Algebra 2: Unit 8

Roots and Radicals

Roots and Radicals

Radicals (also called roots) are directly related to exponents.

Roots and Radicals

The simplest types of radicals are square roots and cube roots.

Radicals beyond square roots and cube roots exist, but we will not discuss them as in depth.

Roots and Radicals

The rules for radicals that you will learn work for all radicals – not just square roots and cube roots.

Roots and Radicals

The symbol used to indicate a root is the radical symbol - $\sqrt{}$

Roots and Radicals

Every radical expression has three parts…

- Radical symbol
- Index
- Radicand

Roots and Radicals

Every radical expression has three parts…

Radical

Index

$$\sqrt[2]{49}$$

Radicand

Roots and Radicals

The index of a radical is a whole number greater than or equal to 2.

Roots and Radicals

The index of a square root is always 2.

Roots and Radicals

By convention, *an index of 2 is not written* since it is the smallest possible index.

Roots and Radicals

The square root of 49 could be written as $\sqrt[2]{49}$ …

but is normally written as $\sqrt{49}$.

Roots and Radicals

plural of index

All indices greater than 2 must be written.

The index of a cube root is always 3.

Roots and Radicals

The cube root of 64 is written as

$$\sqrt[3]{64}\ .$$

Roots and Radicals

What does square root mean?

What does cube root mean?

Roots and Radicals

The square root of a number (or expression) is another number (or expression)…

…which when multiplied by itself (squared) gives back the original number (or expression).

Roots and Radicals

The cube root of a number (or expression) is another number (or expression) …

…which when multiplied by itself three times (cubed) gives back the original number (or expression).

Roots and Radicals

Example:

$$\sqrt{49} = 7 \quad \text{because} \quad 7 \cdot 7 = 7^2 = 49$$

Also

$$\sqrt{49} = -7 \quad \text{because} \quad (-7)(-7) = (-7)^2 = 49$$

Roots and Radicals

Example:

$\sqrt{49}$ has two answers:

7 is called the positive or principal square root.

-7 is called the negative square root.

Intermediate Algebra MTH04

Roots and Radicals

Example:

$$\sqrt[3]{64} = 4 \quad \text{because} \quad 4 \cdot 4 \cdot 4 = 4^3 = 64$$

$$\sqrt[3]{-64} = -4 \quad \text{because}$$

$$(-4)(-4)(-4) = (-4)^3 = -64$$

Roots and Radicals

What are the first 10 whole numbers that are perfect squares?

$$1^2, \ 2^2, \ 3^2, \ 4^2, \ 5^2, \ 6^2, \ 7^2, \ 8^2, \ 9^2, \ 10^2$$

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Roots and Radicals

What are the first 10 whole numbers that are perfect cubes?

$1^3, \; 2^3, \; 3^3, \; 4^3, \; 5^3, \; 6^3, \; 7^3, \; 8^3, \; 9^3, \; 10^3$

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

Roots and Radicals

If a number is a perfect square, then you can find its exact square root.

A perfect square is simply a number (or expression) that can be written as the square [raised to 2nd power] of another number (or expression).

Roots and Radicals

principal square root

Examples:

$$16 = 4^2 \qquad \sqrt{16} = 4$$

$$25 = 5^2 \qquad \sqrt{25} = 5$$

$$1.44 = 1.2^2 \qquad \sqrt{1.44} = 1.2$$

$$\frac{9}{121} = \left(\frac{3}{11}\right)^2 \qquad \sqrt{\frac{9}{121}} = \frac{3}{11}$$

Roots and Radicals

Examples:

principal square root

$$36b^2 = (6b)^2$$

$$m^6 = (m^3)^2$$

$$\sqrt{36b^2} = 6b$$

$$\sqrt{m^6} = m^3$$

Roots and Radicals

If a number is a perfect cube, then you can find its exact cube root.

A perfect cube is simply a number (or expression) that can be written as the cube [raised to 3rd power] of another number (or expression).

Roots and Radicals

principal cube root

Examples:

$$64 = 4^3 \qquad \sqrt[3]{64} = 4$$

$$125 = 5^3 \qquad \sqrt[3]{125} = 5$$

$$1.728 = 1.2^3 \qquad \sqrt[3]{1.728} = 1.2$$

$$\frac{216}{125} = \left(\frac{6}{5}\right)^3 \qquad \sqrt[3]{\frac{216}{125}} = \frac{6}{5}$$

Roots and Radicals

Examples:

principal cube root

$$8c^3 = (2c)^3 \qquad \sqrt[3]{8c^3} = 2c$$

$$m^6 = \left(m^2\right)^3 \qquad \sqrt[3]{m^6} = m^2$$

$$-27y^{12} = \left(-3y^4\right)^3 \qquad \sqrt[3]{-27y^{12}} = -3y^4$$

Roots and Radicals

Not all numbers or expressions have an exact square root or cube root as in the previous examples.

Roots and Radicals

If a number is NOT a perfect square, then you CANNOT find its exact square root.

If a number is NOT a perfect cube, then you CANNOT find its exact cube root.

You can approximate these square roots and cube roots of real numbers with a calculator.

Roots and Radicals

Examples:

$$\sqrt{40} \approx 6.325$$

$$\sqrt{135} \approx 11.619$$

$$\sqrt[3]{40} \approx 3.42$$

$$\sqrt[3]{74} \approx 4.198$$

Roots and Radicals

If a number is NOT a perfect square, then you might also be able to SIMPLIFY it.

What is the process to simplify a square root?

Roots and Radicals

If the expression is not a perfect square ...

1. see if you can rewrite the expression as a product of two smaller factors...
2. where one of the factors is a perfect square.

Roots and Radicals

3. Then, extract the the square root of the factor that is a perfect square …

4. and multiply that answer times the other factor still under the radical symbol.

Roots and Radicals

Examples – Simplifying Square Roots:

perfect square

$$\sqrt{40} = \sqrt{4 \cdot 10} = 2\sqrt{10}$$

$$\sqrt{135} = \sqrt{9 \cdot 15} = 3\sqrt{15}$$

$$\sqrt{50x^7} = \sqrt{25x^6 \cdot 2x} = 5x^3\sqrt{2x}$$

Roots and Radicals

If a number is NOT a perfect cube, then you might also be able to SIMPLIFY it.

What is the process to simplify a cube root?

Roots and Radicals

If the expression is not a perfect cube ...

1. **see if you can rewrite the expression as a product of two smaller factors...**

2. **where one of the factors is a perfect cube.**

Roots and Radicals

3. Then, extract the the cube root of the factor that is a perfect cube…

4. and multiply that answer times the other factor still under the radical symbol.

Roots and Radicals

Examples – Simplifying Cube Roots:

perfect cube

$$\sqrt[3]{80} = \sqrt[3]{8 \cdot 10} = 2\sqrt[3]{10}$$

$$\sqrt[3]{405} = \sqrt[3]{27 \cdot 15} = 3\sqrt[3]{15}$$

$$\sqrt[3]{24x^8} = \sqrt[3]{8x^6 \cdot 3x^2} = 2x^2\sqrt[3]{3x^2}$$

Roots and Radicals

Not all square roots can be simplified!

Example: $\sqrt{77}$

cannot be simplified!

- **77 is not a perfect square …**
- **and it does not have a factor that is a perfect square.**

Roots and Radicals

Not all cube roots can be simplified!

Example: $\sqrt[3]{30}$

cannot be simplified!

- **30 is not a perfect cube …**
- **and it does not have a factor that is a perfect cube.**

Roots and Radicals

The Rules (Properties)

Multiplication

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$$

Division

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

b may not be equal to 0.

Roots and Radicals

The Rules (Properties)

Multiplication

$$\sqrt[3]{a} \cdot \sqrt[3]{b} = \sqrt[3]{a \cdot b}$$

Division

$$\frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}}$$

b may not be equal to 0.

Roots and Radicals

Examples:

Multiplication

$$\sqrt{3}\cdot\sqrt{3} = \sqrt{3\cdot 3}$$

$$= \sqrt{9} = 3$$

Division

$$\frac{\sqrt{96}}{\sqrt{6}} = \sqrt{\frac{96}{6}}$$

$$= \sqrt{16} = 4$$

Roots and Radicals

Examples:

Multiplication

$$\sqrt[3]{5}\cdot\sqrt[3]{16}=\sqrt[3]{5\cdot 16}$$
$$=\sqrt[3]{80}$$
$$=\sqrt[3]{8\cdot 10}$$
$$=\sqrt[3]{8}\cdot\sqrt[3]{10}$$
$$=2\sqrt[3]{10}$$

Division

$$\frac{\sqrt[3]{270}}{\sqrt[3]{5}}=\sqrt[3]{\frac{270}{5}}$$
$$=\sqrt[3]{54}=\sqrt[3]{27\cdot 2}$$
$$=\sqrt[3]{27}\cdot\sqrt[3]{2}$$
$$=3\sqrt[3]{2}$$

Roots and Radicals

To **add** or **subtract** square roots or cube roots...

- simplify each radical
- add or subtract **LIKE** radicals by adding their coefficients.

Two radicals are LIKE if they have the **same expression under the radical symbol.**

Roots and Radicals

Examples:

$$3\sqrt{6} + 4\sqrt{6} = 7\sqrt{6}$$

$$2\sqrt[3]{11} - 6\sqrt[3]{11} = -4\sqrt[3]{11}$$

Roots and Radicals

Example:

$$\sqrt{12} + 5\sqrt{3} = \sqrt{4\cdot 3} + 5\sqrt{3}$$
$$= 2\sqrt{3} + 5\sqrt{3}$$
$$= 7\sqrt{3}$$

Roots and Radicals

Example:

$$-3+\sqrt[3]{40}-\sqrt[3]{135}+7$$

$$-3+\sqrt[3]{8\cdot 5}-\sqrt[3]{27\cdot 5}+7$$

$$-3+2\sqrt[3]{5}-3\sqrt[3]{5}+7$$

$$4-\sqrt[3]{5}$$

Roots and Radicals

Conjugates

Radical conjugates are two expressions of the form $a+b\sqrt{c}$ and $a-b\sqrt{c}$.

Conjugates have the property that when you multiply them, you get a rational number – the radical is gone.

Roots and Radicals

Example – Conjugates:

$$\left(5+3\sqrt{7}\right)\left(5-3\sqrt{7}\right) = 25-15\sqrt{7}+15\sqrt{7}-9\sqrt{49}$$

$$= 25-9\cdot 7$$

$$= 25-63$$

$$= -38$$

Roots and Radicals

Rationalizing the Denominator

The process of removing a radical from the denominator of a fraction is called rationalizing the denominator.

Roots and Radicals

Rationalizing the Denominator

To do this, multiply the fraction with the radical in the denominator by "1" as a fraction where the numerator and denominator are either:

- the radical factor that will produce a perfect square in the denominator radical or
- the expression that is the conjugate of the denominator of the fraction to be rationalized.

Roots and Radicals

Examples:

$$\frac{4}{\sqrt{6}} = \frac{4}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{4\sqrt{6}}{\sqrt{36}} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$$

$$\frac{10}{\sqrt[3]{4}} = \frac{10}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{4^2}}{\sqrt[3]{4^2}} = \frac{10\sqrt[3]{4^2}}{\sqrt[3]{4} \cdot \sqrt[3]{4^2}} = \frac{10\sqrt[3]{4^2}}{\sqrt[3]{4^3}} = \frac{10\sqrt[3]{4^2}}{4} = \frac{5\sqrt[3]{4^2}}{2}$$

Roots and Radicals

Example:

$$\frac{-3}{2+\sqrt{10}} = \frac{-3}{2+\sqrt{10}} \cdot \frac{2-\sqrt{10}}{2-\sqrt{10}} = \frac{-3\left(2-\sqrt{10}\right)}{\left(2+\sqrt{10}\right)\left(2-\sqrt{10}\right)}$$

$$= \frac{-3\left(2-\sqrt{10}\right)}{2^2-\sqrt{10^2}} = \frac{-3\left(2-\sqrt{10}\right)}{4-10}$$

$$= \frac{-3\left(2-\sqrt{10}\right)}{-6} = \frac{2-\sqrt{10}}{2}$$

Roots and Radicals

Solving Radical Equations

A radical equation is simply one that has a radical term that contains a variable.

Example: $\sqrt{c} + 2 = 5$

Roots and Radicals

To solve a radical equation

- Get the radical term by itself on one side of the equation.
- Square both sides of the equation.
- Finish solving for the variable, if needed.
- Check your solution. This is critical when solving radical equations.

Roots and Radicals

Example:

$$\sqrt{c} + 2 = 5$$

$$\sqrt{c} = 3$$

$$\left(\sqrt{c}\right)^2 = 3^2$$

$$c = 9$$

Roots and Radicals

Example:

$$\sqrt{2x-3}-7 = -2$$

$$\sqrt{2x-3} = 5$$

$$\left(\sqrt{2x-3}\right)^2 = 5^2$$

$$2x-3 = 25$$

$$2x = 28$$

$$x = 14$$