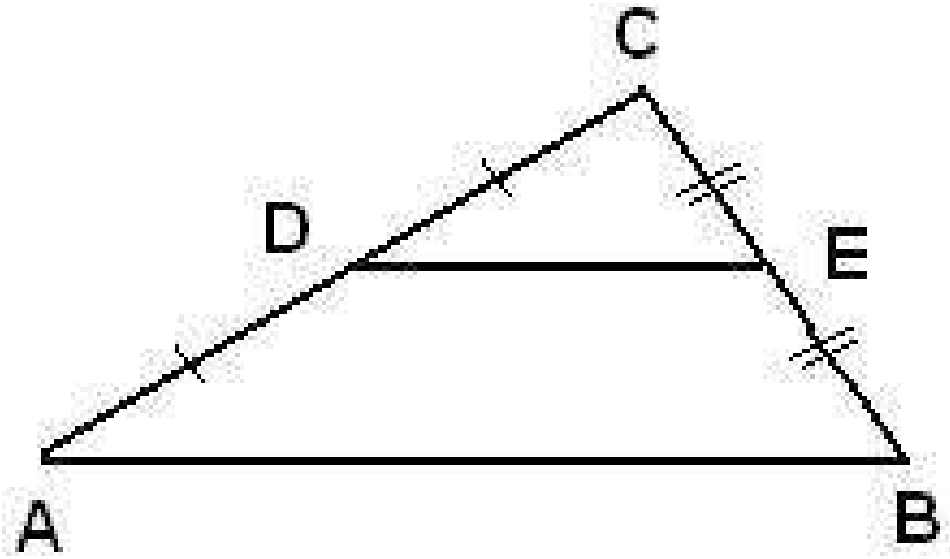
RELATIONSHIPS WITHIN TRIANGLES

Geometry Unit 5

MIDSEGMENTS OF A TRIANGLE

WHAT IS A MIDSEGMENT?

- A midsegment connects the midpoints of two segments of a triangle.
  - Given diagram below:



CONSTRUCTION OF A MIDPOINT

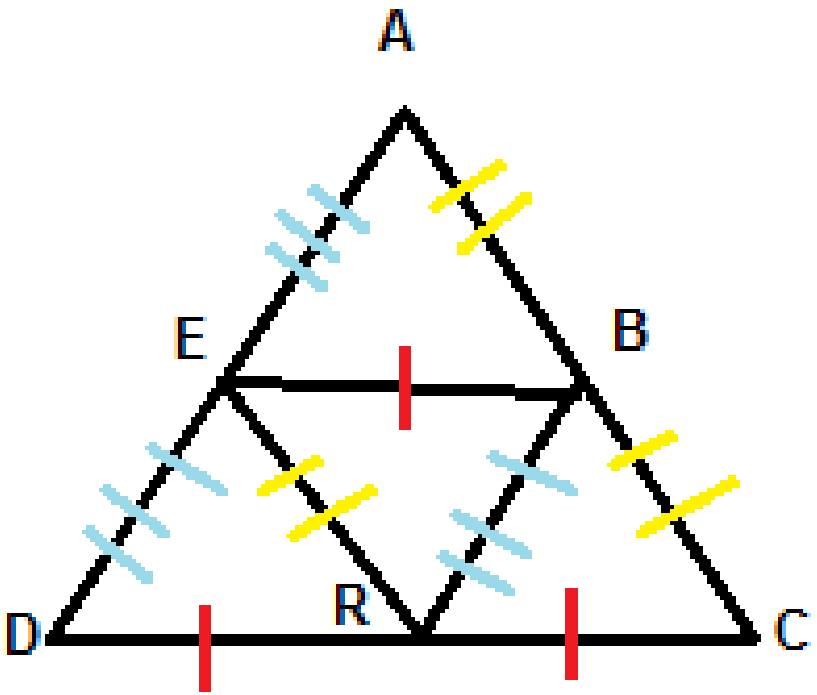
- http://www.mathsisfun.com/geometry/constructions.html

OBSERVATIONS

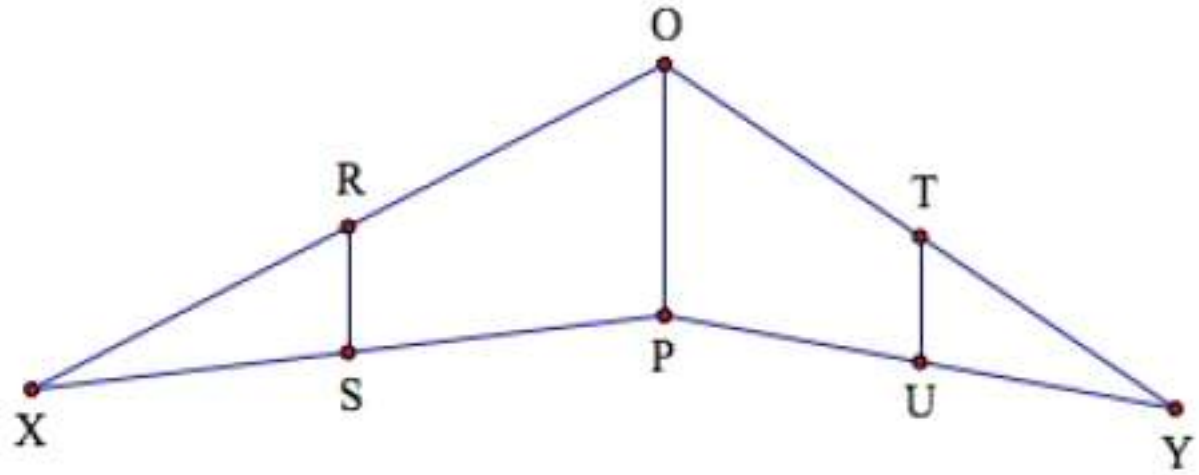
- What do you notice about the midsegments you created?
  - Lengths?
  - Lines?
  - Parallels?
  - Perpendiculars?
  - Intersections?

PROPERTIES OF MIDSEGEMENTS

- Midsegment Theorem:
  - The segment that joins the midpoints of a pair of sides of a triangle is:
    - Parallel to the third side
    - Half as long as the third side



R, S, T, U are midpoints of sides of triangles $\triangle XPO$ and $\triangle YPO$.



Complete the following:

1. If $OP = 12$, then $RS =$ ___ and $TU =$ ___.
2. If $RS = 8$, then $TU =$ ____.
3. If $RS = 2x$ and $OP = 18$, then $x =$ ___ and $TU =$ ___.
4. If $OP = 4x$ and $RS = 6x - 8$, then $x =$ _____.
5. Consider triangle $\triangle XYZ$ with vertices $X(1,1), Y(5,5), Z(3,9)$ and midpoint M on $\overline{XZ}$.

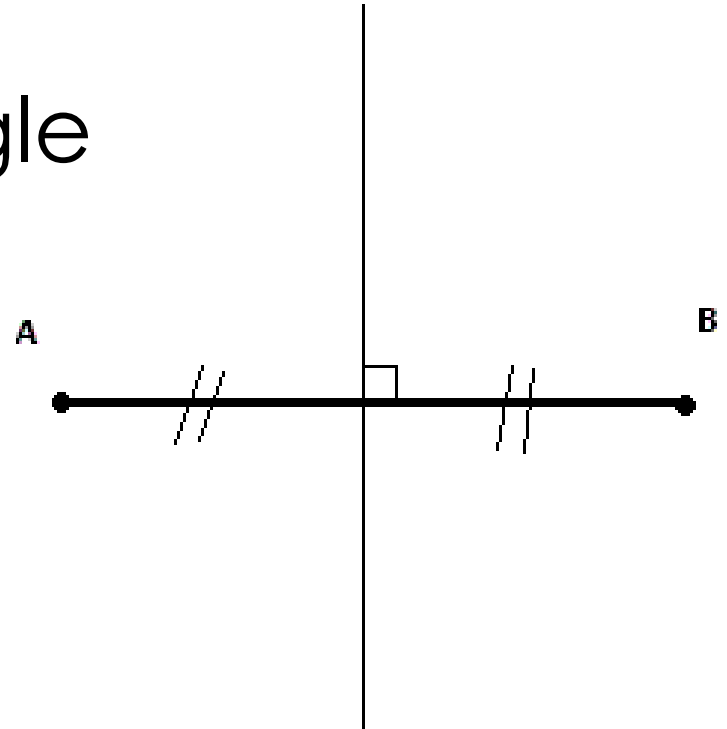
USEFULNESS

- When would you use the midpoints of a triangle?
  - When trying to determine relationships within a triangle.
  - When trying to solve for a variable or a length
  - Application problems when needing to cut across the center and find the distance.

PERPENDICULAR BISECTORS IN A TRIANGLE

WHAT IS A PERPENDICULAR BISECTOR?

- Perpendicular bisectors:
  - Divide the line segment into two congruent parts
  - Intersects the line at a right angle



CONSTRUCTION OF A PERPENDICULAR BISECTOR

- http://www.mathsisfun.com/geometry/constructions.html

PERPENDICULAR BISECTORS

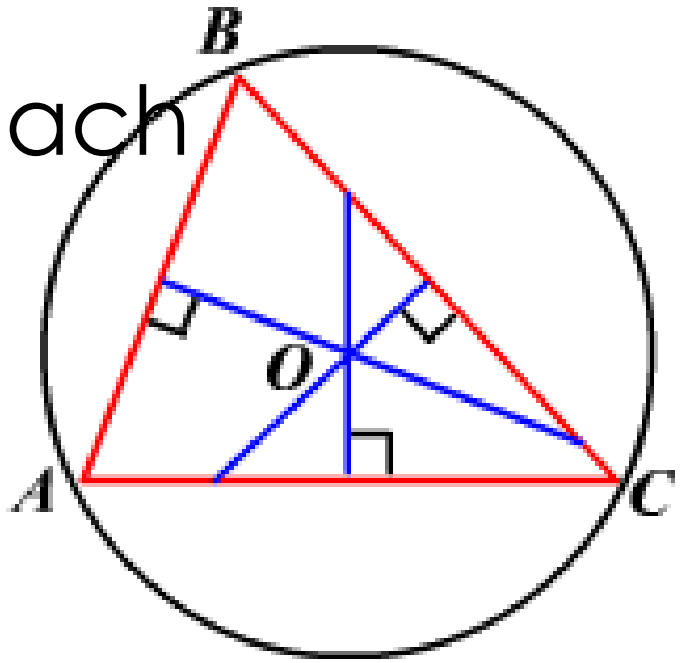
- What do you notice about the perpendicular bisectors of the triangles?
  - Lengths?
  - Lines?
  - Parallels?
  - Perpendiculars?
  - Intersections?

PRACTICE

- On triangles three and four on your worksheet create the perpendicular bisectors for the triangles.

CIRCUMCENTER

- The point of concurrency of the three perpendicular bisectors of the sides of a triangle.
- This point will be equidistant from each vertex.



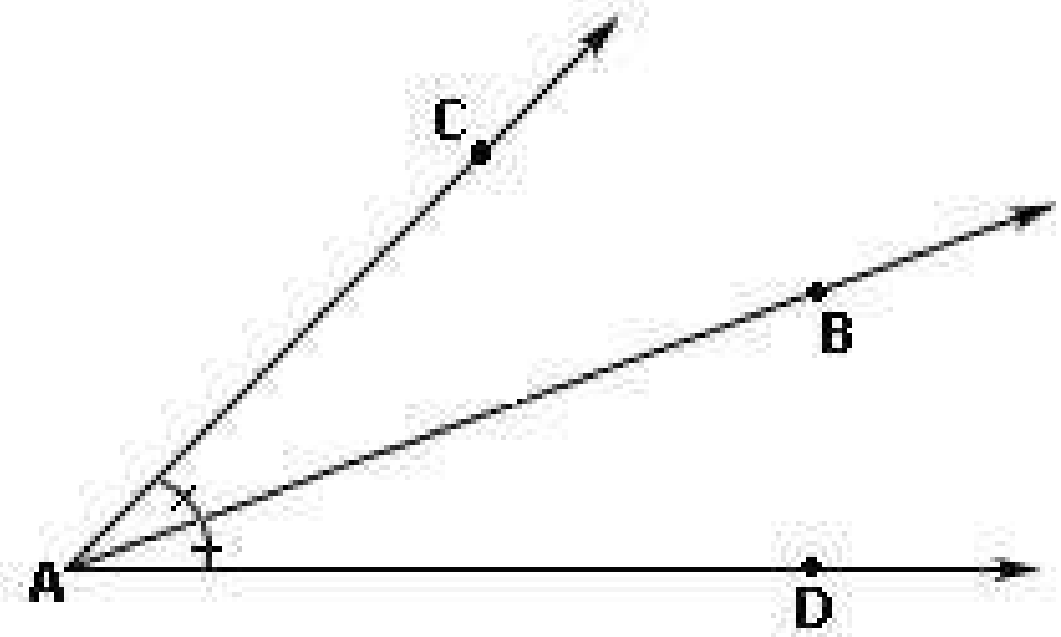
CIRCUMCENTER

- What will the circumcenter look like in the following triangles?
  - Acute?
  - Right?
  - Obtuse?
    - How do you know? Do these findings make sense?

ANGLE BISECTORS IN TRIANGLES

WHAT IS AN ANGLE BISECTOR?

- An angle bisector is a ray or segment which cuts an angle into two congruent angles.



CONSTRUCTION OF AN ANGLE BISECTOR

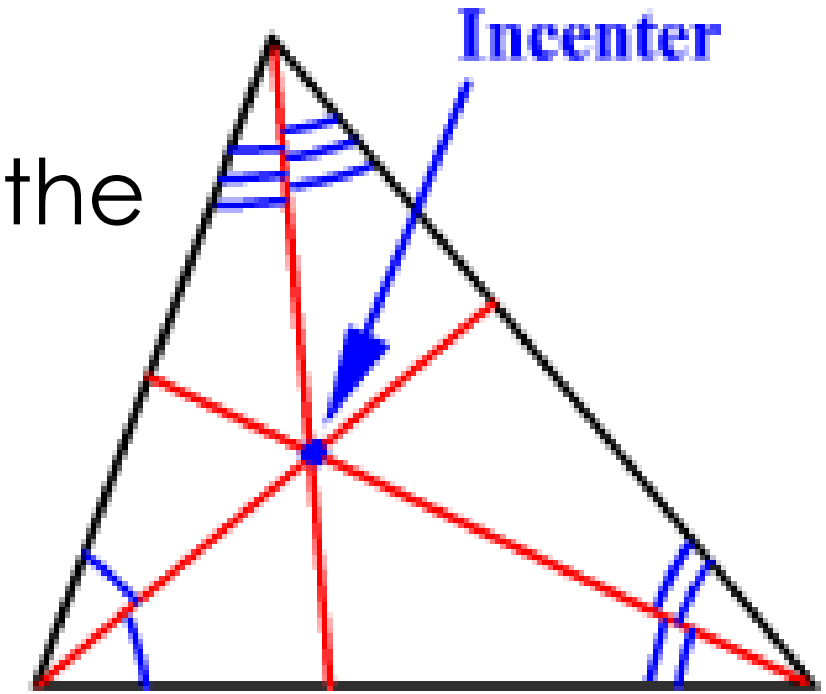
- http://www.mathsisfun.com/geometry/constructions.html

OBSERVATIONS

- What do you notice about the angle bisectors of the triangles?
  - Lengths?
  - Lines?
  - Parallels?
  - Perpendiculars?
  - Intersections?

INCENTER

- The incenter is the point of concurrency of all three angle bisectors of a triangle.
- The incenter is equidistant from all the edges of the triangle.



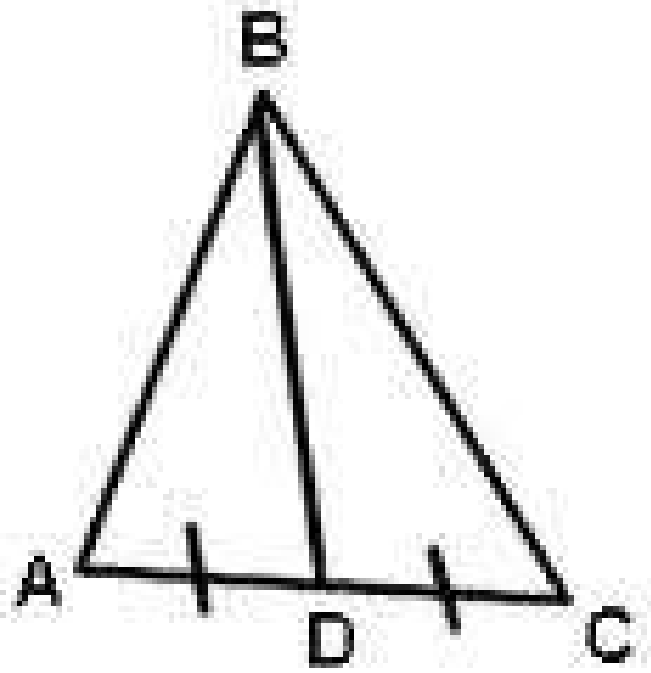
INCENTER

- What will the incenter look like in each of the following triangles?
  - Acute?
  - Right?
  - Obtuse?
  - How do you know?

MEDIANS IN TRIANGLES

WHAT IS A MEDIAN?

- A median of a triangle is the line segment which joins a vertex to the midpoint of the opposite side.



CONSTRUCTION OF A MEDIAN

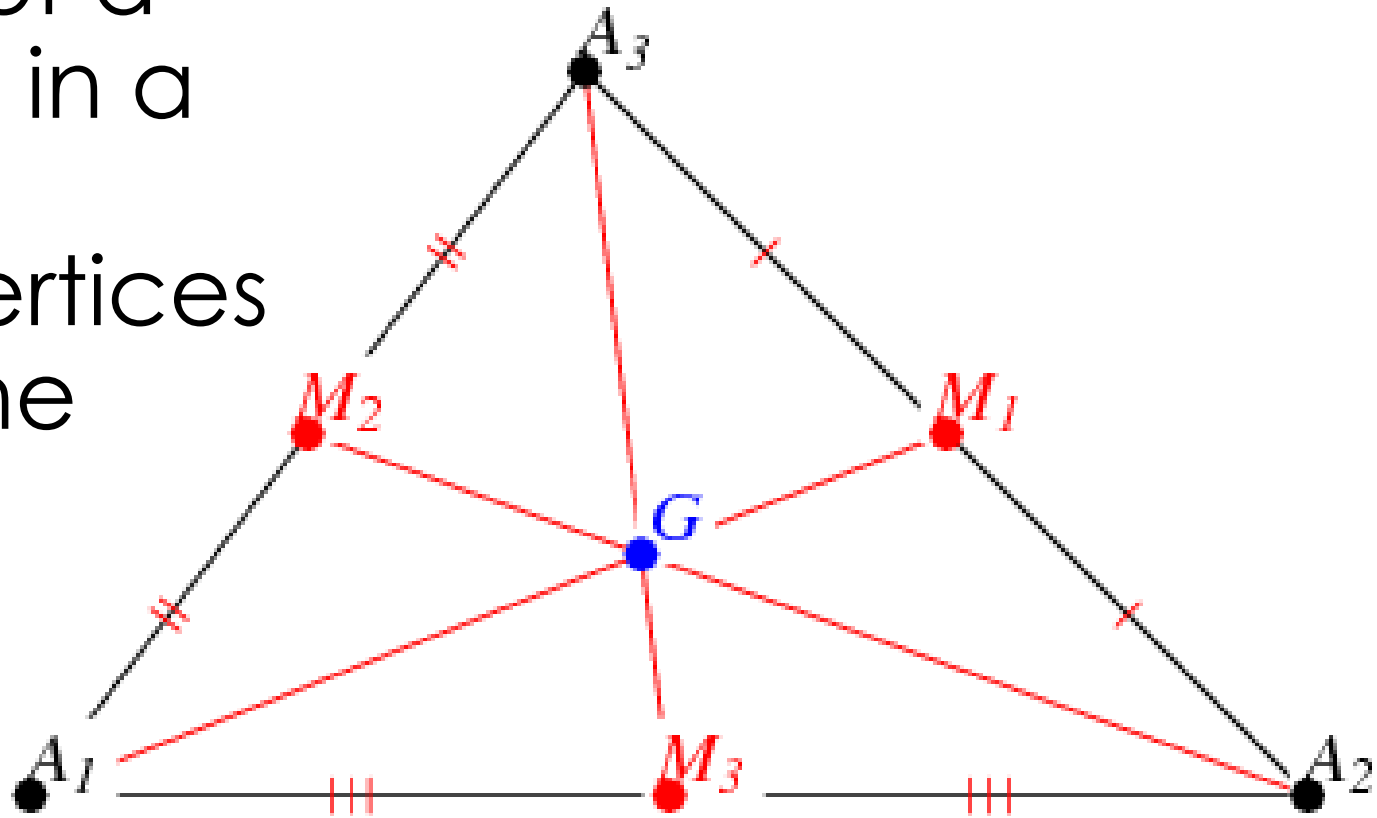
- How could you use your compass and protractor to create a median of a triangle?
  - Lets practice on one of the triangles on your paper.

OBSERVATIONS

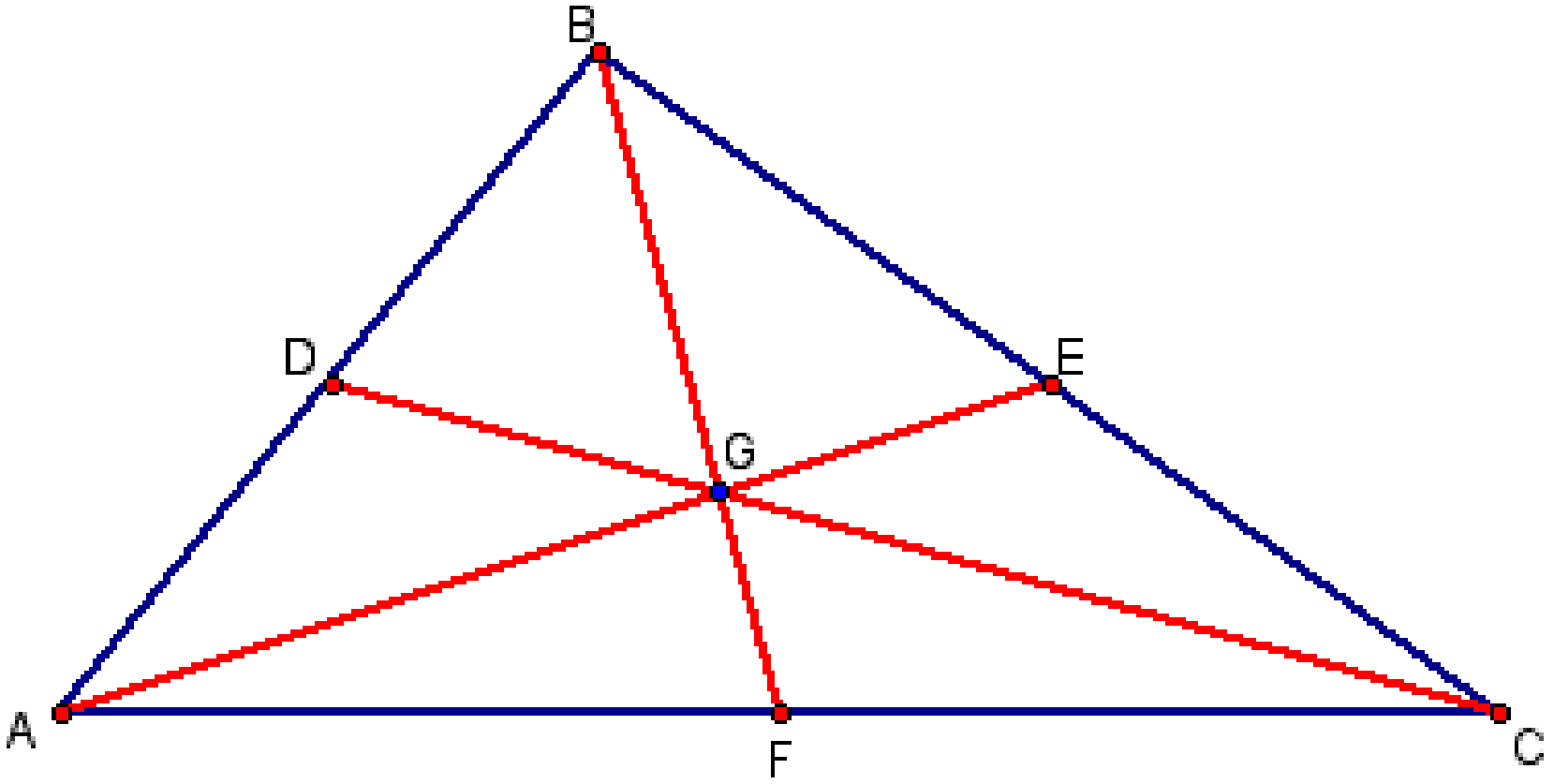
- Do the three medians meet in a point?
  - YES! We call this the centroid of the triangle.
    - What do we notice about the centroid of the triangle?
      - Lengths?
      - Angles?
      - Segments?

CONCURRENCY OF MEDIANS THEOREM

- States the medians of a triangle will intersect in a point that is 2/3 the distance from the vertices to the midpoint of the opposite side.



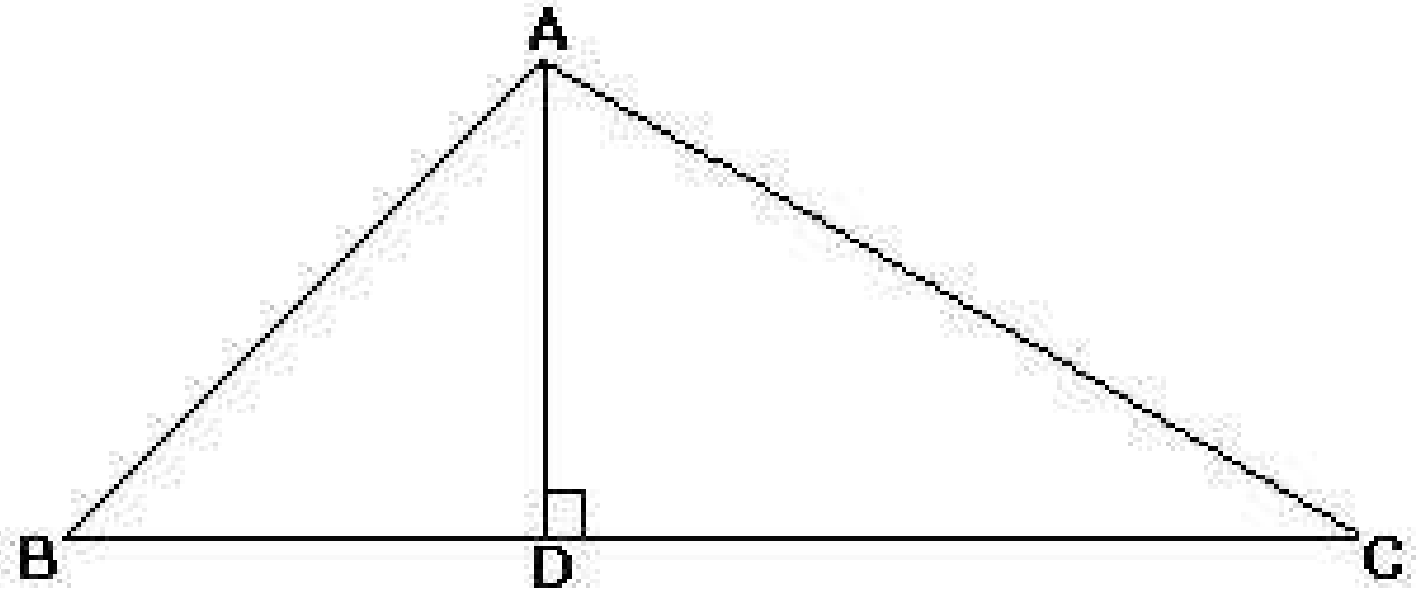
CONCURRENCY OF MEDIANS OF A TRIANGLE



ALTITUDES IN TRIANGLES

WHAT IS AN ALTITUDE?

- The line segment from a vertex perpendicular to the opposite side.



CONSTRUCTION OF AN ALTITUDE

- Can I construct the altitude of a triangle?
- How!!!!!!?????
- http://www.mathsisfun.com/geometry/construct-perpnotline.html

LETS TRY!!!

- Lets create the altitudes of the three angles in one of the triangles on your page.

- What do you notice about the altitudes?

ORTHOCENTER

- The point of concurrency of the altitudes of a triangle is called the orthocenter.
  - What do you notice about the orthocenter of the following triangles:
    - Acute
    - Obtuse
    - Right

OBSERVATIONS

CONSTRUCTIONS

- On your triangle page, find the following for a single triangle. You may want to erase your marks after each.
  - Circumcenter
  - Incenter
  - Centroid
  - Orthocenter

GROUP WORK

- After you have completed each of the constructions and have them clearly labeled.
  - Note all the similarities you can about these

SIMILARITIES

- Acute triangle- all points are inside the triangle
- Obtuse triangle- all points are outside the triangle
- Right triangle- orthocenter on vertex of right angle, circumcenter is on the midpoint of the hypotenuse

SIMILARITIES

- For the general case of a triangle:
  - The orthocenter, circumcenter, and centroid are always collinear
  - The distance from the orthocenter to the centroid is twice the distance from the centroid to the circumcenter.

REVIEW

- Perpendicular bisectors meet at the circumcenter.
- Angle bisectors meet at the incenter.
- Medians meet at the centroid.
- Altitudes meet at the orthocenter.

INEQUALITIES IN TRIANGLES

INEQUALITIES IN TWO TRIANGLES