Review for Quiz on FTC, Rate problems, and Average Value
AP Calculus

Name: Answers

Questions from 2003 and 2008 AP exams

No calculator

1) 7. A particle moves along the x-axis with velocity given by $v(t)=3t^2+6t$ for time $t \geq 0$. If the particle is at position $x=2$ at time $t=0$, what is the position of the particle at $t=1$?

(A) 4 $\quad$ $\boxed{\text{(B) 6}}$ ✓ $\quad$ (C) 9 $\quad$ (D) 11 $\quad$ (E) 12

$$P(1) = P(0) + \int_0^1 v(t)$$
$$= 2 + \left[t^3 + 3t^2\right]_0^1 = 2 + [(1+3)-(0)] = 6$$

03 2) $\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\,dt\right) =$ $\sin((x^2)^3) \cdot 2x = 2x\sin x^6$

(A) $-\cos(x^6)$ $\quad$ (B) $\sin(x^3)$ $\quad$ (C) $\sin(x^6)$ $\quad$ (D) $2x\sin(x^3)$ $\quad$ $\boxed{\text{(E) } 2x\sin(x^6)}$ ✓

03 3). If $G(x)$ is an antiderivative for $f(x)$ and $G(2)=-7$, then $G(4)=$

(A) $f'(4)$

(B) $-7+f'(4)$

(C) $\int_2^4 f(t)\,dt$

(D) $\int_2^4 (-7+f(t))\,dt$

$\boxed{\text{(E) } -7+\int_2^4 f(t)\,dt}$ ✓

$$G(x) = \int_2^4 f(x)\,dx + G(2)$$

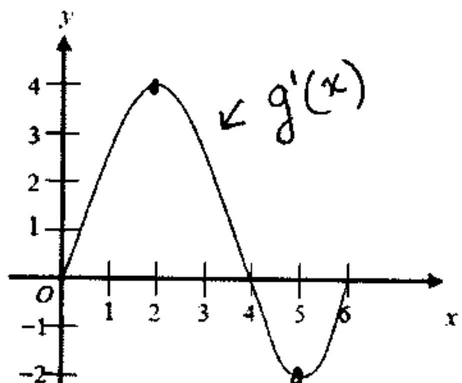
-7

need to switch order

08 **4)** If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

(A) -21 $\quad$ $\boxed{\text{(B) } -13}$ ✓ $\quad$ (C) 0 $\quad$ (D) 13 $\quad$ (E) 21

$$\int_{-5}^{5} f(x)\,dx = \int_{-5}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx = -17 + -(-4)$$
$$= -13$$



Graph of f

08 **5)** The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_{0}^{x} f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only $\quad$ (B) 4 only $\quad$ $\boxed{\text{(C) 2 and 5 only}}$ ✓ $\quad$ (D) 2, 4, and 5 $\quad$ (E) 0, 4, and 6

→ inflection points are when $g''(x) = 0$ <u>and</u> value of $g'(x)$ changes sign

Average w.s

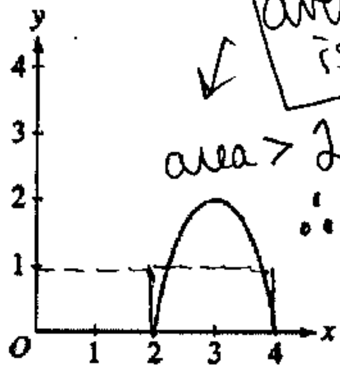
88. On the closed interval [2, 4], which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?

Average value = 1

Compare areas under curve with area of rectangle

$\boxed{=2}$ 1, 2

(A)

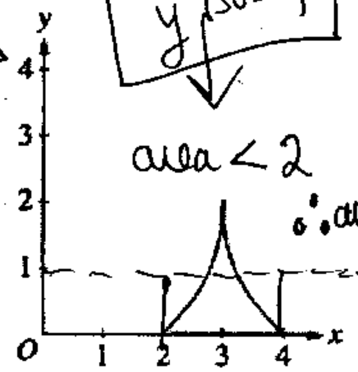


$\boxed{\text{average } y \text{ is } > 1}$

area > 2

$\therefore$ ave > 1

(B)

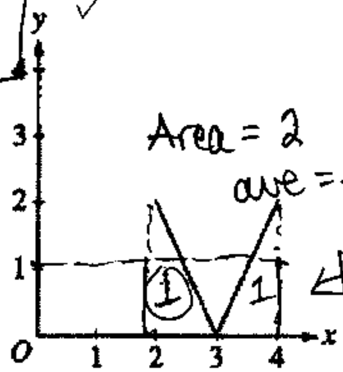


$\boxed{\text{average } y \text{ is } < 1}$

area < 2

$\therefore$ ave < 1

$\boxed{\text{(C)}}$ ✓

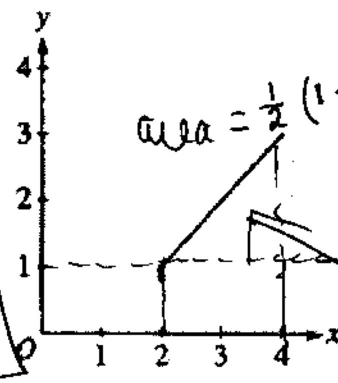


Area = 2

ave $= \frac{1}{2} \cdot 2 = 1$

$\boxed{\text{ave } y \text{ is } 1}$

(D)

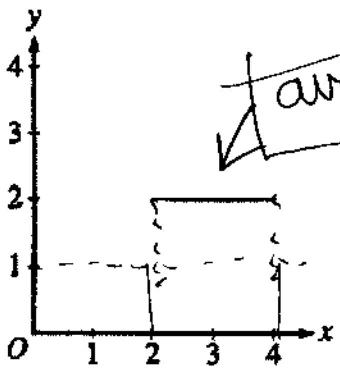


area $= \frac{1}{2}(1+3)2 = 4$

ave $= \frac{1}{2} \cdot 4 = 2$

$\boxed{\text{average } y \text{ is } 2}$

(E)



$\boxed{\text{average } y \text{ value } = 2}$

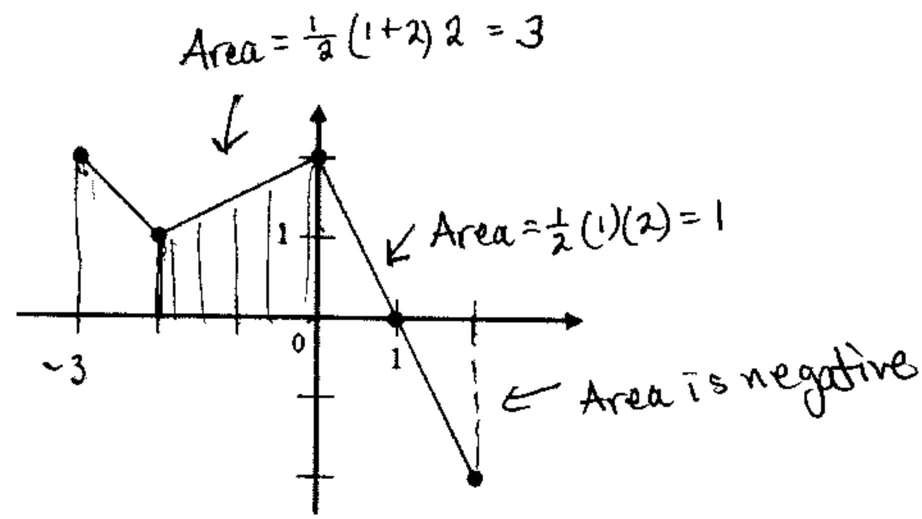
area = 4

ave $= \frac{1}{2} \cdot 4 = 2$

Method 2

Find (or estimate) areas & then multiply by $\frac{1}{2}$

$\boxed{\text{Method 1} \quad \text{Find (or estimate) average } y \text{ values.}}$



Graph of f

6) The graph of the piecewise linear function f is shown in the figure above. If $g(x)=\int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ $\boxed{\text{(D) } g(1)}$ ✓ (E) $g(2)$

$-$ 0 3 4 $4-1$
$= 3$

With calculator

08 **7)** An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0)=2$ and the velocity of the object is $v(t)=\sqrt[3]{1+t^2}$, what is the position of the object at time $t=3$?

(A) 0.431 (B) 2.154 (C) 4.512 $\boxed{\text{(D) } 6.512}$ ✓ (E) 17.408

$$X(3) = X(0) + \int_0^3 v(t)\,dt$$

$$= 2 + \int_0^3 v(t)\,dt = 2 + 4.512$$

$$= 6.512$$

8) A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln\left(1 + 2^t\right)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

(A) 0.462 (B) 1.609 (C) 2.555 (D) 2.886 $\boxed{\text{(E) } 3.346}$ ✓

$$v(2) = v(1) + \int_1^2 a(t)\,dt = 2 + \int_1^2 a(t)\,dt = 3.346$$

9) A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

$\boxed{\text{(A) } 112°\text{F}}$ (B) 119°F (C) 147°F (D) 238°F (E) 335°F

$$F(t) = 350 + \int_0^5 -110e^{-.4t}\,dt = 112.217$$

10) The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$ ✓

(B) $\int_0^8 r(t)\,dt$

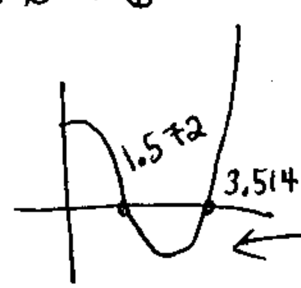
(C) $\int_0^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$ ← careful!

(E) $\int_0^{2.667} r'(t)\,dt$

$y_1 = x^3 - 4x^2 + 6$

$y_2 = 0$



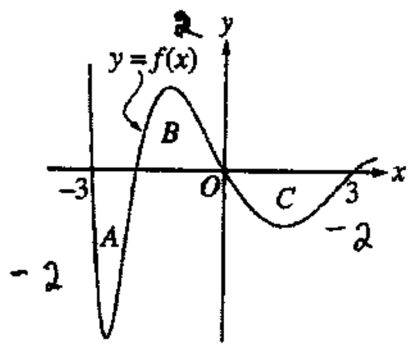
alt. is decr. when $r(t)$ is negative

A8 Average w.s.

31. What is the average value of $y = \dfrac{\cos x}{x^2 + x + 2}$ on the closed interval $[-1, 3]$?

(A) -0.085 (B) 0.090 $\boxed{\text{(C) } 0.183}$ ✓ (D) 0.244 (E) 0.732

$$\text{Ave} = \frac{1}{3 - -1} \int_{-1}^{3} \frac{\cos x}{(x^2+x+2)}\,dx = .183$$



03 17. The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3} (f(x) + 1)\,dx$? $= \int_{-3}^{3} f(x)\,dx + \int_{-3}^{3} 1\,dx$

(A) -2 (B) -1 $\boxed{\text{(C) } 4}$ ✓ (D) 7 (E) 12

$$\int_{-3}^{3} f(x)\,dx = -2$$

don't miss the $\underline{\underline{+1}}$

$$+ \int_{-3}^{3} 1\,dx = 6$$

$$-2 + 6 = 4$$

Average w.s.

83. The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) 20.086 ft/sec ✓

(B) 26.447 ft/sec

(C) 32.809 ft/sec

(D) 40.671 ft/sec

(E) 79.342 ft/sec

$$\text{Ave vel} = \frac{1}{3}\int_0^3 v(t = 20.086$$

Method 1

x	-4	-3	-2	-1
$f(x)$	0.75	-1.5	-2.25	-1.5
$f'(x)$	-3	-1.5	0	1.5

Use FTC, pt 1

$$\int_{-4}^{-1} f'(x)\,dx = f(x)\Big|_{-4}^{-1}$$

08 (2) The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x)\,dx$?

(A) -4.5 (B) -2.25 ✓ (C) 0 (D) 2.25 (E) 4.5

So

$= f(-1) - f(-4)$

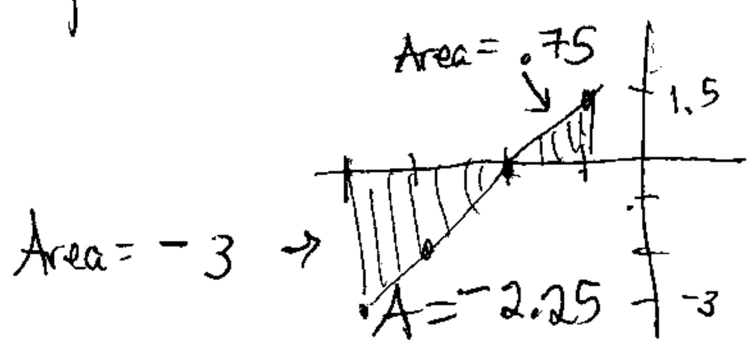
$= -1.5 - .75$

$= -2.25$

Or Method 2

You can plot values of $f'(x)$ in stat plot & find area

or just see that $f'(x)$ is linear, so can find area using Δs



Derivative of Trig Functions AP 2013 2A Name: Answers

AP Calculus

Work on the problem below. Note: We don't know yet how to do part b. Why is that? Think about what we need to learn in order to be able to do part b. You may use a calculator.

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

a) $G'(t) = -45\sin\left(\frac{t^2}{18}\right)\cdot\frac{2t}{18}$

$G'(5) = -45\sin\left(\frac{25}{18}\right)\cdot\frac{2(5)}{18} = -24.588$

The rate at which unproc. gravel is arriving at $t=5$ is decreasing by 24.588 tons/hour each hour

b) $\int_0^8 \left(90 + 45\cos\left(\frac{t^2}{18}\right)\right)dt = \boxed{825.551 \text{ tons}}$

c) $G(5) = 90 + 45\cos\left(\frac{25}{18}\right) = 98.141$ tons/hr

Since plant processes gravel at 100 tons/hr, at $t=5$ the amt. of unprocessed gravel is decreasing.

d) Max. amount:

Amount of gravel $A(x) = 500 + \int_0^x (G(t) - 100)\,dt$

$A'(x) = G(x) - 100$

$0 = 90 + 45\cos\left(\frac{x^2}{18}\right) - 100$

$x = 4.923$

x	$A(x)$
0	500
4.923	635.376
8	525.551

The max. amt. of unprocessed gravel during the workday is 635.376 tons

To find values, can use home screen & change upper limit of $\int$

or use $y_1 = 500 + \int_0^x \left(90 + 45\cos\left(\frac{x^2}{18}\right) - 100\right)dx$

& look at table