**Review 2 for Midterm with FTC, Extrema, Tangent Lines** **Name:** Answers
*AP Calculus*

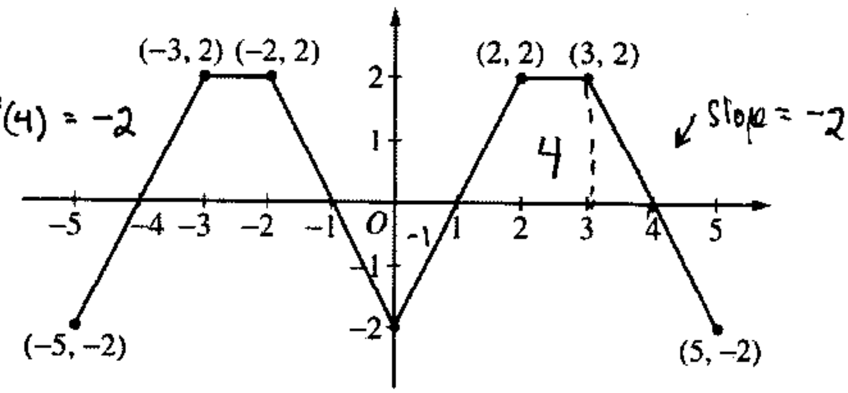
You will not have a calculator for the midterm, so do the problems below without a calculator. Practice showing your work clearly and justifying answers using words.

1)

The graph of the function $f$ shown above consists of six line segments. Let $g$ be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(4)$, $g'(4)$, and $g''(4)$. $g(4) = 3$ $g'(4) = 0$ $g''(4) = -2$

(b) Does $g$ have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer. Rel. min (see below).



Graph of $f$

(c) Suppose that $f$ is defined for all real numbers $x$ and is periodic with a period of length 5. The graph above shows two periods of $f$. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of $g$ at $x = 108$. $y - 44 = 2(x - 108)$ or $y = 2x - 172$

a) $g(4) = \int_0^4 f(t)\,dt = -1 + 4 = 3$ $g'(x) = f(x) \rightarrow g'(4) = f(4) = 0$

$g''(x) = f'(x)$ so $g''(4) = f'(4) = -2$ (slope of $f(x)$ at $x=4$)

b) $g'(1) = 0$ so $x=1$ is a critical point, (since $g'(x) = f(x)$ and $f(1) = 0$). $g'(x)$ is negative before $x = 1$ & $g'(x)$ is positive after $x=1$, which means that $g(x)$ is decreasing, then increasing $\cup$, so $g(x)$ has a <u>rel. min</u> at $x = 1$.

c) $g(10) = \int_0^{10} f(t)\,dt = \int_0^5 f(t)\,dt + \int_5^{10} f(t)\,dt = 2 + 2 = 4$

Note that $f(108)$ will echo the graph at and around $f(3)$ within the 2nd period of $f(x)$

slope of tangent line: $g'(108) = f(108) = 2$

Point: $(108, 44)$

ind point: $g(108) = \int^{108} f(t)\,dt = 21(2) + -1 + 3 = 44$ $\boxed{y - 44 = 2(x - 108)}$

2) Write the equation of the line tangent to the graph of g(x) at x = e given $g(x) = \int_{\sqrt{e}}^{x^2} \frac{1}{t}\,dt$

point: $g(e) = \int_{\sqrt{e}}^{e^2} \frac{1}{t}\,dt = \ln t \Big|_{e^{1/2}}^{e^2}$

$(e, 1.5)$

$= \ln e^2 - \ln e^{1/2}$

$= 2 - \frac{1}{2} = 1.5$

slope: $g'(x) = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$

slope at $x = e$: $g'(e) = \frac{2}{e}$

equation of tangent line:

$$\boxed{y - 1.5 = \frac{2}{e}(x - e)}$$

or $$\boxed{y = \frac{2}{e}x - \frac{1}{2}}$$