Class Practice for Quiz #3 Answers

Find the c value such that -1 < c < 4 that is guaranteed by the Mean Value Theorem for the function below on the interval [-1, 4]

$$f(x) = -2x^2 + 12x - 13$$

average rate = instantaneous rate

$$\frac{f(4) - f(-1)}{4-(-1)} = f'(x)$$

$$\frac{(3) - (-27)}{5} = -4x + 12$$

$$6 = -4x + 12$$

$$-6 = -4x$$

$$\boxed{\frac{3}{2} = x}$$

$$c = \frac{3}{2}$$

At $x = 3/2$, the instantaneous rate = average rate over interval.

Find the c value(s) such that $0 < c < 2\pi$ that is guaranteed by the Mean Value Theorem for the function below on the interval

$[0, 2\pi]$

$$g(x) = \cos(x) - \sin(x)$$

ave rate = $g'(x)$

$$\frac{g(2\pi) - g(0)}{2\pi - 0} = -\sin(x) - \cos(x)$$

$$\frac{1 - 1}{2\pi} = -\sin(x) - \cos x$$

$$0 = -\sin(x) - \cos(x)$$

$$\sin(x) = -\cos(x)$$

$$\boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}}$$

To solve for x:

Method 1: look at trig chart & see what angles result in opposite values for sine & cosine

Method 2: Graph $y_1 = -\sin(x) - \cos(x)$, $y_2 = 0$ & find intersection

How can you be assured that g(x) = 0 at some point over the interval $[0, \frac{\pi}{2}]$?

$$g(x) = \cos(x) - \sin(x)$$

We can use the Intermediate Value Theorem (IVT).

$g(0) = \cos 0 - \sin 0 = 1 - 0 = 1$

$g(\frac{\pi}{2}) = \cos\frac{\pi}{2} - \sin\frac{\pi}{2} = 0 - 1 = -1$

Since $g(0) = 1$ and $g(\frac{\pi}{2}) = -1$ and since $g(x)$ is continuous for all x, we know that at some point, c, $0 < c < \frac{\pi}{2}$, the function $= 0$ since 0 is between -1 and 1.

You will need a calculator for these next two problems.

1) The function f has first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1+x+x^3}$. What is the x-coordinate of the inflection point of the graph of f ?

(A) 1.008 $\quad$ $\boxed{\text{(B) } 0.473}$ $\quad$ (C) 0 $\quad$ (D) -0.278 $\quad$ (E) The graph of f has no inflection point

2) The derivative of the function f is given by $f'(x) = x^2\cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

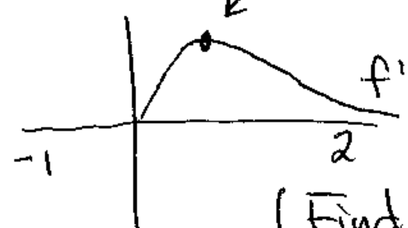
(A) One $\quad$ (B) Two $\quad$ $\boxed{\text{(C) Three}}$ $\quad$ (D) Four $\quad$ (E) Five

1) To find the i.p. of a function we can find the c.p. of $f'(x)$ & check to be sure the slope of $f'(x)$ changes from + to − or − to +.

Let's use graph calc:

$$y_1 = \frac{\sqrt{x}}{1+x+x^3}$$

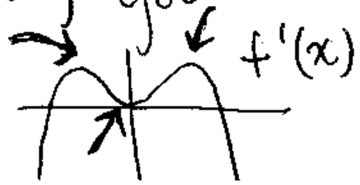
c.p of $f'(x)$ at $x = .4387$



(Find max)

or You could find $f''(x)$ & plug in multiple choice options to see what produces a 0 or undefined value.

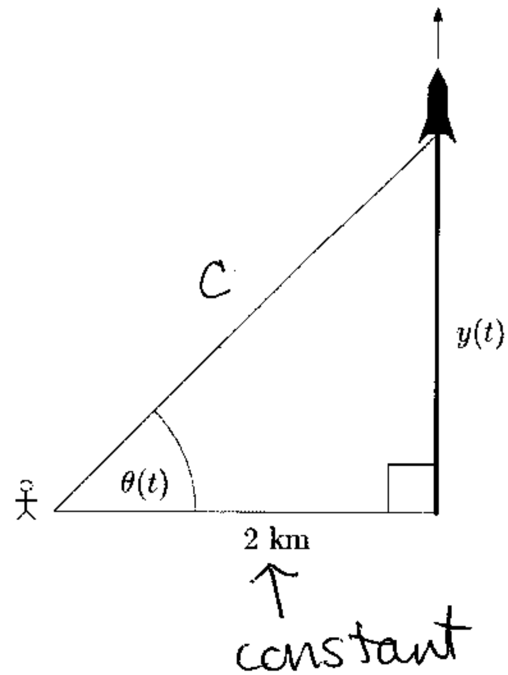
2) You can use the same strategies as above



The c.p. of $f'(x)$ where sign of $f'(x)$ changes are the infl pts of $f(x)$. There are $\boxed{\text{three}}$ of them.

3) An observer watches a rocket launch from a distance of 2 kilometers. The angle of elevation θ is increasing at $\frac{\pi}{60}$ radians per second at the instant when $\theta = \frac{\pi}{4}$.

How fast is the rocket climbing at that instant?



Write an equation that relates the variables

$$\tan\theta = \frac{y}{2}$$

Find deriv.

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{2}\frac{dy}{dt}$$

Substitute in values (& find values you need to sub. in if necessary)

$$2 \cdot \left(\frac{1}{(\cos\frac{\pi}{4})}\right)^2 \cdot \left(\frac{\pi}{60}\right) = \frac{1}{2}\frac{dy}{dt} \cdot 2$$

$$\boxed{\frac{\pi}{15} = \frac{dy}{dt}}$$

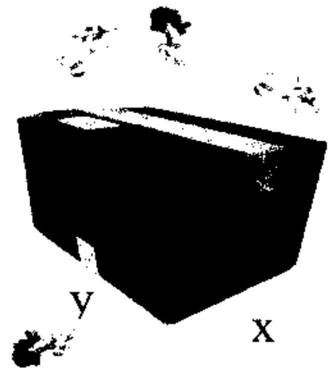
km/sec

A cardboard box with square ends needs to be constructed using 2400 square inches of cardboard. Determine the dimensions that will maximize the amount of popcorn that can be held in this box.

$SA = 2x^2 + 4xy$

$2400 = 2x^2 + 4xy$

$\frac{2400 - 2x^2}{4x} = y$



$V = x^2 y = x^2\left(\frac{2400 - 2x^2}{4x}\right)$

$V = 600x - \frac{1}{2}x^3$

Find c.p. of V(x) to find max.

$V' = 600 - \frac{3}{2}x^2$

$0 = 600 - \frac{3}{2}x^2$

$\frac{3}{2}x^2 = 600$

$x^2 = 400$

$x = 20$

$y = \frac{2400 - 2(20)^2}{4(20)}$

$y = 20$

Box is 20 × 20 × 20 in