Review for Quiz on FTC, Rate problems, and Average Value Name: Answers

AP Calculus

Questions from 2003 and 2008 AP exams

No calculator

1) 7. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \ge 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at $t = 1$?

(A) 4 $\boxed{\text{(B) 6}}$ ✓ (C) 9 (D) 11 (E) 12

$$P(1) = P(0) + \int_0^1 v(t)$$

$$= 2 + \left[t^3 + 3t^2\right]_0^1 = 2 + [(1+3) - (0)] = 6$$

03 2) $\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\,dt\right) =$ $\sin((x^2)^3) \cdot 2x = 2x \sin x^6$

(A) $-\cos(x^6)$ (B) $\sin(x^3)$ (C) $\sin(x^6)$ (D) $2x\sin(x^3)$ $\boxed{\text{(E) } 2x\sin(x^6)}$ ✓

08 3). If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

(A) $f'(4)$

(B) $-7 + f'(4)$

(C) $\int_2^4 f(t)\,dt$

(D) $\int_2^4 (-7 + f(t))\,dt$

$\boxed{\text{(E) } -7 + \int_2^4 f(t)\,dt}$ ✓

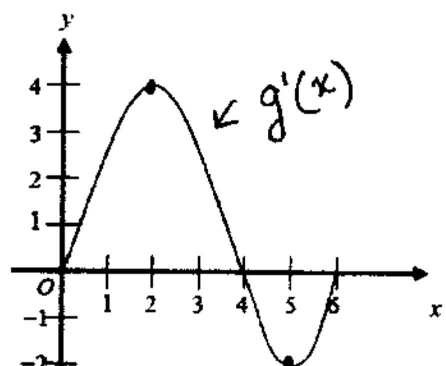
$$G(x) = \int_2^4 f(x)dx + \underset{\uparrow\; -7}{G(2)}$$

need to switch order

084) If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

(A) −21 $\boxed{\text{(B) } -13}$ ✓ (C) 0 (D) 13 (E) 21

$$\int_{-5}^{5} f(x)\,dx = \int_{-5}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx = -17 + -(-4)$$
$$= -13$$

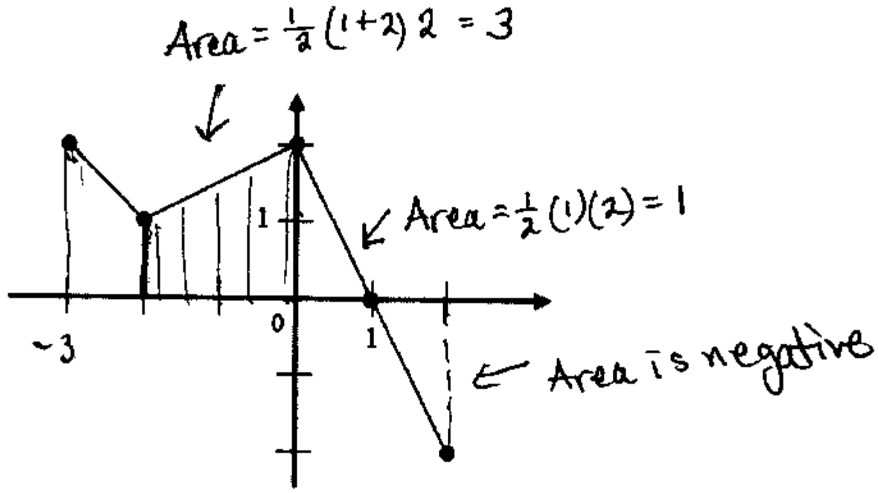


Graph of f

085) 57 The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_{0}^{x} f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only $\boxed{\text{(C) 2 and 5 only}}$ ✓ (D) 2, 4, and 5 (E) 0, 4, and 6

→ inflection points are when $g''(x) = 0$ <u>and</u> value of $g'(x)$ changes sign



Graph of f

6) The graph of the piecewise linear function f is shown in the figure above. If $g(x)=\int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ $\boxed{\text{(D) } g(1)}$ ✓ (E) $g(2)$

$-$ 0 3 4 $4-1 = 3$

With calculator

08 7) An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0)=2$ and the velocity of the object is $v(t)=\sqrt[3]{1+t^2}$, what is the position of the object at time $t=3$?

(A) 0.431 (B) 2.154 (C) 4.512 $\boxed{\text{(D) } 6.512}$ ✓ (E) 17.408

$$X(3) = X(0) + \int_0^3 v(t)\,dt$$

$$= 2 + \int_0^3 v(t)\,dt = 2 + 4.512$$

$$= 6.512$$

03 9) A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln(1 + 2^t)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

(A) 0.462 (B) 1.609 (C) 2.555 (D) 2.886 $\boxed{\text{(E) } 3.346}$ ✓

$$v(2) = v(1) + \int_1^2 a(t)\,dt = 2 + \int_1^2 a(t)\,dt = 3.346$$

03 9) A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

$\boxed{\text{(A) } 112°\text{F}}$ (B) 119°F (C) 147°F (D) 238°F (E) 335°F

$$F(t) = 350 + \int_0^5 -110e^{-.4t}\,dt = 112.217$$

03 10) The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$ ✓

(B) $\int_0^8 r(t)\,dt$

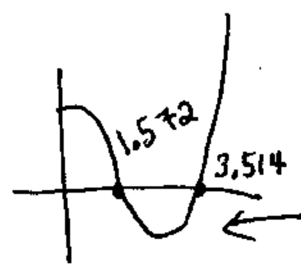
(C) $\int_0^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$ ← careful!

(E) $\int_0^{2.667} r'(t)\,dt$

$y_1 = x^3 - 4x^2 + 6$

$y_2 = 0$



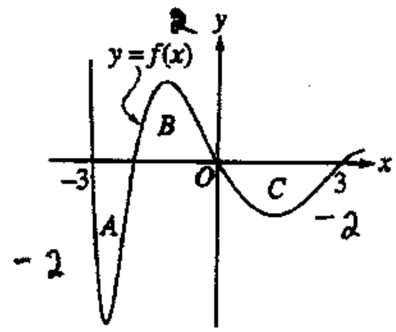
alt. is decr. when $r(t)$ is negative

Average W.S.

98 1. What is the average value of $y = \dfrac{\cos x}{x^2 + x + 2}$ on the closed interval $[-1, 3]$?

(A) -0.085 (B) 0.090 $\boxed{\text{(C) } 0.183}$ ✓ (D) 0.244 (E) 0.732

$$\text{Ave} = \frac{1}{3 - -1} \int_{-1}^{3} \frac{\cos x}{(x^2+x+2)}\,dx = .183$$



03 17. The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3} (f(x) + 1)\,dx$? $= \int_{-3}^{3} f(x)\,dx + \int_{-3}^{3} 1\,dx$

(A) -2 (B) -1 $\boxed{\text{(C) } 4}$ ✓ (D) 7 (E) 12

don't miss the $\underline{\underline{+1}}$

$$\int_{-3}^{3} f(x)\,dx = -2$$

$$+ \int_{-3}^{3} 1\,dx = 6$$

$$-2 + 6 = 4$$

Average W.S.

83. The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) $\boxed{20.086 \text{ ft/sec}}$ ✓

(B) 26.447 ft/sec

(C) 32.809 ft/sec

(D) 40.671 ft/sec

(E) 79.342 ft/sec

$$\text{Ave vel} = \frac{1}{3}\int_0^3 v(t = 20.086$$

$\boxed{\text{Method 1}}$

x	−4	−3	−2	−1
$f(x)$	0.75	−1.5	−2.25	−1.5
$f'(x)$	−3	−1.5	0	1.5

Use FTC, pt 1

$$\int_{-4}^{-1} f'(x)\,dx = f(x)\Big|_{-4}^{-1}$$

So

$= f(-1) - f(-4)$

$= -1.5 - .75$

$= -2.25$

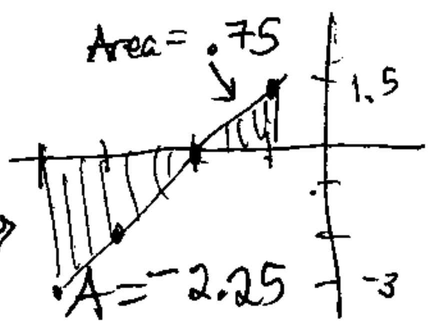
08 (2) The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x)\,dx$?

(A) −4.5 (B) $\boxed{-2.25}$ ✓ (C) 0 (D) 2.25 (E) 4.5

$\boxed{\text{Or}}$ $\boxed{\text{Method 2}}$

You can plot values of f'(x) in stat plot + find area

Area = −3 →



or just see that f'(x) is linear, so can find area using Δs

Average w.s

88. On the closed interval [2, 4], which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?

Average value = 1

Compare areas under curve with area of rectangle

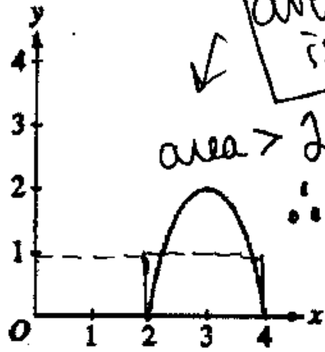
$\boxed{= 2}$

(A)

average y is > 1

area > 2

∴ ave > 1

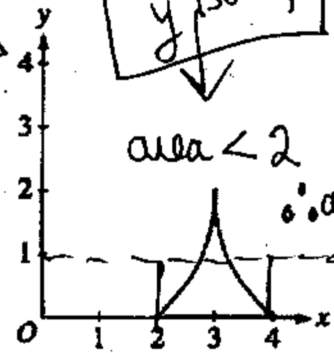


(B)

average y is < 1

area < 2

∴ ave < 1

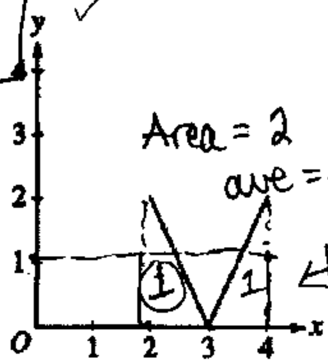


$\boxed{\text{(C)}}$ ✓

Area = 2

ave = $\frac{1}{2}\cdot 2 = 1$

ave y is 1

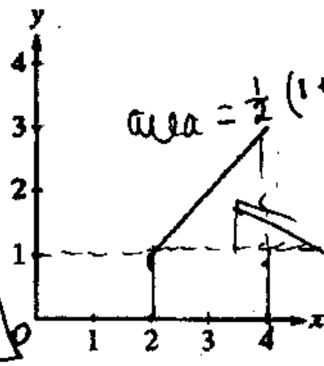


(D)

area = $\frac{1}{2}(1+3)2 = 4$

ave = $\frac{1}{2}\cdot 4 = 2$

average y is 2

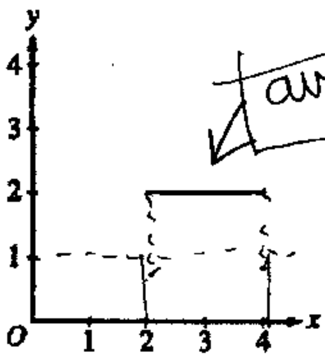


(E)

average y value = 2

area = 4

ave = $\frac{1}{2}\cdot 4 = 2$



Method 2

Find (or estimate) areas & then multiply by $\frac{1}{2}$

Method 1

Find (or estimate) average y values.

Rate Problem -Students entering and leaving school **Name:**

AP Calculus

You will need a calculator for this problem. Give it a try!

should be the "rate"

Suppose the ~~number~~ of students entering the school can be modeled by the function $U(x) = -140\cos(8x+4)+350$ over the domain [0,2] where x = 0 correspond to 7:30am. Suppose also that the function $L(x) = |25\sin(3x)|$ describes the rate at which students are leaving the building during that same interval of time.

a) Given that there are already 25 students in the building at 7:30am, write an integral function in terms of t that would determine the number of students in the school at any time t.

Let f(t) = # students in school on [0,2]

$$f(t) = 25 + \int_0^t U(x) - L(x)\,dx$$

b) Use that function to determine the time the number of students in the building is at a maximum.

Find critical points, if there are any

$$f'(t) = u(t) - L(t) = 0$$

$$u(t) = L(t)$$

U(t) ≠ L(t) so no c.p.

t	f(t)
0	25
2	$25 + \int_0^2 u(x) - L(x)\,dx = 662.777$

Max occurs at time t = 2 hrs.

The max value is about 663 students.