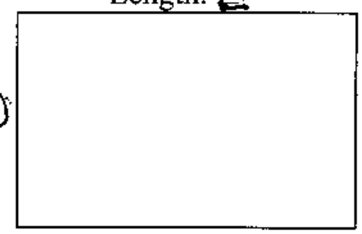
Related Rates Quiz 2015 V1 Name Answers

1) Solve the following related rates problems. You must show work to get credit. The length of a rectangle is increasing at a rate of 8 cm/sec while the width is decreasing at a rate 2 cm/sec. When the length is 12 cm and the width is 7 cm find the rates of change of the following.

$$\frac{dL}{dt} = 8 \qquad \frac{dw}{dt} = -2$$

Length: L (12)

Width: w (7)



a) Perimeter of the rectangle

$$P = 2w + 2L$$

$$\frac{dP}{dt} = 2\frac{dw}{dt} + 2\frac{dL}{dt}$$

$$= 2(-2) + 2(8)$$

$$\boxed{\frac{dP}{dt} = 12 \text{ cm/sec.}}$$

b) The area of the rectangle

$$A = wL$$

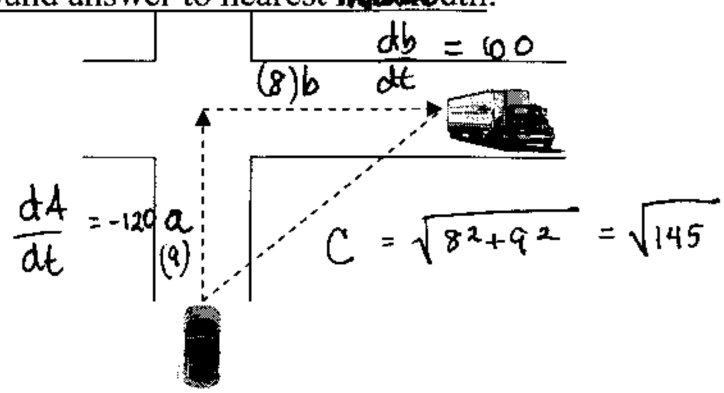
$$\frac{dA}{dt} = \frac{dw}{dt}L + w \cdot \frac{dL}{dt}$$

$$= (-2)(12) + (7)(8)$$

$$= -24 + 56$$

$$\boxed{\frac{dA}{dt} = 32 \text{ cm}^2\text{/sec}}$$

2) A car is traveling north towards an intersection at the rate of 120 mph while a truck is traveling east away from the intersection at the rate 100 mph. Find the rate of change of the distance between the car and the truck when the car is 9 miles south of the intersection and the truck is 8 miles east of intersection. Round answer to nearest ~~thousan~~dth.



$$a^2 + b^2 = c^2$$

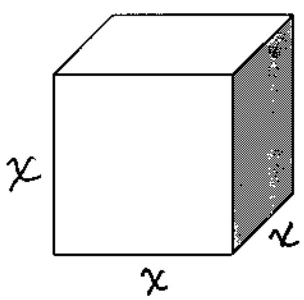
$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2(9)(-120) + 2(8)(100) = 2(\sqrt{145})\frac{dc}{dt}$$

$$\frac{-560}{2\sqrt{145}} = \frac{dc}{dt}$$

$$\boxed{\frac{dc}{dt} = -23.253\text{mph}}$$

3) The sides of a cube are increasing at the rate of 0.5 cm/sec. At what rate is its **surface area** increasing when the **volume** is 1000 cm^3?



$$\frac{dx}{dt} = .5\text{ cm/sec}$$

$$x^3 = 1000$$

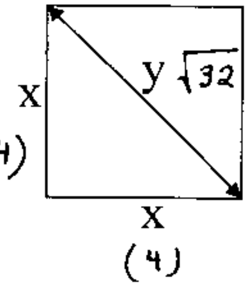
$$x = 10$$

$$S = 6x^2$$

$$\frac{ds}{dt} = 12x\frac{dx}{dt}$$

$$\frac{ds}{dt} = 12(10)(.5) = \boxed{60\text{ cm}^2/\text{sec}}$$

4) The diagonal of a square is increasing at a rate of 3 in/s. How fast is the area of the square changing when a side of the square measures 4 in? <u>Round answer to nearest thousandth.</u>



$$\frac{dy}{dt} = 3$$

$$A = x^2$$

$$\frac{dA}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dA}{dt} = 2(4)(2.12132\ldots)$$

$$\boxed{\frac{dA}{dt} = 16.971}$$

$$x^2 + x^2 = y^2$$

$$2x^2 = y^2$$

$$4x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$$4(4)\frac{dx}{dt} = 2(\sqrt{32})(3)$$

$$\frac{dx}{dt} = \frac{6\sqrt{32}}{16} = 2.1213\ldots$$

5) Mike is pouring sea salt onto his kitchen counter. He notices that the salt falls into a conical pile and that the radius of the cone is always 3 times greater than the height. Mike calculates that he's pouring the salt onto the pile at a rate of 15 cm^3/min. At what rate is the height of the pile changing when the height is 5 cm? <u>Leave answer in terms of pi.</u>

$$V = \frac{1}{3}\pi r^2 h$$

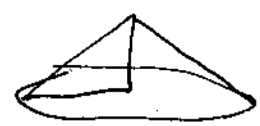
$$V = \frac{1}{3}\pi(3h)^2 h = 3\pi h^3$$

$$\frac{dV}{dt} = 9\pi h^2 \frac{dh}{dt}$$

$$15 = 9\pi(5)^2\frac{dh}{dt}$$

$$\frac{15}{225\pi} = \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{1}{15\pi}\ \text{cm/min}}$$



$$r = 3h$$

$$\frac{dV}{dt} = 15$$

$$h = (5)$$

Extra Credit: Find the derivative of the following implicit function. Fully simplify the derivative.

$$3y^2 = \frac{x}{(2y-1)^4}$$

$$6y\frac{dy}{dx} = \frac{1(2y-1)^4 - x\cdot 4(2y-1)^3\left(2\frac{dy}{dx}\right)}{\left((2y-1)^4\right)^2}$$

$$6y(2y-1)^8\frac{dy}{dx} = (2y-1)^4 - 8x(2y-1)^3\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(6y(2y-1)^8 + 8x(2y-1)^3\right) = (2y-1)^4$$

$$\frac{dy}{dx} = \frac{(2y-1)^{\cancel{4}\,1}}{\cancel{(2y-1)^3}\left(6y(2y-1)^5 + 8x\right)}$$

$$\boxed{\frac{dy}{dx} = \frac{2y-1}{6y(2y-1)^5 + 8x}}$$

Some Handy Formulas

Volume of a cone $V = \frac{1}{3}\pi r^2 h$

Volume of a cylinder $V = \pi r^2 h$

Volume of a cube $V = s^3$

Volume of a sphere $V = \frac{4}{3}\pi r^3$

Surface Area of a cube $A = 6s^2$

Surface Area of a sphere $A = 2\pi r^2$