 50 computers in a day

(d) Graph the function $\overline{C} = \overline{C}(x), 0 < x \leq 80$.

(e) Create a TABLE with TblStart = 1 and ΔTbl = 1.

Which value of x minimizes the average cost?

EXAMPLE Average Cost Function

$$\overline{C}(x) = 0.56x^2 - 34.39x + 1212.57 + \frac{20{,}000}{x}$$

(a) 30 computers in a day = $1351.54

(b) 40 computers in a day = $1232.97

(c) 50 computers in a day = $1293.07

(d) Graph the function $\overline{C} = \overline{C}(x), 0 < x \leq 80.$

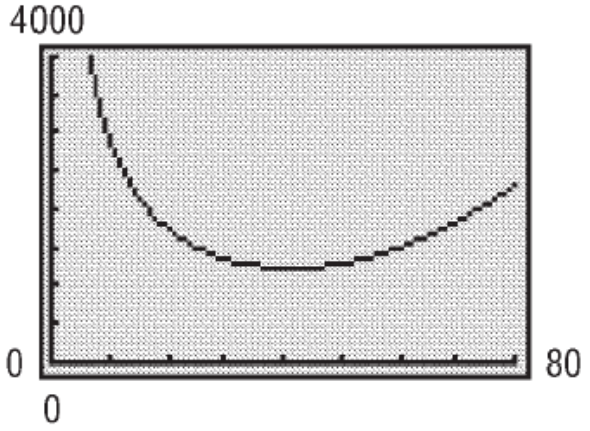
(e) Create a TABLE with TblStart = 1 and ΔTbl = 1

X	Y1
1	21179
2	11146
3	7781.1
4	6084
5	5054.6
6	4359.7
7	3856.4

Y1=.56X²-34.39X...

X	Y1
38	1240.7
39	1235.9
40	1233
41	[illegible]
42	1232.2
43	1234.4
44	1238.1

Y1=1231.74487805



Which value of x minimizes the average cost? $1231.74

Motion of a Golf Ball A golf ball is hit with an initial velocity of 130 feet per second at an inclination of 45° to the horizontal. In physics, it is established that the height h of the golf ball is given by the function

$$h(x) = \frac{-32x^2}{130^2} + x$$

where x is the horizontal distance that the golf ball has traveled.

(a) Determine the height of the golf ball after it has traveled 100 feet.
(b) What is the height after it has traveled 300 feet?
(c) What is the height after it has traveled 500 feet?
(d) How far was the golf ball hit?
(e) Use a graphing utility to graph the function $h = h(x)$.
(f) Use a graphing utility to determine the distance that the ball has traveled when the height of the ball is 90 feet.
(g) Create a TABLE with TblStart = 0 and ΔTbl = 25. To the nearest 25 feet, how far does the ball travel before it reaches a maximum height? What is the maximum height?
(h) Adjust the value of ΔTbl until you determine the distance, to within 1 foot, that the ball travels before it reaches a maximum height.

Free-throw Shots According to physicist Peter Brancazio, the key to a successful foul shot in basketball lies in the arc of the shot. Brancazio determined the optimal angle of the arc from the free-throw line to be 45 degrees. The arc also depends on the velocity with which the ball is shot. If a player shoots a foul shot, releasing the ball at a 45-degree angle from a position 6 feet above the floor, then the path of the ball can be modeled by the function

$$h(x) = -\frac{44x^2}{v^2} + x + 6$$

where h is the height of the ball above the floor, x is the forward distance of the ball in front of the foul line, and v is the initial velocity with which the ball is shot in feet per second. Suppose a player shoots a ball with an initial velocity of 28 feet per second.

(a) Determine the height of the ball after it has traveled 8 feet in front of the foul line.

(b) Determine the height of the ball after it has traveled 12 feet in front of the foul line.

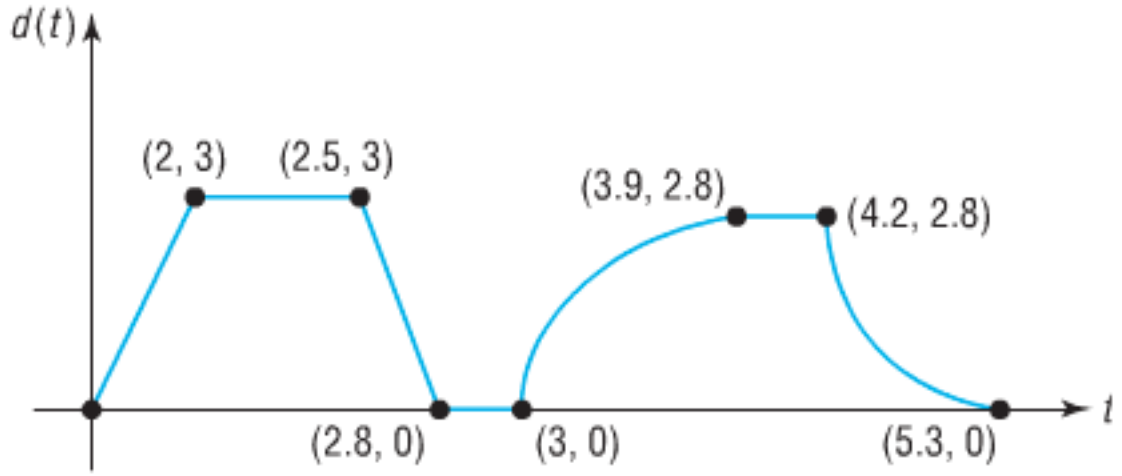
(c) Find additional points and graph the path of the basketball.

(d) The center of the hoop is 10 feet above the floor and 15 feet in front of the foul line. Will the ball go through the hoop? Why or why not? If not, with what initial velocity must the ball be shot in order for the ball to go through the hoop?

Source: *The Physics of Foul Shots,* Discover, *Vol. 21, No. 10, October 2000*

Consider the following scenario: Barbara decides to take a walk. She leaves home, walks 2 blocks in 5 minutes at a constant speed, and realizes that she forgot to lock the door. So Barbara runs home in 1 minute. While at her doorstep, it takes her 1 minute to find her keys and lock the door. Barbara walks 5 blocks in 15 minutes and then decides to jog home. It takes her 7 minutes to get home. Draw a graph of Barbara's distance from home (in blocks) as a function of time.

The following sketch represents the distance d (in miles) that Kevin was from home as a function of time t (in hours). Answer the questions based on the graph. In parts (a)–(g), how many hours elapsed and how far was Kevin from home during this time?



(a) From $t = 0$ to $t = 2$
(b) From $t = 2$ to $t = 2.5$
(c) From $t = 2.5$ to $t = 2.8$
(d) From $t = 2.8$ to $t = 3$
(e) From $t = 3$ to $t = 3.9$
(f) From $t = 3.9$ to $t = 4.2$
(g) From $t = 4.2$ to $t = 5.3$
(h) What is the farthest distance that Kevin was from home?
(i) How many times did Kevin return home?

SUMMARY

Graph of a Function	The collection of points (x, y) that satisfies the equation $y = f(x)$.
Vertical Line Test	A collection of points is the graph of a function provided that every vertical line intersects the graph in at most one point.