

## An Introduction to Riemann Sums

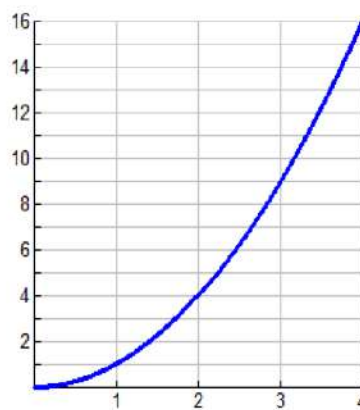
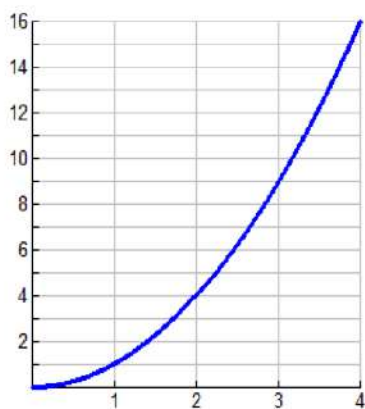
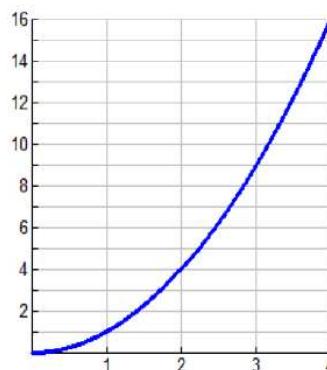
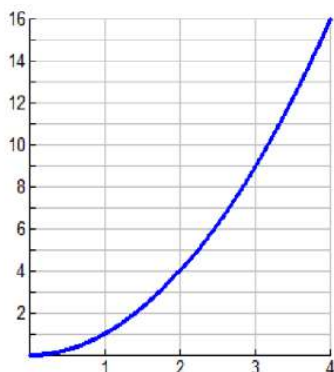
### AP Calculus

Name: \_\_\_\_\_

A **Riemann sum** is an approximation of the area of a region bounded by a function and an axis. One can approximate the area by breaking the space into more manageable geometric shapes, usually rectangles and trapezoids, and adding their areas.

1) Below is the graph of the function  $f(x) = x^2$ .

Determine the approximate area bounded by the function and the x-axis over the interval  $[0,4]$  using four equally spaced sub-intervals and a **left Riemann sum**, a **right Riemann sum**, and a **midpoint Riemann sum**.



Which model do you think gave the most accurate estimate of the actual area between the curve and the x-axis? Which gave an underestimate of the actual area? Which gave an overestimate of the actual area? Will this always be the case? Explain. How could you get even more accurate estimates of the actual area between the curve and the x-axis over the interval  $[0, 4]$ ?

- 2) The values of a differentiable function  $f$  are given in the table below. Approximate the area bounded by this function and the  $x$ -axis from  $x = 0$  to  $x = 8$ , by using a **right** Riemann sum with **four** sub-intervals

<b>x</b>	0	2	4	6	8
<b>f(x)</b>	6	8	14	18	21

- 3) Use the same function as in #2 above but this time estimate the area by using a **midpoint** Riemann sum with **two** equally spaced sub-intervals

**Trapezoids are also often used to help approximate the area under a curve.**

$$f(x) = \frac{(x-2)^3}{4} + 2$$

Given the function from  $[0,4]$  estimate the area bounded above by the function and below by the x-axis using the trapezoidal rule.

- a) Estimate the area using 2 trapezoids.
- b) Then estimate the area using 4 trapezoids.

Note: The trapezoidal rule is similar to using Riemann sums but instead of using rectangles we use

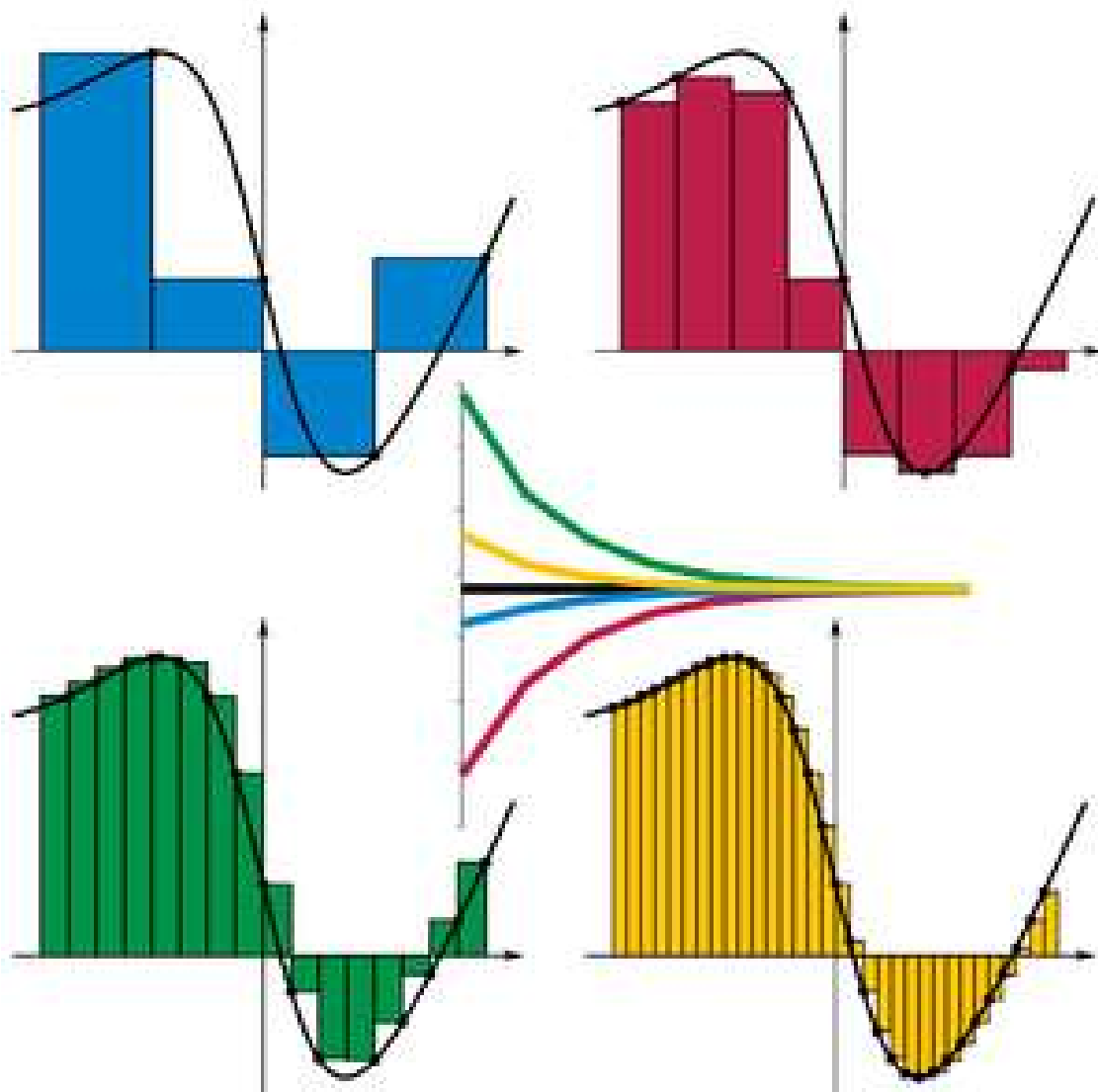
trapezoids. Remember that the area of a trapezoid is  $\frac{1}{2}(b_1 + b_2)h$  and the bases of trapezoids are the two parallel sides.

**There are some other types of Riemann sums as well...**

### **Types of Riemann sums**

- 1) a lower sum – the height of each rectangle is the lowest it could be within its intervals
- 2) an upper sum – the height of each rectangle is the highest it could be within its intervals
- 3) a right Riemann sum – the height of each rectangle comes from the right of the intervals
- 4) a left Riemann sum – the height of each rectangle comes from the left of each intervals
- 5) a midpoint sum – the height of each rectangle comes from the exact middle of the intervals

Look at the images here and decide which graph illustrates which Riemann sum described above. Then draw an example of the Riemann sum described above that's not illustrated here.



Note: In calculus, we often talk about “signed area”, where the area is positive when the bounded area is above the x-axis ( i.e. when  $f(x) > 0$ ) and area is negative when the bounded area is below the x-axis (i.e.  $f(x) < 0$ ). Riemann sums give us both positive areas and negative areas...When we sum them together, then, the total area is not necessarily the same as the sum of the absolute values of all the areas. Usually if you’re asked to find the area under a curve, you want the absolute value of all the areas summed together.