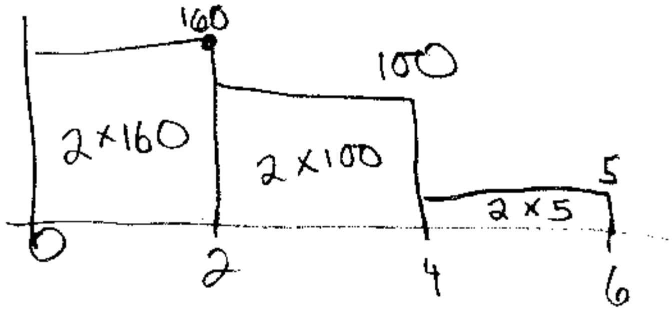
Review for Quiz – Riemann Sums, the Definite Integral, and FTC, part 1 **Name:** Answers
AP Calculus

1) Use a right-hand Riemann Sum, using three intervals, to find the approximate value of $\int_0^6 f(x)dx$

x	0	2	4	6
f(x)	200	160	100	5



$320+200+10$

$=\boxed{530}$

2) Is the right-hand sum you found above an **overestimate** or **underestimate** of $\int_0^6 f(x)dx$, given that f(x) is decreasing and differentiable over its whole domain? Explain your answer clearly and completely.

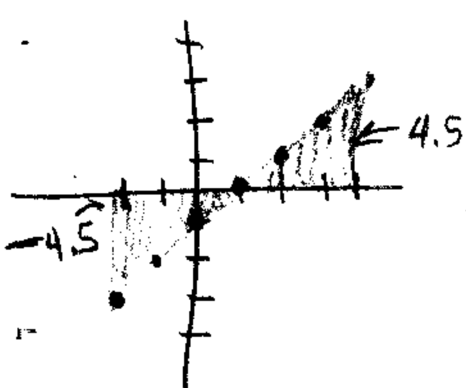
Since f(x) is decreasing a right Riemann sum will be an underestimate

3) Show a sketch of what $\int_{-2}^{4}(x-1)\,dx$ represents and then find the value of the integral.

$\int_{-2}^{4}(x-1)dx$

$= \frac{1}{2}x^2 - x\Big]_{-2}^{4}$

$= \frac{1}{2}(16) - 4 - \left(\frac{1}{2}(4)+2\right) = \boxed{0}$



$\int_{-2}^{4}(x-1)dx = \boxed{0}$

using graph

4) The function shown is f(x). Find the values of each definite integral below:

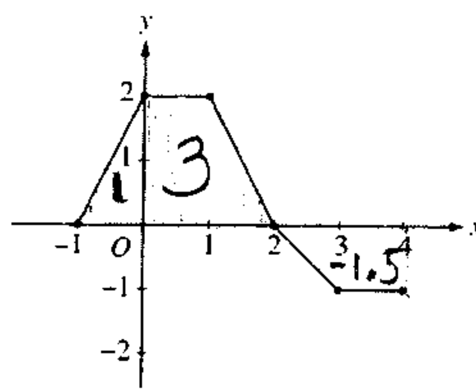
a) $\int_0^2 f(x)dx = \boxed{3}$

b) $\int_0^{-1} f(x)dx = \boxed{-1}$

negative since going "back in time"

c) $\int_{-1}^{4} f(x)dx = 4 - 1.5$

$= \boxed{2.5}$



4)

a) $$\int_1^{\sqrt{e}} \frac{3x^2-2x}{x^2}\,dx = \int_1^{e^{\frac{1}{2}}} 3-2x^{-1}\,dx$$

$$= \Big[3x - 2\ln|x|\Big]_1^{e^{\frac{1}{2}}}$$

$$= (3e^{\frac{1}{2}} - 2\ln e^{\frac{1}{2}}) - (3 - 2\ln 1)$$

$$= 3e^{\frac{1}{2}} - \ln e - 3 + 2(0)$$

$$= 3e^{\frac{1}{2}} - 1 - 3 \quad = \boxed{3e^{\frac{1}{2}} - 4}$$

note: remember that $\log b^a \Rightarrow a \log b$

so $\ln e^{\frac{1}{2}} \Rightarrow \frac{1}{2}\ln e$

remember, too, that $\ln e = 1$ and $\ln 1 = 0$

since $e^1 = e$ since $e^0 = 1$

b) $$\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \cos(2x)\,dx = \frac{1}{2}\sin 2x\Big]_{\frac{\pi}{8}}^{\frac{\pi}{6}} = \frac{1}{2}\sin\frac{\pi}{3} - \frac{1}{2}\sin\frac{\pi}{4}$$

$$= \frac{1}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \boxed{\frac{\sqrt{3}-\sqrt{2}}{4}}$$

c) $\displaystyle\int_0^{\frac{\pi}{3}} 2\sec^2 x\,(\tan x)^{\frac{1}{2}}\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

change bounds:

0 in x land becomes 0 since $u = \tan 0 = 0$

$\frac{\pi}{3}$ in x land becomes $\sqrt{3}$ since $u = \tan\frac{\pi}{3} = \sqrt{3}$

$$2\int_0^{\sqrt{3}} u^{\frac{1}{2}}\,du$$

$$= 2\,u^{\frac{3}{2}}\cdot\frac{2}{3}\Big]_0^{\sqrt{3}}$$

$$= \frac{4}{3}\left(\sqrt{3}\right)^{\frac{3}{2}} - \frac{4}{3}(0)^{\frac{3}{2}}$$

$$= \frac{4}{3}\left(3^{\frac{1}{2}}\right)^{\frac{3}{2}}$$

$$= \boxed{\frac{4}{3}(3)^{\frac{3}{4}}}$$

4d) $\displaystyle\int_0^{\sqrt{7}} x\left(x^2+1\right)^{\frac{1}{3}} dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$0 \to 0^2+1 \to 1$

$\sqrt{7} \to \sqrt{7}^2+1 \to 8$

$$\frac{1}{2}\int_1^8 u^{\frac{1}{3}}\,du = \frac{1}{2}\cdot u^{\frac{4}{3}}\cdot\frac{3}{4}\Big]_1^8$$

$$= \frac{3}{8}(8)^{\frac{4}{3}} - \frac{3}{8}(1)^{\frac{4}{3}}$$

$$= \frac{3}{8}\sqrt[3]{8}^{\,4} - \frac{3}{8}\sqrt[3]{1}^{\,4}$$

$$= \frac{3}{8}(2)^4 - \frac{3}{8}(1) = \frac{3}{8}\cdot 16 - \frac{3}{8}$$

$$= \boxed{6 - \frac{3}{8}}$$

or $\boxed{\dfrac{45}{8}}$

5) Find K

$$\int_0^4 x\,dx = \int_0^K y^2\,dy$$

$$\frac{1}{2}x^2\Big]_0^4 = \frac{1}{3}y^3\Big]_0^K$$

$$\frac{1}{2}(4)^2 - \frac{1}{2}(0)^2 = \frac{1}{3}(K)^3 - \frac{1}{3}(0)^3$$

$$8 = \frac{1}{3}K^3$$

$$24 = K^3$$

$$\boxed{\sqrt[3]{24} = K}$$

or $\boxed{K = 2\sqrt[3]{3}}$