Review for Differential Equations Quiz 2018 Name: Answers

AP Calculus

Solve each differential equation below. If an initial condition is given, find the particular solution; if an initial solution is not given, find the general solution. Verify that your solutions are correct.

1) $\frac{dy}{dx} = x + 2$

$$\int 1\,dy = \int (x+2)\,dx$$

$$\boxed{y = \frac{1}{2}x^2 + 2x + C}$$

Verify:

$y' = x + 2$ ✓

2) $\frac{dy}{dx} = y\sqrt{x}$ given (0, 7) is a point on f(x)

$$\int \frac{dy}{y} = \int x^{\frac{1}{2}}\,dx$$

$$\ln|y| = \frac{2}{3}x^{\frac{3}{2}} + C$$

$$e^{\frac{2}{3}x^{\frac{3}{2}} + C} = |y|$$

$$|y| = e^{\frac{2}{3}x^{\frac{3}{2}}} \cdot e^{C}$$

$$y = Ce^{\frac{2}{3}x^{\frac{3}{2}}}$$

given (0, 7):

$$7 = Ce^{\frac{2}{3}(0)^{3/2}}$$

$$7 = C$$

$$\boxed{y = 7e^{\frac{2}{3}x^{\frac{3}{2}}}}$$

Verify: $y' = 7e^{\frac{2}{3}x^{3/2}} \cdot x^{\frac{1}{2}}$

$y' = y\sqrt{x}$ ✓

3) $y' + xy = 100x$

$$\frac{dy}{dx} = x(100-y)$$

$$\int \frac{dy}{100-y} = \int x\,dx$$

$$\ln|100-y| = \frac{1}{2}x^2 + C_1$$

$$e^{\frac{1}{2}x^2 + C_1} = |100-y|$$

$$\pm e^{\frac{1}{2}x^2} \cdot e^{C_1} = 100-y$$

$$C_2 e^{\frac{1}{2}x^2} = 100-y$$

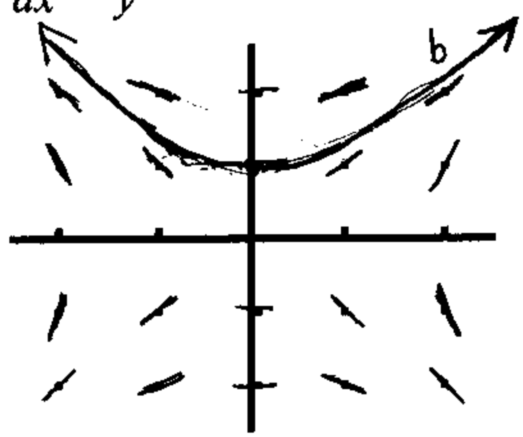
$$\boxed{y = Ce^{\frac{1}{2}x^2} + 100}$$

Note:
Since "C" is an arbitrary constant, the sign in front of it isn't relevant here.

4 & 5 →

6 a) Sketch the slope field for the equation $\frac{dy}{dx} = \frac{x}{y}$ on the grid below.

x	y	$\frac{x}{y}$	x	y	$\frac{x}{y}$
0	1	0	2	1	2
0	2	0	2	2	1
1	1	1	-2	1	-2
-1	1	-1	-2	-1	2
-1	-1	1	2	-1	-2
1	-1	-1			
1	2	$\frac{1}{2}$			
-1	2	$-\frac{1}{2}$			
-1	-2	$\frac{1}{2}$			
1	-2	$-\frac{1}{2}$			



b) Given that $y = f(x)$ is a solution to the differential equation above and given that $1 = f(0)$ is a point on $f(x)$, sketch $f(x)$, on the grid above.

See above

4) $\frac{dy}{dx} = \frac{5x^4}{\sin(y)}$

$$\int \sin y \, dy = \int 5x^4 \, dx$$

$$-\cos y = x^5 + C_1$$

$$\cos y = -x^5 + C_2$$

$$\boxed{y = \arccos(-x^5 + C)}$$

5) $y' = \frac{3xy}{\sqrt{1+x^2}}$

$$\int \frac{1}{y} dy = \int \frac{3x}{\sqrt{1+x^2}} dx$$

$$u = 1 + x^2$$

$$du = 2x\,dx$$

$$\frac{3}{2} du = 3x\,dx$$

$$\ln|y| = \frac{3}{2} \int u^{-\frac{1}{2}} du$$

$$\ln|y| = \frac{3}{2} \cdot 2u^{\frac{1}{2}} + C$$

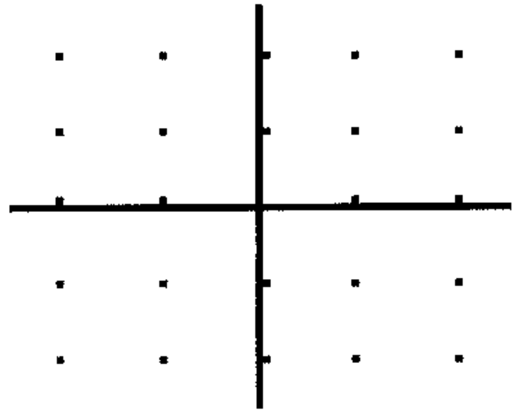
$$\ln|y| = 3\sqrt{1+x^2} + C$$

$$|y| = e^{3\sqrt{1+x^2} + C}$$

$$= e^{3\sqrt{1+x^2}} \cdot e^{C}$$

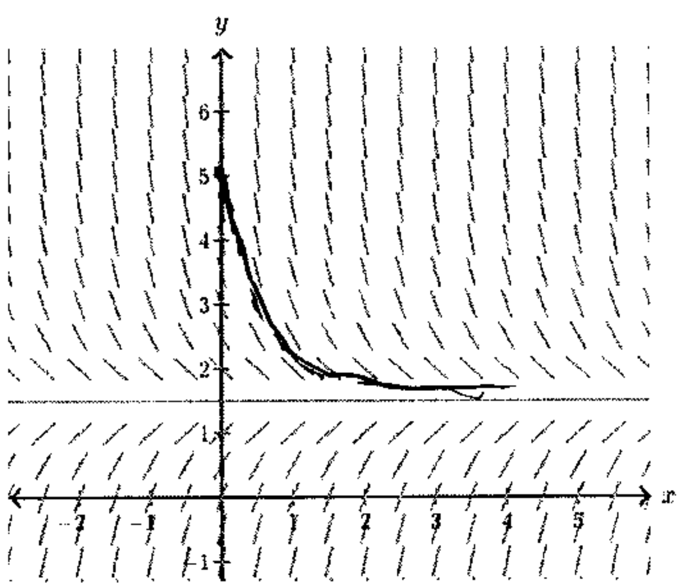
$$\boxed{y = Ce^{3\sqrt{1+x^2}}}$$

6) a) Sketch the slope field for the equation $\dfrac{dy}{dx} = \dfrac{x}{y}$ on the grid below.



b) Given that $y = f(x)$ is a solution to the differential equation above and given that $1 = f(0)$ is a point on $f(x)$, sketch $f(x)$, on the grid above.

7) If the initial condition is $(0, 5)$, what is the range of the solution curve $y = f(x)$ for $x \geq 0$?



range $(0, 5]$

8) Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

a) tangent line: point $(-1,1)$ slope $= \frac{1}{3(1)^2-(-1)} \Rightarrow \frac{1}{4}$

$$\boxed{y-1 = \frac{1}{4}(x+1)}$$

b) vertical tangents $\frac{dy}{dx}$ is undefined, so $3y^2 - x = 0$

solve system: $\begin{cases} y^3 - xy = 2 \\ 3y^2 - x = 0 \end{cases} \rightarrow x = 3y^2$

$y^3 - (3y^2)y = 2$

$y^3 - 3y^3 = 2$

$-2y^3 = 2$

$y^3 = -1$

$y = -1$

$x = 3(-1)^2 = 3$

$$\boxed{(3,-1)}$$

c) see next page

$$\boxed{\frac{d^2y}{dx^2} = \frac{1}{32}}$$

9) Let $y = f(x)$ be the particular solution to the differential equation $y' = x + 3y$ with the initial condition $f(1) = -2$. Does f have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.

Slope at $(1,-2) = y' = 1 + 3(-2) = -5$

Since slope $\neq 0$ and slope $\neq$ undef.

$(1,-2)$ is NOT a critical point & is therefore neither a relative max nor a relative min.

c) $$\frac{d^2y}{dx^2} = \frac{\frac{dy}{dx}(3y^2-x) - y\left(6y\frac{dy}{dx} - 1\right)}{(3y^2-x)^2}$$

$$= \frac{\frac{y}{\cancel{3y^2-x}}\left(\frac{\cancel{3y^2-x}}{1}\right) - y\left(6y\cdot\frac{y}{3y^2-x} - 1\right)}{(3y^2-x)^2}$$

$$= \frac{y - \frac{6y^3}{3y^2-x} + y}{(3y^2-x)^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(-1,1)} = \frac{1 - \frac{6}{4} + 1}{(4)^2}$$

$$= \frac{\frac{1}{2}}{16}$$

$$= \boxed{\frac{1}{32}}$$